RENDICONTI
del
SEMINARIO MATEMATICO
della
UNIVERSITÀ DI PADOVA

R. K. RAINA

MAMTA BOLIA

Characterization properties for starlike and convex functions involving a class of fractional integral operators

Rendiconti del Seminario Matematico della Università di Padova, tome 97 (1997), p. 61-71

<http://www.numdam.org/item?id=RSMUP_1997__97__61_0>

REND. SEM. MAT. UNIV. PADOVA, Vol. 97 (1997)

Characterization Properties for Starlike and Convex Functions Involving a Class of Fractional Integral Operators (*).

R. K. RAINA (**) - MAMTA BOLIA (***)

ABSTRACT - This paper studies the characterization properties satisfied by a class of fractional integrals of certain analytic functions in the open unit disk to be starlike or convex. Further characterization theorems associated with the Hadamard product (or convolution) are also investigated

1. - Introduction.

A new class of fractional integral operators with a particular case of Fox's H-function in the kernal was introduced by Kiryakova [4] and [5] (see also [3]). Subsequently, this generalized fractional integral operator was extended and studied in a wider context on McBride spaces $F_{p,\mu}$ and $F'_{p,\mu}$ by Raina and Saigo [9] (see also [12]). Recently, several classes of distortion theorems have been obtained involving certain fractional integral operators by Srivastava, Saigo and Owa [15]. Distortion inequalities associated with the new class of fractional integral operators [5] have very recently been considered by Raina and Bolia [10].

This paper is devoted to studying the sufficiency conditions satisfied by a class of fractional integral operators (defined by eq. (5) below)

(*) Dedicated to the memory of Late Professor Bertram Ross of New Haven University, U.S.A.

(**) Indirizzo dell'A.: Department of Mathematics, C.T.A.E. Campus Udaipur, Udaipur, 313 001 Rajasthan, India.

(***) Indirizzo dell'A.: Department of Mathematics, College of Science, Sukhadia University, Udaipur, 313 001 Rajasthan, India.

AMS Subject Classification (1991): 26A33, 30C45, 30C99, 33C40

of certain analytic functions in the open unit disk to be starlike or convex. Further characterization properties associated with the Hadamard product (or convolution) are also considered. The class of fractional integral operators incorporates several well-known integral operators like; the Riemann-Liouville operator, Kober fractional integral, Erdélyi fractional integral, Love fractional integral, Saigo fractional integral and their various generalizations. One may refer to [9] for comprehensive details of these above special cases. The results of this paper are widely applicable to several fractional integral operators including the one discussed in [7].

The paper is organised as follows: Section 2 gives preliminary details and definitions of starlike and convex analytic functions, and generalized fractional integral operators. In Section 3 we state first the results which are required in our sequel and then establish our main characterization properties in the form of Theorems 1 and 2. Lastly, Section 4 considers the characterization properties associated with the Hadamard product.

2. - Preliminaries and definitions.

Let $T(n)$ denote the class of functions of the form

$$f(z) = z + \sum_{k=n+1}^{\infty} a_k z^k \qquad (n \in N), \tag{1}$$

which are analytic in the unit disk $U = \{z: |z| < 1\}$. Then a function $f(z) \in T(n)$ is said to be in the class $S(n)$ if and only if

$$\mathrm{Re}\left\{\frac{zf'(z)}{f(z)}\right\} > 0 \qquad (z \in U). \tag{2}$$

Further, a function $f(z) \in T(n)$ is said to be in the class $K(n)$ if and only if

$$\mathrm{Re}\left\{1 + \frac{zf''(z)}{f'(z)}\right\} > 0 \qquad (z \in U). \tag{3}$$

It is easily verified that

$$f(z) \in K(n) \Leftrightarrow zf'(z) \in S(n) \qquad (\forall n \in N), \tag{4}$$

and that $S(1)$ and $K(1)$ are the well known classes of starlike and convex functions.

