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Completion of r.t. extensions of local fields (II)

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Completion of r.t. Extensions of Local Fields (II).

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Let (K, v) be a local field and let $(K(X), w)$ be a residual transcendental extension of it. The following question arises: describe the completion of $(K(X), w)$ (see [5]. ch VI Sect. 6) using (K, v) and related concepts. In the first part of this work (see [1] had been considered the particular, but important case, when $w = w_0$ is the Gauss r.t. extension of v to $K(X)$. Thus $(\widetilde{K(X)}, \tilde{w}_0)$ is described as the subfield of $(K\{\{X\}\}, u)$ consisting of all Laurent power series whose positive parts are almost periodic (see [1], Theorem 2.4).

In this paper we consider the general case when $(K(X), w)$ is any r.t. extension of (K, v). In Section 2 we try to give a «combinatorial» description of elements from $(\widetilde{K(X)}, \tilde{w})$, using a natural inclusion $\widetilde{K(X)} \to K\{\{X - a, \delta\}\}$. For that we use some elementary remarks in section 1 and a characterization of r.t. extensions of a valuation given in [3]. In Section 3 we describe $(\widetilde{K(X)}, \tilde{w})$ as a finite extension of $(\widetilde{K(r)}, \tilde{w}_0)$, where r is a suitable element of $K(X)$ and w_0 is the Gauss r.t. extension of v to $K(r)$. Finally, in Section 4 we study the conditions when $(\widetilde{K(X)}, \tilde{w})$ is coincident to the completion of $(K'(X), w')$, where K' is a suitable finite extension of K and w' an extension of w to $K'(X)$ and moreover w' is a slightly modified Gauss r.t. extension of v' to $K'(X)$ (here v' is the unique extension of v to K').

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1. – Taylor's type embeding of rational function field.

1) Let K be a field and let $K[X]$ be the polynomial ring of one variable over K. Let $\overline{K}$ be an algebraic closure of K, a an element in $\overline{K}$ and let $f(X)$ be the monic minimal polynomial of a with respect to K. Denote $K' = K(a)$. For any $g(X) \in K[X]$, we consider the Taylor's expansion $T(g) = \sum_{i=0}^{n} (g^{(i)}/i!)(a)(X-a)^i$ in $K'[X]$, where $n = \deg g$. In this way we obtain a ring embeding T: $K[X] \to K'[X-a] = K'[X]$. Extend T in an usual manner to a field embeding T: $K(X) \to K'(X-a) = K'(X)$ and call a rational function from the range of T a *K-rational function in $K'(X)$ with respect to a*. One calls *T the Taylor's embeding of $K(X)$ in $K'(X)$.*

2) For any natural number $k \geqslant 0$ one consider the following upper triangular $(k+1) \times (k+1)$-matrix:

$$\mathcal{a}_k = (a_{ij}) = \begin{cases} 0, & \text{if } i > j, \\ 1, & \text{if } i = j, \\ \begin{pmatrix} i \\ j-1 \end{pmatrix} a^{j-i}, & \text{if } i < j, \end{cases}$$

where $i, j \in \{0, 1, \ldots, k\}$ and $\begin{pmatrix} n \\ m \end{pmatrix} = \dfrac{n!}{m!(n-m)!}$, $n, m \in \boldsymbol{N}$, $m \leqslant n$.

PROPOSITION. 1.1. *a) A polynomial in $K'[X]$ of the form $G(X) = \sum_{i=0}^{n} A_i (X-a)^i$ is a K-polynomial in $K'[X]$ if and only if the column vector $\mathcal{a}_n(A_0, A_1, \ldots, A_n)^t = (a_0, a_1 \ldots, a_n)^t$ has its components in K, i.e. $a_0, \ldots, a_n \in K$. (Here "t" means the transposed). One has $T\left(\sum_{i=0}^{n} a_i X^i\right) = g(X)$.*

b) A rational function in $K'(X)$,

$$\frac{P(X)}{Q(X)} = \frac{\sum_{i=0}^{m} P_i (X-a)^i}{\sum_{j=0}^{n} Q_j (X-a)^j}$$

is a K-function in $K'(X)$ if and only if P and Q are K-polynomials in $K'(X)$.

