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Darboux and Goursat Type Problems in the Trihedral Angle for Hyperbolic Type Equations of Third Order.

OTARI JOKHADZE (*)

1. - Statement of the problem and some notations.

In the space of independent variables $x \equiv (x_1, x_2, x_3) \in \mathbb{R}^3$, $\mathbb{R} \equiv \equiv (-\infty, \infty)$, $\mathbb{R}^3 \equiv \mathbb{R} \times \mathbb{R} \times \mathbb{R}$ let us consider a partial differential equation of third order of the kind

$$u_{x_1 x_2 x_3} = F, \tag{1.1}$$

where F is a given function and u is an unknown real function.

Equation (1.1) in the Euclidean space $\mathbb{R}^3$ is of hyperbolic type for which a family of planes $x_1 = \text{const}$, $x_2 = \text{const}$, $x_3 = \text{const}$ is characteristic, while the directions determined by the unit vectors $e_1 \equiv (1, 0, 0)$, $e_2 \equiv (0, 1, 0)$, $e_3 \equiv (0, 0, 1)$ of the coordinate axes are bicharacteristic.

In the space $\mathbb{R}^3$ let $S_i^0: p_i^0(x) \equiv \alpha_i^0 x_1 + \beta_i^0 x_2 + \gamma_i^0 x_3 = 0$, $i = 1, 2, 3$, be arbitrarily given planes, without restriction of generality, passing through the origin.

Assume that

$$\operatorname{rank}(\nu_1^0, \nu_2^0, \nu_3^0) = 3, \tag{1.2}$$

where $\nu_i^0 \equiv (\alpha_i^0, \beta_i^0, \gamma_i^0)$, $i = 1, 2, 3$. The space $\mathbb{R}^3$ is partitioned by the planes S_i^0, $i = 1, 2, 3$, into eight trihedral angles. We consider equation (1.1) in one of these trihedral angles D_0, which, without restriction of

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generality, is assumed to be given in the form

$$D_0 \equiv \{x \in \mathbb{R}^3 : p_i^0(x) > 0,\ i = 1, 2, 3\}.$$

Regarding the domain D_0, we suppose that:

Each bicharacteristic of equation (1.1) is parallel to only one of the faces of the trihedral angle D_0. Without loss of generality, we suppose that $e_i \| S_i^0$, $i = 1, 2, 3$. This is equivalent to the requirement that

$$\alpha_1^0 = 0, \qquad \beta_2^0 = 0, \qquad \gamma_3^0 = 0.$$

In particular, this implies that

a) the edge $\Gamma_k^0 \equiv \{x \in \mathbb{R}^3 : p_i^0(x) = 0,\ p_j^0(x) = 0,\ i, j = 1, 2, 3,\ i < j,\ k \neq i, j\}$, $k = 1, 2, 3$, of the trihedral angle D_0 has no bicharacteristic direction, i.e., $\nu^k \not\parallel e_i$, $k, i = 1, 2, 3$, where $\nu^k \equiv \nu_i^0 \times \nu_j^0$, $i, j, k = 1, 2, 3$, $i < j$, $k \neq i, j$ is the vector product of the vectors ν_i^0 and ν_j^0;

b) the bicharacteristics passing through the edge Γ_k^0, $k = 1, 2, 3$, do not pass into the domain D_0.

For convenience of the invesigation of the boundary value problem for equation (1.1) we transform the domain D_0 into the domain $D \equiv \{y \in \mathbb{R}^3 : y_1 > 0,\ y_2 > 0,\ y_3 > 0\}$ of the space of variables y_1, y_2, y_3. To this end, let us introduce new independent variables defined by the equalities

$$y_i = p_i^0(x), \qquad i = 1, 2, 3. \tag{1.3}$$

Owing to (1.2), the linear transform (1.3) is obviously non-degenerate, it establishes the one-to-one correspondence between the domains D_0 and D.

Retaining the previous notations for u and F equation (1.1) in the domain D for the variables y_1, y_2, y_3 can be rewritten as

$$\frac{\partial^3 u}{\partial \mu_1 \partial \mu_2 \partial \mu_3} = F. \tag{1.4}$$

Here

$$\frac{\partial}{\partial \mu_1} \equiv \alpha_2^0 \frac{\partial}{\partial y_2} + \alpha_3^0 \frac{\partial}{\partial y_3}, \qquad \frac{\partial}{\partial \mu_2} \equiv \beta_1^0 \frac{\partial}{\partial y_1} + \beta_3^0 \frac{\partial}{\partial y_3},$$

$$\frac{\partial}{\partial \mu_3} \equiv \gamma_1^0 \frac{\partial}{\partial y_1} + \gamma_2^0 \frac{\partial}{\partial y_2}.$$

In the domain D let us consider instead of equation (1.4) a more general equation

$$\frac{\partial^3 u}{\partial l_1\, \partial l_2\, \partial l_3} = F. \tag{1.5}$$

Here for the variables y_1, y_2, y_3 we use the previous notations x_1, x_2, x_3; $(\partial/\partial l_i) \equiv \alpha_i(\partial/\partial x_1) + \beta_i(\partial/\partial x_2) + \gamma_i(\partial/\partial x_3)$ is a derivative with respect to the direction $l_i \equiv (\alpha_i, \beta_i, \gamma_i)$, $\operatorname{rank}(l_1, l_2, l_3) = 3$, $i = 1, 2, 3$, F is a given function, and u is an unknown real function. Moreover, the bicharacteristics of equation (1.5) and the domain D will be assumed to satisfy the condition formulated above for equation (1.1) in the domain $D_0(l_i \,\|\, S_i,\ i = 1, 2, 3)$, which in this case takes the form: $\alpha_1 = 0$, $\beta_2 = 0$, $\gamma_3 = 0$.

Let $P = P(x) \in \overline{D}$ be an arbitrary point of the closed domain $\overline{D}$, and let S_k, $k = 1, 2, 3$, be plane faces of the angle D, i.e., $\partial D = \bigcup_{k=1}^{3} S_k$, $S_k \equiv \{x \in \mathbb{R}^3 : x_k = 0, (x_i, x_j) \in \overline{\mathbb{R}}^2_+\}$, Γ_k, $k = 1, 2, 3$, edge of the angle D, i.e., $\Gamma_k \equiv \{x \in \mathbb{R}^3 : x_k \in \overline{\mathbb{R}}_+, x_i = x_j = 0\}$, $i, j, k = 1, 2, 3$, $i < j$, $k \neq i, j$, $\mathbb{R}_+ \equiv (0, \infty)$, $\overline{\mathbb{R}}^2_+ \equiv \overline{\mathbb{R}}_+ \times \overline{\mathbb{R}}_+$. Let the bicharacteristic beams $L_i(P)$, corresponding to the bicharacteristics directions l_i of equation (1.5) be drawn from the point P to the intersection with one of the faces S_i at the points P_i, $i = 1, 2, 3$. Without loss of generalitly it is assumed that $P_3 \in S_1$, $P_2 \in S_3$, $P_1 \in S_2$.