We give now below the definition of a class of fractional integral operators [3] and [4] (see also [5], [9] and [12]):

DEFINITION. Let $m \in N$, $\beta_k \in R_+$ and $\gamma_k, \delta_k \in C$, $\forall k = 1, \ldots, m$; with $\sum_{k=1}^{m} \operatorname{Re}(\delta_k) > 0$. Then the integral operator

$$(5)\quad \begin{cases} I_{(\beta_m);\, m}^{(\gamma_m),(\delta_m)} f(z) = I_{(\beta_1, \ldots, \beta_m);\, m}^{(\gamma_1, \ldots, \gamma_m),(\delta_1, \ldots, \delta_m)} f(z) = \\ \qquad = \dfrac{1}{z} \displaystyle\int_0^z H_{m, m}^{m, 0}\left[\frac{t}{z} \,\middle|\, \begin{matrix} (\gamma_k + \delta_k + 1 - 1/\beta_k,\, 1/\beta_k)_{1, m} \\ (\gamma_k + 1 - 1/\beta_k,\, 1/\beta_k)_{1, m} \end{matrix} \right] f(t)\, dt, \\ \qquad\qquad\qquad\qquad\qquad\qquad \text{for } \sum_1^m \delta_k > 0; \\ \qquad = f(z), \qquad \delta_1 = \ldots = \delta_m = 0, \end{cases}$$

is said to be a multiple fractional integral operator of Riemann-Liouville type of multiorder $\delta = (\delta_1, \ldots, \delta_m)$. Here and elsewhere, the set of nonnegative integers is denoted by $N \cdot R$ means the real field and $R_+ = (0, \infty)$, and C denotes the complex number field.

The H-function involving in (5) is a special case of Fox's $H_{P,Q}^{M,N}$-function [2] (see also [8, Sect. 8.3]) which is defined as follows:

Let $m, n, p, q \in N$ such that $0 \leqslant m \leqslant q$, $0 \leqslant n \leqslant p$, and $a_j, b_i \in C$ and $\alpha_i, \beta_j \in R_+$ ($j = 1, \ldots, q$; $i = 1, \ldots, p$). The H-function occurring in the paper is defined by ([2, p. 408]):

$$(6)\quad H_{p,q}^{m,n}[z] = H_{p,q}^{m,n}\left[z \,\middle|\, \begin{matrix} (a_j, \alpha_j)_{1, p} \\ (b_j, \beta_j)_{1, q} \end{matrix} \right] = \frac{1}{2\pi i} \int_L \theta(s) z^s\, ds, \qquad i = \sqrt{-1},$$

where

$$(7)\quad \theta(s) = \frac{\prod_{j=1}^{m} \Gamma(b_j - \beta_j s) \prod_{j=1}^{n} \Gamma(1 - a_j + \alpha_j s)}{\prod_{j=m+1}^{q} \Gamma(1 - b_j + \beta_j s) \prod_{j=n+1}^{p} \Gamma(a_j - \alpha_j s)},$$

the contour L is suitably chosen and an empty product, if it occurs, is taken to be one. The details of this function may be found in [1], [6, Chapter 1], [8, Sect. 8.3] and [14, Chapter 2]. The symbol $(\lambda)_k$ denotes the usual Pochhammer symbol

$$(\lambda)_k = \frac{\Gamma(\lambda + k)}{\Gamma(\lambda)} = \begin{cases} 1, & \text{if } k = 0, \\ k(k+1)\ldots(\lambda + k - 1), & \forall k \in N. \end{cases}$$

3. - Chracterization properties.

We first introduce a class of fractional integral operators as follows:

The fractional integral operator $R^{(\gamma_m),(\delta_m)}_{(\beta_m);\, m} f(z)$ is defined by

$$R^{(\gamma_m),(\delta_m)}_{(\beta_m);\, m} f(z) = \prod_{j=1}^{m} \left\{ \frac{\Gamma(\beta_j + \gamma_j + \delta_j + 1)}{\Gamma(\beta_j + \gamma_j + 1)} \right\} I^{(\gamma_m),(\delta_m)}_{(\beta_m^{-1});\, m} f(z), \tag{8}$$

where $m \in N$, $\beta_j, \delta_j \in R_+$ and $\gamma_j \in R$, $\forall j = 1, \ldots, m$; such that

$$\min\{1 + \beta_j + \gamma_j + \delta_j,\, 1 + \beta_j + \gamma_j\} > 0 \quad (\forall j = 1, \ldots, m), \tag{9}$$

and n is a positive integer so chosen that

$$\prod_{j=1}^{m} \left\{ \frac{(1 + \gamma_j + \beta_j(n+1))_{\beta_j}}{(1 + \gamma_j + \delta_j + \beta_j(n+1))_{\beta_j}} \right\} \leqslant 1. \tag{10}$$

In order to establish the characterization properties satisfied by the operator (8) of certain analytic functions, we require the following results:

LEMMA 1. *If the function $f(z)$ defined by* (1) *satisfies*

$$\sum_{k=n+1}^{\infty} k|a_k| \leqslant 1 \quad (n \in N), \tag{11}$$

then $f(z) \in S(n)$. The equality in (11) *is attained by the function*

$$g_1(z) = z + \frac{z^k}{k} \quad (k \geqslant n+1;\, n \in N). \tag{12}$$

LEMMA 2. *If the function $f(z)$ defined by* (1) *satisfies*

$$\sum_{k=n+1}^{\infty} k^2|a_k| \leqslant 1 \quad (n \in N), \tag{13}$$

then $f(z) \in K(n)$. *The equality in* (13) *is attained by the function*

$$g_2(z) = z + \frac{z^k}{k^2} \qquad (k \geqslant n+1;\ n \in N). \tag{14}$$

Lemmas 1 and 2 stated in [7, p. 420] are easy consequences of the corresponding results due to Silverman [13].

LEMMA 3 [5, p. 261]. *Let*

$$\gamma_j > -\frac{\mu}{\beta_j} - 1, \qquad \delta_j \geqslant 0, \qquad \forall j = 1, \ldots, m. \tag{15}$$

Then the operator $I^{(\gamma_m),(\delta_m)}_{(\beta_m);\, m}$ *maps the class* $\mathcal{H}_\mu(G)$ *into itself preserving the power functions* $f(z) = z^p$ *up to a constant multiplier, namely*:

$$I^{(\gamma_m),(\delta_m)}_{(\beta_m);\, m}\{z^p\} = \prod_{j=1}^{m} \left\{ \frac{\Gamma(p/\beta_j + \gamma_j + 1)}{\Gamma(p/\beta_j + \gamma_j + \delta_j + 1)} \right\} z^p, \qquad p \geqslant \mu. \tag{16}$$

We prove now the following:

THEOREM 1. *Under the conditions stated in* (8), (9), *and* (10), *let the function* $f(z)$ *defined by* (1) *satisfy*

$$\sum_{k=n+1}^{\infty} k|a_k| \leqslant \prod_{j=1}^{m} \left\{ \frac{(\beta_j + \gamma_j + \delta_j + 1)_{n\beta_j}}{(\beta_j + \gamma_j + 1)_{n\beta_j}} \right\} \qquad (n \in N). \tag{17}$$

Then

$$R^{(\gamma_m),(\delta_m)}_{(\beta_m);\, m} f(z) \in S(n).$$

PROOF. Applying suitably Lemma 3, we have from (1) and (8):

$$R^{(\gamma_m),(\delta_m)}_{(\beta_m);\, m} f(z) = z + \sum_{k=n+1}^{\infty} \psi(k)\, a_k z^k, \tag{18}$$

where

$$\psi(k) = \prod_{j=1}^{m} \left\{ \frac{(\beta_j + \gamma_j + 1)_{(k-1)\beta_j}}{(\beta_j + \gamma_j + \delta_j + 1)_{(k-1)\beta_j}} \right\} \qquad (k \geqslant n+1;\ n \in N). \tag{19}$$

The function $\psi(k)$ is a non-increasing function of k, since we observe that

$$(20) \qquad 0 < \psi(k) \leqslant \psi(n+1) = \prod_{j=1}^{m} \left\{ \frac{(\beta_j + \gamma_j + 1)_{n\beta_j}}{(\beta_j + \gamma_j + \delta_j + 1)_{n\beta_j}} \right\} \qquad (n \in N),$$

under the conditions stated in (8), (9), and (10). Now (17) and (20) gives

$$(21) \qquad \sum_{k=n+1}^{\infty} k\psi(k) |a_k| \leqslant \psi(n+1) \sum_{k=n+1}^{\infty} k |a_k| \leqslant 1 .$$

Therefore, by Lemma 1, we conclude that

$$R_{(\beta_m);\, m}^{(\gamma_m),(\delta_m)} f(z) \in S(n),$$

and the theorem is proved.