The proof is based on the *Newton*'s binomial formula for $(X-a)^i$, $i=0, 1\dots$

3) A sequence $Q_n, Q_{n-1}, \dots, Q_0$ of elements of K' not all zero, is called a K-sequence if there exists a K-rational function P/Q in $K'(X)$ such that $Q(X) = \sum_{i=0}^{n} Q_i(X-a)^i$.

PROPOSITION 1.2. *Let P be an element of $K'(X)$ and let $\sum_{i \geq i_0} A_i(X-a)^i$ be the associated power series in an usual manner. This power series represent a rational function in $K(X)$ if and only if there exists a natural number m and a K-sequence $Q_n, \dots, Q_0$ in K' such that:*

1) $\sum_{i=0}^{n} Q_i A_{k-i} = 0$, *for every $k > m$, and*

2) *if one denote $P_j = \sum_{i=0}^{n} Q_i A_{j-i}$, $j=0, 1 \dots, m$, the components of the matrix $\mathcal{A}_m(P_0, \dots, P_m)^t$ are in K.*

PROOF. It is enough to identify the coefficients in the equality $P(X) = Q(X) \sum_{i \geq i_0} A_i(X-a)^i$ and apply Proposition 1.1.

2. – $w_{(a, \delta)}$-completion of $K(X)$.

1) Let (K, v) be a local field (see [9] i.e a valued field K complete relative to a rank one volution v. Denote by $\overline{K}$ a fixed algebraic closure of K and denote by $\bar{v}$ the unique extension of v to $\overline{K}$. If $a \in \overline{K}$ denote $\deg a = [K(a): K]$, the *degree* of a with respect to K. Let $a \in \overline{K}$ and let $\delta \in \boldsymbol{Q}$, the additive group of rational numbers. We say that (a, δ) is *minimal pair* if for any $b \in \overline{K}$, the condition $\bar{v}(a-b) \geq \delta$ implies $\deg b \geq \deg a$. According to [8], the number $\delta(a) = \sup_b (\bar{v}(a-b), b \in \overline{K}, \deg b < \deg a)$ is called the *main invariant* of a. Then (a, δ) is a minimal pair if and only if $\delta > \delta(a)$.

An extension w of v to $K(X)$ is called residual transcendental (r.t.- extension) if the residue field of w is a transcendental extension of the residue field of v.

Let (a, δ) be a minimal pair. Denote $w = w_{(a, \delta)}$ the extension of v to $K(X)$ defined as follows: if $P(X) \in K[X]$, then

$$w(P(X)) = \inf_{i=0}^{m} \left\{ \overline{v}\left(\frac{P^{(i)}(a)}{i!} \right) + i\delta \right\}$$

where $m = \deg P(X)$, According to [3], $w_{(a, \delta)}$ (extended in an usual manner to $K(X)$) is a r.t. extension of v to $K(X)$ and by ([3], Theorem 2.1) any r.t. extension of u to $K(X)$ is the form $w_{(a, \delta)}$ for a suitable minimal pair (a, δ).

2) Let $a \in \overline{K}$ and $K' = K(a)$. Denote v' the restriction of $\overline{v}$ to K'. According to [1] denote by $K'\{\{X - a, \delta\}\}$ the valued field of δ-formal Laurent series over the local field (K', v'), where $\delta > \delta(a)$ is a rational number. According to ([1], § 2) we can also consider the embeding ω: $K'(X) = K'(X - a) \to K'\{\{X - a, \delta\}\}$. In this way $K'(X - a)$ can be viewed as a subfield of $K'\{\{X - a, \delta\}\}$ and by the composition $\omega \circ T$ we can view $K(X)$ as a subfield of $K'\{\{X - a, \delta\}\}$. As usual (see [1]) denote u the natural valuation of $K'\{\{X - a, \delta\}\}$. Now if $w = w_{(a, \delta)}$ is the valuation on $K(X)$ defined in the previous point, then it is easy to see that for any $\alpha \in K(X)$ one has $w(\alpha) = u((T \circ \omega)(\alpha))$. Then, through $\omega \circ T$, $K'\{\{X - a, \delta\}\}$ becomes an extension of the valued field $(K(X), w)$. In what follows we consider $(K(X), w)$ as a subfield of $K'\{\{X - a, \delta\}\}$ and the mapping $\omega \circ T$ does not appear explicitely.