In the domain D let us consider a Darboux type problem for the equation (1.5) which is formulated as follows: Find in the domain D a regular solution u of equation (1.5) satisfying the following boundary conditions:

$$\left(M_i \frac{\partial^2 u}{\partial l_1\, \partial l_2} + N_i \frac{\partial^2 u}{\partial l_1\, \partial l_3} + Q_i \frac{\partial^2 u}{\partial l_2\, \partial l_3} \right)\Bigg|_{S_i} = f_i, \quad i = 1, 2, 3, \tag{1.6}$$

where M_i, N_i, Q_i, f_i, $i = 1, 2, 3$, are the given real functions.

A regular solution of equation (1.5) is said to the function u which is continuous in D together with its partial derivatives $(\partial^{i+j+k} u)/(\partial l_1^i\, \partial l_2^j\, \partial l_3^k)$, $i, j, k = 0, 1$, and satisfies equation (1.5) in D.

It should be noted that the boundary value problem (1.5), (1.6) is a natural continuation of the well-known classical statements of the Goursat and Darboux problems (see, e.g., [1]-[4]) for linear hyporbolic equations of second and third order with two independent variables on a plane. Multi-dimensional analogs of the Goursat and Darboux problems for hyperbolic type equations of second and third order in a dihedral angle have been studied in a number of papers (see, e.g., [2], [5]-[9]).

Many works are devoted to the initial boundary value and characteristic problems for a wide class of hyporbolic equations of third and higher orders in multi-dimensional domains with dominated lower terms (see, e.g., [10], [11]).

REMARK 1.1. Note that the hyperbolicity of problem (1.5), (1.6) is taken into account in conditions (1.6) because of the presence of derivatives of second order dominated by $\partial^3 u/(\partial l_1 \partial l_2 \partial l_3)$.

In the domains D and $\mathbb{R}^2_+$ let us introduce into consideration the following functional spaces:

$$\overset{0}{C}_\alpha(\overline{D}) \equiv \Big\{ v \in C(\overline{D}) \colon v\big|_\Gamma = 0, \sup_{x \in D_n} (\varrho_1 \varrho_2 \varrho_3)^{-\alpha} |v(x)| < \infty, n \in \mathbb{N} \Big\},$$

$$\overset{0}{C}_\alpha(\overline{\mathbb{R}}^2_+) \equiv \Big\{ \varphi \in C(\overline{\mathbb{R}}^2_+) \colon \varphi\big|_{\Gamma^*} = 0, \sup_{(\xi,\eta) \in \Omega_n} (\xi\eta\varrho)^{-\alpha} |\varphi(\xi,\eta)| < \infty, n \in \mathbb{N} \Big\},$$

where

$$\Gamma \equiv \Gamma_1 \cup \Gamma_2 \cup \Gamma_3, \quad \Gamma^* \equiv \Gamma_1^* \cup \Gamma_2^*, \quad \Gamma_1^* \equiv \{(\xi,\eta) \in \overline{\mathbb{R}}^2_+ : \xi \in \overline{\mathbb{R}}_+, \eta = 0\},$$

$$\Gamma_2^* \equiv \{(\xi,\eta) \in \overline{\mathbb{R}}^2_+ : \eta \in \overline{\mathbb{R}}_+, \xi = 0\},$$

$$r \equiv |x| \equiv \sqrt{x_1^2 + x_2^2 + x_3^2}, \qquad \varrho = \varrho(\xi,\eta) = \sqrt{\xi^2 + \eta^2},$$

$$D_n \equiv D \cap \{0 < r < n\}, \qquad \Omega_n \equiv \mathbb{R}^2_+ \cap \{0 < \varrho < n\}, \qquad n \in \mathbb{N},$$

ϱ_i is the distance from the point $x \in \overline{D}$ to the edge Γ_i of the domain D, i.e., $\varrho_i = \varrho(x_k, x_j)$, $i, j, k = 1, 2, 3$, $k < j$, $i \neq k, j$, and the parameters $\alpha \equiv \text{const} \geqslant 0$, $\mathbb{N} \equiv \{1, 2, \dots\}$.

Obviously, for the semi-norms

$$\|v\|_{\overset{0}{C}_\alpha(\overline{D}_n)} = \sup_{x \in \overline{D}_n \setminus \Gamma} (\varrho_1 \varrho_2 \varrho_3)^{-\alpha} |v(x)|,$$

$$\|\varphi\|_{\overset{0}{C}_\alpha(\overline{\Omega}_n)} = \sup_{(\xi,\eta) \in \overline{\Omega}_n \setminus \Gamma^*} (\xi\eta\varrho)^{-\alpha} |v(\xi,\eta)|, \qquad n \in \mathbb{N},$$

the spaces $\overset{0}{C}_\alpha(\overline{D})$ and $\overset{0}{C}_\alpha(\overline{\mathbb{R}}^2_+)$ are the countable normed Frechet spaces.