Because $K'\{\{X - a, \delta\}\}$ is complet, the topological adherence of $K(X)$ in it is exactly the completion of $(K(X), w)$. Denote by $(\widetilde{K}(X), \widetilde{w})$ this completion.

3) Using now the results of [1], we try to describe the elements of $K\{\{X - a, \delta\}\}$ which lie in $\widetilde{K(X)}$.

If $S = \sum_{i \in \boldsymbol{Z}} b_i (X - a)^i$ is a Laurent series in $K'\{\{X - a, \delta\}\}$ and n is a naturd number, the partial sum: $S_{i(n)} = \sum_{i \geqslant i(n)} b_i (X - a)^i$ is called a n-section of S if $u(S - S_{i(n)}) > n$. To every series $S_{i_0} = \sum_{i \geqslant i_0} a_i (X - a)^i$ we associate the sequence $S_{i_0}^* = \{a_i^*\}_{i \in Z}$ such that $a_i^* = a_i$ if $i \geqslant i_0$, and $a_j^* = 0$ if $j < i_0$.

If $D = \{d_0, d_1, \dots, d_q\}$ is a finite sequence of elements in K' and $A = \{a_i\}_{i \in \boldsymbol{Z}}$ is an infinite set of elements in K', we define a «multiplication» between D and A in the following way:

$$[D, A] = C = \{c_k\}_{k \in \boldsymbol{Z}}, \qquad \text{where } c_k = \sum_{i=0}^{q} d_i a_{k-i}.$$

If $C = \{c_k\}_{k \in Z}$, $B = \{b_k\}_{k \in Z}$ are subsets of K', denote $u(C, B) = \inf_{k \in \boldsymbol{Z}} \{v(c_k - b_k) + k\delta\}$. It is possible that $u(C, B) = \infty$.

A Laurent series $S = \sum_{i \in \boldsymbol{Z}} a_i (X-a)^i$ in $K'\{\{X - a, \delta\}\}$ is called *K-local periodic series* if for any $n \in N$, there exists a n-section $S_{i(n)} = \sum_{i \geq i(n)} a_i (X-a)^i$ of it and two sequencs in K', $B = \{b_0, b_1, \dots, b_p\}$ and $D = \{d_0, d_1, \dots, d_q\}$, $D \neq 0$ such that $\mathcal{A}_p(b_0, b_1, \dots, b_p)^t$ and $\mathcal{A}_q(d_0, \dots, d_q)^t$ are vectors with components in K (for the definition of $\mathcal{A}_p$ see § 1) and that $u([D, S^*_{i(n)}], B) > n + u(D)$, where $u(D) = \inf_i \{v(d_i) + i\delta\}$.

THEOREM 2.1. *An element $S = \sum_{i \in Z} a_i (X-a)^i$ of $K'\{\{X - a, \delta\}\}$ belongs to $\widetilde{K(X)}$ if and only if it is a K-local periodical series.*

PROOF. Let S be an element in $\widetilde{K(X)}$, n a natural number and $S_{i(n)}$ a n-section of S. Let

$$\alpha = \frac{P}{Q} = \frac{b_0 + b_1(x-a) + \dots + b_p(x-a)^p}{d_0 + d_1(x-a) + \dots + d_q(x-a)^q}$$

be an element in $K(X)$ such that $u(S - \alpha) > n$. Since $u(S - S_{i(n)}) > n$, then $u(S_{i(n)} - \alpha) > n$. If $B = (b_k)_k$, $D = (d_k)_k$, then it is easy to see that S is K-local periodic.

Conversely, let us assume that S is a K-local periodic element in $K'\{\{X - a, \delta\}\}$. If $B = (b_0, \dots, b_p)$ and $D = (d_0, \dots, d_q)$ are as in the definition of a local periodic series S, for a natural number n, then

$$\alpha = \frac{P}{Q} = \frac{a_0 + a_1(X-a) + \dots + b_p(X-a)^p}{d_0 + d_1(X-a) + \dots + d_q(X-a)^q}$$

belongs to $K(X)$ and $u(S - \alpha) > n$ etc.

REMARK 2.2. The above results try to give a «combinatorial» description of elements of $K'\{\{X' - a, \delta\}\}$ which lie in $\widetilde{K(X)}$. We remark that even in the case $a = 0$ this result is not as powerful as the one given in ([1], Theorem 2.3).

3. – Another type of characterization of the completion of $(K(X), w_{(a, \delta)})$.

1) The notation and the hypotheses are as in the section 2. Let f be the minimal and monic polynomial of a with respect to K. According to ([3], Theorem 2.1) and [7], there exists a natural number e, and a polynomial $h(X)$ in $K[X]$, with $\deg h < n = \deg f$ such that:

i) The rational function $r = r(X) = (f^e/h) \in K(X)$ is such that $w(r) = 0$ and t, the image of r in the residue field of w, is transcendental over the residue field of v, and

ii) r is of minimal degree with these properties:

Let us denote by w_0 the restriction of w to $K(r)$ and by $e(w|w_0)$, $f(w|w_0)$ the ramification index and respectively the inertia degree of w over w_0. Using ([3], Theorem 2.1, Corollaries 2.4, 2.5, and 2.6) and ([4]), Theorem 4.5) we conclude that $[K(X)\colon K(r)] = ne = e(w|w_0)\cdot f(w(w_0)$, so w is the unique extension of w_0 to $K(X)$. The valuation w_0 on $K(r)$ is exactly the Gauss extension of v to $K(r)$ and from [1] we know a characterization for the completion $\widetilde{K(r)}$. It is easy to see that $K(X)$ and $\widetilde{K(r)}$ are linear disjoint over $K(r)$, i.e. $f^e(Y) - h(Y)\, r(X)$, the minimal polynomial of X over $K(r)$ is also irreducible over $\widetilde{K(r)}$, so $\widetilde{K(X)}$, the completion of $K(X)$ with respect to w, is exactly $\widetilde{K(r)}(X)$.

It is clear that any element in $K(X)$ is of the form $\sum_{i=0}^{ne-1} a_i(r)\, X^i$, where $a_i(r) \in K(r)$. Let us consider now the embedding ω (see [1]) of $K(r)$ in $K\{\{r\}\}$, the local field of 0-formal Laurent series in the variable r over K, (see [1]), and describe any element from $\widetilde{K(X)}$ as being of the from $\sum_{i=0}^{ne-1} b_i(r)\, X^i$, where $b_i(r) \in \widetilde{K(r)}$.

Using these last remarks above it is not difficult to see that any element in $\widetilde{K(X)}$ may be described as being a Laurent series of the type $\sum_{i \in Z} s_i(X)\, r^i$, where $s_i(X) \in K[X]$, $\deg s_i < ne$, the values of coefficients of $s_i(X)$ are inferior bounded (for all $i \in Z$) and when $i \to -\infty$ the values of them tendes to ∞. Even the sequence $\{s_i(X)\}_i$ is almost periodic and we have the equality

$$w\left(\sum_{i\in Z} s_i(X)\, r^i\right) = \inf_{i\in Z} w(s_i(X))\,.$$

4. – Some properties of the algebraic extension $K(X) \hookrightarrow K'(X)$.

The notations and definitions are as in section 1. In this section we are interested in some properties of the extension $\widetilde{K'(X)}/\widetilde{K(X)}$. Particularly we derive some conditions such that $\widetilde{K(X)}$ is coincident to $\widetilde{K'(X)}$.

1) Let $a \in \overline{K}$ and f the monic minimal polynomial of a with respect to K. Let $S = \{a = a_1, \ldots, a_n\}$ be all roots of f in $\overline{K}$. Let δ be a rational number such that (a, δ) is a minimal pair, i.e $\delta > \delta(a)$. Denote $\gamma = = \sum_{i=1}^{n} \inf(\delta, \overline{v}(a - a_i))$ and let e be the smallest non-zero natural number such that $e\gamma \in G_{v'}$, the value group of v' and by e' denote the smallest non-zero natural number such that $e'\delta \in G_{v'}$.