It can be easily seen that the belonging of the functions $v \in \overset{0}{C}(\overline{D})$ and $\varphi \in \overset{0}{C}(\overline{\mathbb{R}}^2_+)$ respectively to the spaces $\overset{0}{C}_\alpha(\overline{D})$ and $\overset{0}{C}_\alpha(\overline{\mathbb{R}}^2_+)$ is equivalent to the fulfilment of the following inequalities:

$$(1.7) \qquad |v(x)| \leqslant c_1 (\varrho_1 \varrho_2 \varrho_3)^\alpha, \quad x \in \overline{D}_n, \quad |\varphi(\xi,\eta)| \leqslant c(\xi\eta\varrho)^\alpha,$$
$$(\xi,\eta) \in \overline{\Omega}_n, \quad n \in \mathbb{N}.$$

We investigate the boundary value problem (1.5), (1.6) in the Frechet space

$$\overset{0}{C}{}^{l}_{\alpha}(\overline{D}) \equiv \Big\{ u:\ \frac{\partial^{i+j+k} u}{\partial l_1^i\, \partial l_2^j\, \partial l_3^k} \in \overset{0}{C}_{\alpha}(\overline{D}),\ i, j, k = 0, 1,$$

$$i + j + k = 2, 3,\ \frac{\partial u}{\partial l_i}\Big|_{\Gamma_i} = 0,\ i = 1, 2, 3,\ u(O) = 0 \Big\},$$

$$\boldsymbol{l} \equiv (l_1, l_2, l_3), \qquad O \equiv (0, 0, 0),$$

with respect to the semi-norms

$$\|u\|_{\overset{0}{C}{}^{l}_{\alpha}(\overline{D}_n)} = \sum_{i+j+k=2}^{3} \left\| \frac{\partial^{i+j+k} u}{\partial l_1^i\, \partial l_2^j\, \partial l_3^k} \right\|_{\overset{0}{C}_{\alpha}(\overline{D}_n)}, \qquad n \in \mathbb{N}.$$

In considering the boundary value problem (1.5), (1.6) in the class $\overset{0}{C}{}^{l}_{\alpha}(\overline{D})$, we require that the functions $F \in \overset{0}{C}_{\alpha}(\overline{D})$, $f_i \in \overset{0}{C}_{\alpha}(\overline{\mathbb{R}^2_+})$, $i = 1, 2, 3$.

REMARK 1.2. When $\alpha = 0$, we omit the subscript index α used above in the notations of functional classes.

2. - Equivalent reduction of problem (1.5), (1.6) to a functional equation.

Using the notations

$$\frac{\partial^2 u}{\partial l_1\, \partial l_2} \equiv v_3\,, \qquad \frac{\partial^2 u}{\partial l_1\, \partial l_3} \equiv v_2\,, \qquad \frac{\partial^2 u}{\partial l_2\, \partial l_3} \equiv v_1$$

problem (1.5), (1.6) in the domain D can be rewritten equivalently as a boundary value problem for a system of partial differential equations of first order with respect to the unknown functions v_1, v_2, v_3

$$\text{(2.1)} \qquad \frac{\partial v_3}{\partial l_3} = F\,, \qquad \frac{\partial v_2}{\partial l_2} = F\,, \qquad \frac{\partial v_1}{\partial l_1} = F\,,$$

$$\text{(2.2)} \qquad (M_i v_3 + N_i v_2 + Q_i v_1)|_{S_i} = f_i\,, \qquad i = 1, 2, 3\,.$$

The equivalence of the initial problem (1.5), (1.6) and problem (2.1), (2.2) is an obvious consequence of

LEMMA 2.1. *In the closed domain $\overline{D}_0$, which is trihedral angle introduced in § 1, there exists a unique function*

$$u \in \left\{ u\colon \frac{\partial^{i+j+k} u}{\partial x_1^i \partial x_2^j \partial x_3^k} \in C(\overline{D}_0),\ i, j, k = 0, 1 \right\},$$

satisfying both the redefined system of partial differential equations of second order

$$u_{x_1 x_2} = v_3\,, \qquad u_{x_1 x_3} = v_2\,, \qquad u_{x_2 x_3} = v_1 \tag{2.3}$$

and the conditions

$$u(O) = 0\,, \qquad u_{x_1}|_{\Gamma_1^0} = 0\,, \qquad u_{x_2}|_{\Gamma_2^0} = 0\,, u_{x_3}|_{\Gamma_3^0} = 0\,. \tag{2.4}$$

Here v_1, v_2, v_3 are given functions such that,

$$\frac{\partial v_i}{\partial x_i} \in C(\overline{D}_0)\,, \quad i = 1, 2, 3\,; \quad \frac{\partial v_1}{\partial x_1}(x) = \frac{\partial v_2}{\partial x_2}(x) = \frac{\partial v_3}{\partial x_3}(x)\,, \quad x \in \overline{D}_0\,.$$

PROOF. Let $P_0 = P_0(x_1^0, x_2^0, x_3^0)$ be an arbitrary point of the closed domain $\overline{D}_0$. It is obvious that owing to the requirement for the domain D_0 in §1, the plane $x_1 = x_1^0$ has the unique point of intersection of P_0^* with the edge Γ_1^0.

Since $(u_{x_1}(x_1^0, x_2, x_3))_{x_2} = v_3(x_1^0, x_2, x_3)$, $(u_{x_1}(x_1^0, x_2, x_3))_{x_3} = v_2(x_1^0, x_2, x_3)$ and $u_{x_1}(P_0^*) = 0$, the function $u_{x_1}(x_1^0, x_2, x_3)$ is defined uniquely at the point $P_0(x_1^0, x_2^0, x_3^0)$ by the formula

$$u_{x_1}(x_1^0, x_2^0, x_3^0) = \int\limits_{(x_2^*(x_1^0),\, x_3^*(x_1^0))}^{(x_2^0,\, x_3^0)} v_3(x_1^0, x_2, x_3)\,dx_2 + v_2(x_1^0, x_2, x_3)\,dx_3\,. \tag{2.5}$$

Here the curvilinear integral is taken along any simple smooth curve connecting the points $(x_2^*(x_1^0), x_3^*(x_1^0))$ and (x_2^0, x_3^0) of the plane $x_1 = x_1^0$ and lying wholly in $\overline{D}_0$. Since the point P_0 is chosen arbitrarily, formula (2.5) gives in a closed domain $\overline{D}_0$ the representation of the function u_{x_1} which is written in terms of the given functions v_3 and v_2. Analogously, the representation formulas for the functions u_{x_2} and u_{x_3} in $\overline{D}_0$ are given respectively by the known functions v_1, v_3 and v_1, v_2. It remains only to note that the function u defined by the formula

$$u(x) = \int\limits_{OP} u_{x_1} dx_1 + u_{x_2} dx_2 + u_{x_3} dx_3 = \tag{2.6}$$

$$= \int\limits_{OP} \left\{ \int\limits_{P^* P^1} v_3(x_1, \eta, \zeta)\, d\eta + v_2(x_1, \eta, \zeta)\, d\zeta \right\} dx_1 +$$

$$+ \left\{ \int\limits_{P^{**} P^2} v_3(\xi, x_2, \zeta)\, d\xi + v_1(\xi, x_2, \zeta)\, d\zeta \right\} dx_2 +$$

$$+ \left\{ \int\limits_{P^{***} P^3} v_2(\xi, \eta, x_3)\, d\xi + v_1(\xi, \eta, x_3)\, d\eta \right\} dx_3$$

defines a unique solution of problem (2.3), (2.4). Here

$$P \equiv P(x_1, x_2, x_3), \qquad P^1 \equiv P^1(x_2, x_3), \qquad P^2 \equiv P^2(x_1, x_3),$$

$$P^3 \equiv P^3(x_1, x_2), \qquad P^* \equiv P^*(x_2^*(x_1), x_3^*(x_1)),$$

$$P^{**} \equiv P^{**}(x_1^{**}(x_2), x_3^{**}(x_2)), \qquad P^{***} \equiv P^{***}(x_1^{***}(x_3), x_2^{***}(x_3)).$$