Denote $w = w_{(a,\delta)}$ the r.t. extension of v to $K(X)$ defined by the minimal pair (a, δ). According to ([3], Theorem 2.1) relative to the value group G_w of w one has:

$$\gamma = w(f)\,; \qquad G_w = G_{v'} + Z\gamma\,. \tag{1}$$

In what follows we assume that a is separable over k. We remark, according to ([4], Theorem 3.1) that any r.t. extension w of v to $K(X)$ is of the form $w = w_{(a,\delta)}$ where a is separable over K.

Denote s the number of all conjugates a' of a over K such that $\overline{v}(a - a') \geqslant \delta$. As usual by $e(v' \mid v)$ and $f(v' \mid v)$ we denote the ramification index and the residual degree of v' with respect to v (see [10]).

THEOREM 4.1. *With the above notations and hypothesis one has*:

1) *s divides $n = \deg f = [K' : K]$,*

2) *There exists exactly n/s distinct extensions of w to $K'(X)$.*

3) *If w' is an extension of w to $K'(x)$ and $d = e(w' \mid w)$ then d is the greatest common divisor between s and e', and $f(w' \mid w) = s/d$.*

PROOF. Let $K'' = K(a_1, \ldots, a_n)$. Then K''/K is a Galois extension and denote by $G = \mathrm{Gal}(K''/K)$. Let $H = \{g \in G / \overline{v}(a - g(a)) \geq \delta\}$ and $S(H) = \{g(a), g \in H\}$. Then H is a subgroup of G and $S(H)$ contains exactly s elements. Since G acts transitivly on $S(H)$ then s devides n, as claimed.

2) and 3). Let $t = n/s$ and let $H = H_1, \ldots, H_t$ be all conjugates of H in G. Denote by $S(H_i) = \{g(a) \mid g \in H_i\}$, $1 \leq i \leq t$ and $S(H_i) = \{a_i^{(1)}, \ldots, a_i^{(s)}\}$. Denote $\overline{w_i}$, $1 \leq i \leq t$, the r.t. extension of $\overline{v}$ to $\overline{K}(X)$, defined by the minimal pair $(a_i^{(1)}, \delta)$. Then according to ([4], Theorem 2.2), the extensions $\overline{w_i}$, $1 \leq i \leq t$ are pair distinct and are all extensions of w to $\overline{K}(X)$. Denote w_i the restriction of $\widetilde{w}_i$ to $K'(X)$. It is clear that the set of w_i, $1 \leq i \leq t$ contains all possible non - equivalent extension of w to $K'(X)$. We assert that w_i, $1 \leq i \leq t$ are all pair distinct. Using the above notations and relations it will be enough to show the following equalities

$$e(w_i \mid w) = d\,, \qquad f(w_i \mid w) = s/d\,, \qquad 1 \leq i \leq t\,.$$

Because of symmetry it is enough to shaw the above equalities for $w_1 = w'$. Now we denote $\varepsilon = \gamma - s\delta$. It is clear that $\varepsilon = \sum v(a - a_j)$, where a_j runs all conjugates a_j of a such that $v(a - a_j) < \delta$. By a permutation of a_i, $1 \leq i \leq n$ we can assume that $\overline{v}(a - a_i) \geq \delta$ if $1 \leq i \leq s$ and $v(a - a_i) < \delta$ if $s < i \leq n$. We assert that $\varepsilon \in G_{v'}$. Indeed, one has $(f^{(s)}/s!)(a) \in K''$. But $(f^{(s)}/s!)(a) = \sum_{2 \leq i_1 < i_2 < \ldots < i_{n-s} \leq n} (a - a_{i_1}) \ldots (a - a_{i_{n-s}})$. Then according to definition of s one has $v'((f^{(s)}/s!)(a)) = \overline{v}((a - a_{s+1}) \ldots (a - a_n)) = \varepsilon \in G_{v'}$, as claimed.