REMARK 2.1. If instead of system (2.3) we consider the system

$$\frac{\partial^2 u}{\partial l_1 \partial l_2} = v_3, \qquad \frac{\partial^2 u}{\partial l_1 \partial l_3} = v_2, \qquad \frac{\partial^2 u}{\partial l_2 \partial l_3} = v_1, \tag{2.7}$$

in the trihedral angle $\overline{D}$ of a space of independent variables $x \in \mathbb{R}^3$, then similarly to the requirement domain D_0 from § 1 one should require that $l_i \| S_i$, $i = 1, 2, 3$.

Note that system (2.7) reduces to system (2.3) in the variables $(\xi, \eta, \zeta) \in \mathbb{R}^3$ by means of the following nondegenerate transform of variables x_1, x_2, x_3:

$$x_1 = \alpha_1 \xi + \alpha_2 \eta + \alpha_3 \zeta, \quad x_2 = \beta_1 \xi + \beta_2 \eta + \beta_3 \zeta, \quad x_3 = \gamma_1 \xi + \gamma_2 \eta + \gamma_3 \zeta,$$

assuming that the vectors l_1, l_2 and l_3 are linearly independent.

Let the bicharecteristic beams $L_i(P)$, $i = \overline{1, 2, 3}$, of equation (1.5) be drawn from an arbitrary point $P = P(x) \in \overline{D}$ to the intersection with the faces at the points $P_3 \in S_1$, $P_2 \in S_3$, $P_1 \in S_2$.

Denoting

$$v_3(x)|_{x_1=0} \equiv \varphi_3(x_2, x_3), \quad v_2(x)|_{x_3=0} \equiv \varphi_2(x_1, x_2), \quad v_1(x)|_{x_2=0} \equiv \varphi_1(x_1, x_3),$$

$$(x_2, x_3), \quad (x_1, x_2), \quad (x_1, x_3) \in \overline{\mathbb{R}^2_+}$$

and integrating the equations of system (2.1) along the corresponding bicharacteristics, we get

$$(2.8) \qquad \begin{cases} v_1(x) = \varphi_1(x_1, x_3 - \beta_1^{-1}\gamma_1 x_2) + F_1(x), \\ v_2(x) = \varphi_2(x_1 - \alpha_2\gamma_2^{-1}x_3, x_2) + F_2(x), \; x \in \overline{D}, \\ v_3(x) = \varphi_3(x_2 - \alpha_3^{-1}\beta_3 x_1, x_3) + F_3(x), \end{cases}$$

where F_i, $i = 1, 2, 3$, are the known functions from the class $\overset{0}{C}_\alpha(\overline{\mathbb{R}^2_+})$, and the superscript -1 here and below denotes an inverse value.

Substituting the expressions for v_1, v_2 and v_3 from equalities (2.8) into the boundary conditions (2.2), we obtain

$$M_1(x_2, x_3)\varphi_3(x_2, x_3) + N_1(x_2, x_3)\varphi_2(-\alpha_2\gamma_2^{-1}x_3, x_2) = f_4(x_2, x_3),$$

$$(x_2, x_3) \in \overline{\mathbb{R}^2_+},$$

$$M_2(x_1, x_3)\varphi_3(-\alpha_3^{-1}\beta_3 x_1, x_3) + Q_2(x_1, x_3)\varphi_1(x_1, x_3) = f_5(x_1, x_3),$$

$$(x_1, x_3) \in \overline{\mathbb{R}^2_+},$$

$$N_3(x_1, x_2)\varphi_2(x_1, x_2) + Q_3(x_1, x_2)\varphi_1(x_1, -\beta_1^{-1}\gamma_1 x_2) = f_6(x_1, x_2),$$

$$(x_1, x_2) \in \overline{\mathbb{R}^2_+},$$

where the known functions f_i, $i = 4, 5, 6$ belong to the class $\overset{0}{C}_\alpha(\overline{\mathbb{R}^2_+})$.

By variables ξ, η the last system has the form

$$(2.9) \qquad \begin{cases} M_1(\xi, \eta)\varphi_3(\xi, \eta) + N_1(\xi, \eta)\varphi_2(-\alpha_2\gamma_2^{-1}\eta, \xi) = f_4(\xi, \eta), \\ M_2(\xi, \eta)\varphi_3(-\alpha_3^{-1}\beta_3\xi, \eta) + Q_2(\xi, \eta)\varphi_1(\xi, \eta) = f_5(\xi, \eta), \\ \qquad\qquad\qquad\qquad\qquad\qquad (\xi, \eta) \in \overline{\mathbb{R}^2_+}, \\ N_3(\xi, \eta)\varphi_2(\xi, \eta) + Q_3(\xi, \eta)\varphi_1(\xi, -\beta_1^{-1}\gamma_1\eta) = f_6(\xi, \eta). \end{cases}$$

Let the following conditions be fulfilled:

$$(2.10) \qquad M_1(\xi, \eta) \neq 0, \quad Q_2(\xi, \eta) \neq 0, \quad N_3(\xi, \eta) \neq 0, \quad (\xi, \eta) \in \overline{\mathbb{R}^2_+}.$$

By eliminating successively the unknown values, for the function φ_2 we obtain the following functional equation from system (2.9)

$$\varphi_2(\xi, \eta) - a(\xi, \eta)\varphi_2(\tau_2\eta, \tau_1\xi) = f(\xi, \eta), \quad (\xi, \eta) \in \overline{\mathbb{R}}^2_+ , \tag{2.11}$$

where

$$a(\xi, \eta) \equiv -N_3^{-1}(\xi, \eta)Q_2^{-1}(\xi, -\beta_1^{-1}\gamma_1\eta)M_1^{-1}(-\alpha_3^{-1}\beta_3\xi, -\beta_1^{-1}\gamma_1\eta) \times$$

$$\times Q_3(\xi, \eta)M_2(\xi, -\beta_1^{-1}\gamma_1\eta)N_1(-\alpha_3^{-1}\beta_3\xi, -\beta_1^{-1}\gamma_1\eta),$$

$$\tau_2 \equiv \alpha_2\beta_1^{-1}\gamma_1\gamma_2^{-1}, \quad \tau_1 \equiv -\alpha_3^{-1}\beta_3 ,$$

and the known function f belongs to the class $\overset{0}{C}_\alpha(\overline{\mathbb{R}}^2_+)$.

REMARK 2.2. It is obvious that when conditions (2.10) are fulfilled, problem (1.5), (1.6) in the class $\overset{0}{C}{}^l_\alpha(\overline{D})$ is equivalently reduced to equation (2.11) with respect to the unknown function φ_2 of the class $\overset{0}{C}_\alpha(\overline{\mathbb{R}}^2_+)$. Furthermore, if $u \in \overset{0}{C}{}^l_\alpha(\overline{D})$ then $\varphi_2 \in \overset{0}{C}_\alpha(\overline{\mathbb{R}}^2_+)$, and vice versa, if $\varphi_2 \in \overset{0}{C}_\alpha(\overline{\mathbb{R}}^2_+)$, then taking into account inequalities (1.7), we find from equalities (2.9), (2.8), (2.6) that $u \in \overset{0}{C}{}^l_\alpha(\overline{D})$.

3. – Investigation of the functional equation (2.11).

Let $K: X \to X$ be a linear operator acting on the linear space X. Let us consider the equation

$$\varphi - K\varphi = \psi \tag{3.1}$$

and the iterated equation corresponding to (3.1)

$$\varphi - K^n\varphi = \psi + \dots + K^{n-1}\psi , \tag{3.2}$$

here φ is an unknown element and ψ is a given element of the space X, $n \geq 2$.

We have the following simple lemma.

LEMMA 3.1. *If the homogeneous equation corresponding to* (3.2) *has only a trivial solution, then equations* (3.1) *and* (3.2) *are equivalent.*

PROOF. It is clear that any solution of equation (3.1) is a solution of equation (3.2) as well. Let now φ be an arbitrary solution of equation

(3.2). Rewrite equations (3.1) and (3.2) as

$$\varphi = K_0 \varphi, \tag{3.3}$$

$$\varphi = K_0^n \varphi, \tag{3.4}$$

where $K_0 \varphi = K\varphi + \psi$. Since φ is a solution of equation (3.4), we have $K_0 \varphi = K_0^n (K_0 \varphi)$. From this, in view of the uniqueness of the solvability of equation (3.4), we find that $\varphi = K_0 \varphi$, i.e., φ satisfies equation (3.3).

Using Lemma 3.1, we will prove the uniqueness of the solvability of equation (2.11) in the class $\overset{0}{C}_\alpha(\overline{\mathbb{R}^2_+})$. Indeed, define the operator K appearing in Lemma 3.1 by the equality

$$(K\varphi_2)(\xi, \eta) = a(\xi, \eta)\, \varphi_2(T_1(\xi, \eta)),$$

where $T_1: (\xi, \eta) \to (\tau_2 \eta, \tau_1 \xi)$, $(\xi, \eta) \in \overline{\mathbb{R}^2_+}$. Then for $n = 2$ the iterated equation corresponding to (3.2) will be of the form

$$\varphi_2(\xi, \eta) - a_*(\xi, \eta)\varphi_2\big(T(\xi, \eta)\big) = f_*(\xi, \eta), (\xi, \eta) \in \overline{\mathbb{R}^2_+}; \tag{3.5}$$

here

$$a_*(\xi, \eta) \equiv a(\xi, \eta)\, a(T_1(\xi, \eta)), \; f_*(\xi, \eta) \equiv f(\xi, \eta) + a(\xi, \eta)\, f(T_1(\xi, \eta)),$$

$$T_1^2 \equiv T: (\xi, \eta) \to (\tau_0 \xi, \tau_0 \eta), \text{ where } \tau_0 \equiv \tau_1 \tau_2, \; (\xi, \eta) \in \overline{\mathbb{R}^2_+}.$$

Assume that conditions (2.10) are fulfilled and $0 < \tau_0 < 1$. Set

$$(\Lambda\varphi_2)(\xi, \eta) \equiv \varphi_2(\xi, \eta) - a_*(\xi, \eta)\, \varphi_2(T(\xi, \eta)), \quad (\xi, \eta) \in \overline{\mathbb{R}^2_+},$$

$$\sigma \equiv a_*(O_*), \quad \alpha_0 \equiv -\frac{\log|\sigma|}{3\log \tau_0}, \quad (\sigma \neq 0), \quad O_* \equiv (0, 0).$$

LEMMA 3.2. *If $\alpha > \alpha_0$, then equation (3.5) is uniquely solvable in the space $\overset{0}{C}_\alpha(\overline{\mathbb{R}^2_+})$ and for the solution $\varphi_2 = \Lambda^{-1} f_*$ the following estimate*

$$|\varphi_2(\xi, \eta)| = |(\Lambda^{-1} f_*)(\xi, \eta)| \leq c_*(\xi\eta\varrho)^\alpha \|f_*\|_{\overset{0}{C}_\alpha(\overline{\Omega}_\varrho)}, \tag{3.6}$$

holds; here and below the superscript -1 of the operators denotes an inverse operator, $\overline{\Omega}_\varrho \equiv \{(\xi_1, \eta_1) \in \overline{\mathbb{R}^2_+} : \varrho(\xi_1, \eta_1) \leq \varrho\}$ and the positive constant c_* does not depend on the function f_*.

PROOF. We introduce into consideration the operators

$$(3.7)\qquad \Gamma \equiv K^2\,, \qquad \text{i.e., } (\Gamma\varphi_2)(\xi, \eta) \equiv a_*(\xi, \eta)\,\varphi_2(T(\xi, \eta))\,,$$

$$(\xi, \eta) \in \overline{\mathbb{R}}^2_+ \text{ and } \Lambda^{-1} = I + \sum_{j=1}^{\infty} \Gamma^j\,,$$

where I is the identical operator. It is easy to see that the operator Λ^{-1} is formally inverse to the operator Λ. Thus, by Lemma 3.3 proved below, it is enough for us to prove that the Neumann series $\Lambda^{-1} = I + \sum_{j=1}^{\infty} \Gamma^j$ converges in the space $\overset{0}{C}_\alpha(\overline{\Omega}_n)$, $n \in \mathbb{N}$.