Now we use the definitions of e and e' and the equality $e'\gamma = se'\delta + e'\varepsilon$ in order to derive that $e'\gamma \in G_{u'}$. It follows that $e' = ed$ for a natural number d. Since by (1), $G_w = G_{v'} + Z\gamma$ and $G_{w'} = G_v + Z\delta$, it follows that $e(w' \mid w) = d$. But d is a divisor of s since $se\delta = e\gamma - e\varepsilon \in G_{v'}$ and so se is a multiple of $e' = ed$. It is easy to see that d is exactly the greatest common divisor between s and e' (use the difinition of e').

Now let us compute $f(w' \mid w)$. Denote by b an element in $\overline{K}$ such that $\overline{v}(b) = \delta$ and by $k_{\overline{w}}$, $k_{\overline{v}}$, etc. the residue field of $\overline{w}$; $\overline{v}$, etc. Then θ, the image of $(X - a)/b$ in $k_{\overline{w}}$, is transcendental over $k_{\overline{u}}$ and $k_{\overline{w}} = k_{\overline{v}}(\theta)$ (see [3]. Theorem 2.1). Now consider the element $r \in k(X)$ defined in section 3. Then $k_w = k_{v'}(t) \subset k_{w'} \subset k_{\overline{w}}$. It is not dificult to see that t is a polynomial of θ of degree se. If $l(X) \in K'[X]$ is such that $w'(l(X)) = e'\delta$ then t', the image in $k_{w'}$ of $(X - a)^{e'}/l(X)$, is transcendental over $k_{v'}$. The degree of t'

as polynomial of θ is exactly e'. Then $[k_{w'}: k_w] = f(w|w') = se/e' = s/d$. The proof of Theorem 4.1 is complete.

COROLLARY 4.2. *The extension* $(K(X), w) \subset (K'(X), w')$ *is immediate (see* [11]) *if and only if* $s = 1$. *In this case* $X - a \in (\widetilde{K(X)}, \tilde{w}) = (\widetilde{K(X)}, \tilde{w}')$. *This is true* if and only if $\delta > \omega(a) = \sup (\bar{v}(a - a'))$, *where* a' *runs all conjugates of* a *distinct to* a.

Now we indicate some cases when the conditions of Corollary 4.2 are verified

COROLLARY 4.3. *Let* $p = \mathrm{char}\, k_u > 0$. *If* $n = [K' : K]$ *is relative prime to* p *then* $s = 1$.

PROOF. Indeed, in this case $\delta(a) = \omega(a)$. For that let $S(a) = \{a' \in S, \bar{v}(a - a') \geq \omega(a)\}$ and let h be the cardinal of $S(a)$. Let H be the subgroup of $G = \mathrm{Gal}(K''|K)$ defined by: $H = \{g \in G | g(a) \in S(a)\}$. If $m = |G/H|$ then m divides n and let $H = H_1, \ldots, H_m$ be all conjugates of H in G. If $H_i = g_i H g_i^{-1}$, let $a_i = g_i(a)$ and let $S(a_i)$ defined as $S(a)$. Then the $\{S(a_i)\}_i$ gives a partition of S. Denote $b_i = \sum a'$, where a' runs over $S(a_i)$, $1 \leq i \leq m$. It is clear that G acts transitively on the set $\{b_1, \ldots, b_m\}$. It follows that the coefficients of the polynomial $\varphi(X) = (X - b_1)\ldots(X - b_m)$ belongs to K. Hence the element b_1 is such that $[K(b_1): K] \leq m \leq n$. Since one has $\bar{v}(a - b_i/h) \geq \omega(a)$ then, according to definition of $\delta(a)$, one has necessarily $\delta(a) = \omega(a)$.

REMARK 4.4. It can be proved that the polynomial φ is irreducible and the pair (a, b_1) is admissible (see [8]).

COROLLARY 4.5. *If* a *is an uniformising element of* K' *and* $e(v/v')$ *is relatively prime to* p, *or* $f(v|v') = n = [k' : k]$, $\bar{v}(a) = 0$ *and* $k_{v'} = k_v(a)$, *where* a *is the image of* a *in* $k_{v'}$, *then* $s = 1$.

The proof follows by [9], since in both cases one has $\omega(a) = \delta(a)$.

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