By the definition of the operator Γ from (3.7) we have

$$(\Gamma^j\varphi_2)(\xi, \eta) = a_*(\xi, \eta)\,a_*(T(\xi, \eta)) \dots a_*(T^{j-1}(\xi, \eta))\,\varphi_2(T^j(\xi, \eta))\,,$$

$$T^j \equiv T(T^{j-1})\,, \qquad j \geq 1\,, \qquad T^0 \equiv I\,, \qquad (\xi, \eta) \in \overline{\mathbb{R}}^2_+\,.$$

The condition $\alpha > \alpha_0$ is equivalent to the inequality $\tau_0^{3\alpha}|\sigma| < 1$. Therefore by virtue of the continuity of the function a_* and the equality $a_*(O_*) = \sigma$ there are positive numbers $\varepsilon(\varepsilon < n)$, δ and q, such that the inequalities

$$(3.8)\qquad |a_*(\xi, \eta)| \leq |\sigma| + \delta\,, \qquad \tau_0^{3\alpha}(|\sigma| + \delta) \equiv q < 1\,,$$

hold for $0 \leq \varrho \leq \varepsilon$.

It is obvious that the sequence of points $\{T^j(\xi, \eta)\}_{j=0}^{\infty}$, $(\xi, \eta) \in \overline{\Omega}_n$ uniformly converges to the point O_* as $j \to \infty$ on a set $\overline{\Omega}_n$, $n \in \mathbb{N}$. Therefore there is a natural number j_0, such that

$$(3.9)\qquad \varrho(T^j(\xi, \eta)) \leq \varepsilon\,, \qquad \text{for } (\xi, \eta) \in \overline{\Omega}_n\,, \qquad j \geq j_0\,.$$

By virtue of the obvious equality $\varrho(T^j(\xi, \eta)) = \tau_0^j\varrho$ inequality (3.9) takes the form $\tau_0^j\varrho \leq \varepsilon$; so, as j_0 one can take, for example, $j_0 = [\log \varepsilon\varrho^{-1}/\log \tau_0] + 1$, where $[p]$ denotes the integral part of the number p.

Let $\max_{(\xi, \eta) \in \overline{\Omega}_n} |a_*(\xi, \eta)| \equiv \beta$. By virtue of (3.8), (3.9) the following estimates hold for $j > j_0$, $f_* \in \overset{0}{C}_\alpha(\overline{\mathbb{R}}^2_+)$:

$$(3.10)\qquad |(\Gamma^j f_*)(\xi, \eta)| = |a_*(\xi, \eta)\,a_*(T(\xi, \eta)) \dots a_*(T^{j_0-1}(\xi, \eta))| \times$$

$$\times |a_*(T^{j_0}(\xi, \eta)) \dots a_*(T^{j-1}(\xi, \eta))|\,|f_*(T^j(\xi, \eta))| \leq$$

$$\leqslant \beta^{j_0}(|\sigma| + \delta)^{j-j_0}(\xi\eta\tau_0^{3j}\varrho)^\alpha \|f_*\|_{\overset{0}{C}_\alpha(\overline{\Omega}_\varrho)} \leqslant$$

$$\leqslant \beta^{j_0}(|\sigma| + \delta)^{-j_0}[\tau_0^{3\alpha}(|\sigma| + \delta)]^j(\xi\eta\varrho)^\alpha \|f_*\|_{\overset{0}{C}_\alpha(\overline{\Omega}_\varrho)} = c_0 q^j (\xi\eta\varrho)^\alpha \|f_*\|_{\overset{0}{C}_\alpha(\overline{\Omega}_\varrho)},$$

where $c_0 \equiv \beta^{j_0}(|\sigma| + \delta)^{-j_0}$.

For $1 \leqslant j \leqslant j_0$ we have

$$|(\Gamma^j f_*)(\xi, \eta)| \leqslant \beta^j(\xi\eta\tau_0^{3j}\varrho)^\alpha \|f_*\|_{\overset{0}{C}_\alpha(\overline{\Omega}_\varrho)} \leqslant \beta^j(\xi\eta\varrho)^\alpha \|f_*\|_{\overset{0}{C}_\alpha(\overline{\Omega}_\varrho)}. \tag{3.11}$$

Now by (3.10) and (3.11) we eventually have

$$|\varphi_2(\xi, \eta)| = |(\Lambda^{-1}f_*)(\xi, \eta)| \leqslant |f_*(\xi, \eta)| + \left|\sum_{j=1}^{j_0}(\Gamma^j f_*)(\xi, \eta)\right| +$$

$$+\left|\sum_{j=j_0+1}^{\infty}(\Gamma^j f_*)(\xi, \eta)\right| \leqslant \left(1 + \sum_{j=1}^{j_0}\beta^j + c_0\sum_{j=j_0+1}^{\infty} q^j\right)(\xi\eta\varrho)^\alpha \|f_*\|_{\overset{0}{C}_\alpha(\overline{\Omega}_\varrho)} =$$

$$= \left(1 + \sum_{j=1}^{j_0}\beta^j + c_0\frac{q^{j_0+1}}{1-q}\right)(\xi\eta\varrho)^\alpha \|f_*\|_{\overset{0}{C}_\alpha(\overline{\Omega}_\varrho)} = c_*(\xi\eta\varrho)^\alpha \|f_*\|_{\overset{0}{C}_\alpha(\overline{\Omega}_\varrho)},$$

where

$$c_* \equiv 1 + \sum_{j=1}^{j_0}\beta^j + c_0\frac{q^{j_0+1}}{1-q},$$

from which we obtain the continuity of the operator Λ^{-1} in the space $\overset{0}{C}_\alpha(\overline{\Omega}_n)$, $n \in \mathbb{N}$ and the validity of estimate (3.6).

REMARK 3.1. If $\sigma = 0$, then the inequality $\tau_0^{3\alpha}|\sigma| < 1$ is fulfilled for any $\alpha \geqslant 0$ and, as seen from the proof, in that case Lemma 3.2 holds for all $\alpha \geqslant 0$.

Thus the unique solvability of equation (3.5) on $\overline{\Omega}_n$ is proved for any $n \in \mathbb{N}$. The unique solvability of this equation on the whole $\overline{\mathbb{R}}_+^2$ in the class $\overset{0}{C}_\alpha(\overline{\mathbb{R}}_+^2)$ follows from

LEMMA 3.3. *If the equation*

$$(\Lambda\varphi_2)(\xi, \eta) = f_*(\xi, \eta), \quad (\xi, \eta) \in \overline{\mathbb{R}}_+^2, \tag{3.12}$$

is uniquely solvable on $\overline{\Omega}_n$, *for any* $n \in \mathbb{N}$, *then equation* (3.12) *is uniquely solvable on the whole* $\overline{\mathbb{R}}_+^2$.

Indeed, let $\varphi_{2,n}(\xi, \eta)$ *be the unique solution of equation* (3.12) *on* $\overline{\Omega}_n$, *whose existence has been proved above. Owing to the above establi-*

shed uniqueness of the solution, we have $\varphi_{2,n}(\xi,\eta)=\varphi_{2,m}(\xi,\eta)$, *if* $(\xi,\eta)\in\overline{\Omega}_n$ *and* $m>n$. *Then it is obvious that, for* $(\xi,\eta)\in\overline{\Omega}_n$, $\varphi_2(\xi,\eta)=$ $=\varphi_{2,n}(\xi,\eta)$ *is a unique solution of equation* (3.12). *Thus Lemma* 3.3 *is proved. Finally, by Lemmas* 3.1-3.3 *the following Lemma holds.*

LEMMA 3.4. *If* $\alpha>\alpha_0$, *then equation* (2.11) *is uniquely solvably in the space* $\overset{0}{C}_\alpha(\overline{\mathbb{R}}^2_+)$ *and estimate* (3.6) *holds for the solution.*

By Lemma 3.4 and Remark 2.2, there holds the following

THEOREM 3.1. *Let conditions* (2.10) *be fulfilled and* $0<\tau_0<1$. *If the equality* $\sigma=0$ *holds, then problem* (1.5), (1.6) *is uniquely solvable in the class* $\overset{0}{C}{}^l_\alpha(\overline{D})$ *for all* $\alpha\geqslant 0$. *If however* $\sigma\neq 0$, *then problem* (1.5), (1.6) *is uniquely solvably in the class* $\overset{0}{C}{}^l_\alpha(\overline{D})$ *for* $\alpha>\alpha_0$.

On account of inequality (3.6) and the function f_*, written in terms of the functions f_i, $i=4,5,6$, which, in their turn, are the linear combinations of the functions f_i, $i=1,2,3$, and F, we can easily show that

$$\|\varphi_2\|_{\overset{0}{C}_\alpha(\overline{\Omega}_n)}\leqslant c\left(\sum_{i=1}^{3}\|f_i\|_{\overset{0}{C}_\alpha(\overline{\Omega}_n)}+\|F\|_{\overset{0}{C}_\alpha(\overline{D}_n)}\right), \tag{3.13}$$

where c is a positive constant not depending on the functions f_i, $i=1$, 2, 3 and F.

Moreover, because of equalities (2.9), (2.8) it follows that estimates analogous to (3.13) are also valid for the functions φ_i, $i=1,3$, and v_i, $i=1,2,3$. Finally, by virtue of formula (2.6) we can easily see that for a regular solution of problem (1.5), (1.6) of the class $\overset{0}{C}{}^l_\alpha(\overline{D})$, $\alpha>\alpha_0$ the estimate

$$\|u\|_{\overset{0}{C}{}^l_\alpha(\overline{D}_n)}\leqslant c\left(\sum_{i=1}^{3}\|f_i\|_{\overset{0}{C}_\alpha(\overline{\Omega}_n)}+\|F\|_{\overset{0}{C}_\alpha(\overline{D}_n)}\right), \tag{3.14}$$

holds, where c is a positive constant not depending on the functions f_i, $i=1,2,3$ and F. These estimate (3.14) imply that a regular solution of problem (1.5), (1.6) is stable in the space $\overset{0}{C}{}^l_\alpha(\overline{D})$, $\alpha>\alpha_0$.

The following question arises naturally. Is it possible or not to investigate problem (1.5), (1.6) by the method considered above when equation (1.5) or the boundary conditions (1.6) contains the dominating lower terms? In general, this problem seems to be quite difficult. In this case, in the investigation of the problem arise significant difficulties. The reason is that the integral operators, which appear when one reduces problem (1.5), (1.6) to a system of integro-functional equations by the method offered above, are not of Volterra type. As we shall show below, when the plane faces S_i, $i=1,2,3$, of the trihedral angle D are characteristic for the equation under consideration and the dominating lower terms occur in the boundary conditions only, the investigation of this problem is somewhat easier. In the general case, i.e., when the dominating lower terms occur in equation (1.5) as well, one can probably use succesfully the Riemann method [11].

4. – Goursat type problem for equation (1.1).

Denote by $\overset{0}{C}{}^{1,1,1}(\overline{D})$ the space $\overset{0}{C}{}^{l}(\overline{D})$ from § 1, for $l_i=e_i$, $i=1,2,3$. In the domain D let us consider a Goursat type problem for equation (1.1) formulated as follows: in the domain D, find a regular solution u of (1.1) from the class $\overset{0}{C}{}^{1,1,1}(\overline{D})$ satisfyng the following boundary conditions:

$$\left(\sum_{i,j=1,\,i<j}^{3} M_{ij}^{k} u_{x_i x_j} + \sum_{i=1}^{3} M_i^k u_{x_i} + M^k u\right)\Bigg|_{S_k} = f_k\,, \tag{4.1}$$

where F, M_{ij}^k, M_i^k, M^k and f_k, $i,j,k=1,2,3$, $i<j$ are the given real functions.

By considering problem (1.1), (4.1) in the space $\overset{0}{C}{}^{1,1,1}(\overline{D})$ we require that $F\in\overset{0}{C}(\overline{D})$, $f_k\in\overset{0}{C}(\overline{S}_k)$, M_{ij}^k, M_i^k, $M^k\in C(\overline{S}_k)$, $i,j,k=1,2,3$, $i<j$.

It is well-known that for regular solutions of equations (1.1) of the class $\overset{0}{C}{}^{1,1,1}(\overline{D})$ we have the following integral representation (see, e.g., [10]):

$$u(x)=\int_0^{x_1}\!\!\int_0^{x_2}\varphi(\xi_1,\xi_2)\,d\xi_1\,d\xi_2+\int_0^{x_1}\!\!\int_0^{x_3}\psi(\xi_1,\xi_3)\,d\xi_1\,d\xi_3+ \tag{4.2}$$

$$+\int_0^{x_2}\!\!\int_0^{x_3}\chi(\xi_2,\xi_3)\,d\xi_2\,d\xi_3+\int_0^{x_1}\!\!\int_0^{x_2}\!\!\int_0^{x_3}F(\xi_1,\xi_2,\xi_3)\,d\xi_1\,d\xi_2\,d\xi_3\,,$$

where

$$u_{x_1x_2}(x_1,x_2,0)\equiv\varphi(x_1,x_2)\,,\qquad u_{x_1x_3}(x_1,0,x_3)\equiv\psi(x_1,x_3)\,,$$

$$u_{x_2x_3}(0,x_2,x_3)\equiv\chi(x_2,x_3)\,,\quad (x_1,x_2),\quad (x_1,x_3),\quad (x_2,x_3)\in\overline{\mathbb{R}}^2\,.$$

Substituting the integral representation (4.2) into the boundary conditions (4.1), with the respect to the unknoun functions φ, ψ, χ from the class $\overset{0}{C}(\overline{\mathbb{R}^2_+})$ we obtain the following splitted system of Volterra integral equations of third kind:

$$
(4.3)\quad \begin{cases}
M^1_{23}(x_2, x_3)\chi(x_2, x_3) + \\
+M^1_2(x_2, x_3)\int\limits_0^{x_3}\chi(x_2, \xi_3)\,d\xi_3 + M^1_3(x_2, x_3)\int\limits_0^{x_2}\chi(\xi_2, x_3)\,d\xi_2 + \\
+M^1(x_2, x_3)\int\limits_0^{x_2}\int\limits_0^{x_3}\chi(\xi_2, \xi_3)\,d\xi_2\,d\xi_3 = \tilde{f}_1(x_2, x_3),\ (x_2, x_3)\in\overline{\mathbb{R}^2_+}, \\
M^2_{13}(x_1, x_3)\psi(x_1, x_3) + \\
+M^2_1(x_1, x_3)\int\limits_0^{x_3}\psi(x_1, \xi_3)\,d\xi_3 + M^2_3(x_1, x_3)\int\limits_0^{x_1}\psi(\xi_1, x_3)\,d\xi_1 + \\
+M^2(x_1, x_3)\int\limits_0^{x_1}\int\limits_0^{x_3}\psi(\xi_1, \xi_3)\,d\xi_1\,d\xi_3 = \tilde{f}_2(x_1, x_3),\ (x_1, x_3)\in\overline{\mathbb{R}^2_+}, \\
M^3_{12}(x_1, x_2)\varphi(x_1, x_2) + \\
+M^3_1(x_1, x_2)\int\limits_0^{x_2}\varphi(x_1, \xi_2)\,d\xi_2 + M^3_2(x_1, x_2)\int\limits_0^{x_1}\varphi(\xi_1, x_2)\,d\xi_1 + \\
+M^3(x_1, x_2)\int\limits_0^{x_1}\int\limits_0^{x_2}\varphi(\xi_1, \xi_2)\,d\xi_1\,d\xi_2 = \tilde{f}_3(x_1, x_2),\ (x_1, x_2)\in\overline{\mathbb{R}^2_+},
\end{cases}
$$

where $\tilde{f}_i \in \overset{0}{C}(\overline{\mathbb{R}^2_+})$ are the known functions expressed by F and f_i, $i = 1, 2, 3$.

Let the following conditions be fulfilled:

$$
(4.4)\quad M^k_{ij}(\xi, \eta) \neq 0, \quad (\xi, \eta)\in\overline{\mathbb{R}^2_+}, \quad i, j, k = 1, 2, 3,\ i<j,\ k\neq i,\ j.
$$

Then (4.3) is a system of Volterra integral equations of second kind, whose solution exists and is unique. Obviously, by hypotheses (4.4) problem (1.1), (4.1) is equivalent to system (4.3). From this we conclude that under hypotheses (4.4) problem (1.1), (4.1) is uniquely solvable in the class $\overset{0}{C}{}^{1,1,1}(\overline{D})$.

5. – Influence of the lower terms of the boundary conditions on the correctness of the statement of problem (1.1), (4.1) when conditions (4.4) are violated.

As the example of the equation $u_{x_1x_2x_3} = 0$ shows, problem (1.1), (4.1) may appear to be ill-posed when conditions (4.4) are violated. Below we shall show that the existence of lower terms in the boundary conditions (4.1) may affect the correctness of the statement of problem (1.1), (4.1).

For simplicity let

$$M_1^3\,,\, M_2^3\,,\, M^3 = \text{const}\,, \qquad M_{12}^3 \equiv 0\,,$$

$$M_{23}^1\,,\, M_{13}^2 \neq 0\,, \qquad |M_1^3| + |M_2^3| + |M^3| \neq 0\,.$$

Without loss of generality we may assume $|M_1^3| + |M_2^3| \neq 0$, since, otherwise, this can be achieved by differentiating the boundary condition (4.1) with respect to x_1 or x_2.

Denoting

$$\int_0^{x_1}\int_0^{x_2} \varphi(\xi_1, \xi_2)\, d\xi_1\, d\xi_2 \equiv \Phi(x_1, x_2)\,, \qquad (x_1, x_2) \in \overline{\mathbb{R}}_+^2\,,$$

by virtue of the third equation of (4.3), with respect to the function Φ we obtain a partial differential equation of first order

$$\frac{\partial \Phi}{\partial l} + M^3 \Phi = \tilde{f}_3 \tag{5.1}$$

with the boundary conditions

$$\Phi(x_1, 0) = 0\,, \qquad \Phi(0, x_2) = 0\,, \qquad x_1\,,\, x_2 \in \overline{\mathbb{R}}_+\,, \tag{5.2}$$

where

$$\frac{\partial \Phi}{\partial l} \equiv M_1^3\, \Phi_{x_1} + M_2^3\, \Phi_{x_2}\,.$$

It is well-known (see, e.g., [12]), that problem (5.1), (5.2) is correct, for $M_1^3 M_2^3 > 0$, and may not be correct otherwise.

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