REMARK 1. A function $f(z)$ satisfying (17) can be considered to be of the form

$$(22) \qquad g_3(z) = z + \frac{1}{k} \prod_{j=1}^{m} \left\{ \frac{(\beta_j + \gamma_j + \delta_j + 1)_{(k-1)\beta_j}}{(\beta_j + \gamma_j + 1)_{(k-1)\beta_j}} \right\} z^k$$

$$(k \geqslant n+1;\ n \in N,\ z \in U).$$

In an analogous manner, we can prove with the help of Lemma 2 the following result which characterizes the class $K(n)$:

THEOREM 2. *Under the constraints stated in* (8), (9) *and* (10), *let the function* $f(z)$ *defined by* (1) *satisfy*

$$(23) \qquad \sum_{k=n+1}^{\infty} k^2 |a_k| \leqslant \prod_{j=1}^{m} \left\{ \frac{(\beta_j + \gamma_j + \delta_j + 1)_{n\beta_j}}{(\beta_j + \gamma_j + 1)_{n\beta_j}} \right\} \qquad (n \in N).$$

Then

$$R_{(\beta_m);\, m}^{(\gamma_m),(\delta_m)} f(z) \in K(n).$$

REMARK 2. A function $f(z)$ satisfying (23) can be considered to be of the form

$$(24)\qquad g_4(z) = z + \frac{1}{k^2} \prod_{j=1}^{m} \left\{ \frac{(\beta_j + \gamma_j + \delta_j + 1)_{(k-1)\beta_j}}{(\beta_j + \gamma_j + 1)_{(k-1)\beta_j}} \right\} z^k$$

$$(k \geqq n+1;\ n \in N,\ z \in U).$$

4. – Characterization properties associated with the Hadamard product.

Let $f_i(z) \in T_n$ $(i = 1, 2)$ be given by

$$(25)\qquad f_i(z) = z + \sum_{k=n+1}^{\infty} a_{i,k} z^k \qquad (n \in N).$$

Then the Hadamard product or convolution

$$(26)\qquad (f_i * f_2)(z) = z + \sum_{k=n+1}^{\infty} a_{1,k} a_{2,k} z^k \qquad (n \in N).$$

We recall here the following result due to Ruscheweyh and Sheil-Small [11]:

LEMMA 4. *Let $g(z)$, $h(z)$ be analytic in U and satisfy the condition*

$$g(0) = h(0) = 0.$$

Suppose also that

$$(27)\qquad g(z) * \left\{ \frac{1 + abz}{1 - bz} h(z) \right\} \neq 0 \qquad (z \in U - \{0\}),$$

for a and b on the unit circle. Then, for a function $F(z)$ analytic in U and satisfying the inequality:

$$(28)\qquad \begin{cases} \operatorname{Re}\{F(z)\} > 0 & (z \in U), \\ \operatorname{Re}\left\{ \dfrac{(g * Fh)(z)}{(g * h)(z)} \right\} > 0 & (z \in U). \end{cases}$$

By making use of Lemma 4, we prove the following:

THEOREM 3. *Let the conditions stated in* (8), (9) *and* (10) *hold, and suppose that the function* $f(z)$ *defined by* (1) *is such that* $f(z) \in S(n)$ *and satisfies*

$$w(z) * \left\{\frac{1+abz}{1-bz} f(z)\right\} \neq 0 \qquad (z \in U - \{0\}), \tag{29}$$

for a and b on the unit circle, where

$$w(z) = z + \sum_{k=n+1}^{\infty} \prod_{j=1}^{m} \left\{ \frac{(\beta_j + \gamma_j + 1)_{(k-1)\beta_j}}{(\beta_j + \gamma_j + \delta_j + 1)_{(k-1)\beta_j}} \right\} z^k \qquad (n \in N). \tag{30}$$

then $R^{(\gamma_m),(\delta_m)}_{(\beta_m);\, m} f(z) \in S(n)$ *also.*

PROOF. We observe from (18) and (30) that

$$R^{(\gamma_m),(\delta_m)}_{(\beta_m);\, m} f(z) = \tag{31}$$

$$= z + \sum_{k=n+1}^{\infty} \prod_{j=1}^{m} \left\{ \frac{(\beta_j + \gamma_j + 1)_{(k-1)\beta_j}}{(\beta_j + \gamma_j + \delta_j + 1)_{(k-1)\beta_j}} \right\} a_k z^k = (w * f)(z).$$

This gives

$$\frac{z(R^{(\gamma_m),(\delta_m)}_{(\beta_m);\, m} f(z))'}{R^{(\gamma_m),(\delta_m)}_{(\beta_m);\, m} f(z)} = \frac{z(w*f)'(z)}{(w*f)(z)} = \frac{((w*(zf'))(z)}{(w*f)(z)} . \tag{32}$$

By putting $g(z) = w(z)$, $h(z) = f(z)$, and $F(z) = zf'(z)/f(z)$ in Lemma 4 above, we find that

$$\operatorname{Re}\left\{ \frac{z(R^{(\gamma_m),(\delta_m)}_{(\beta_m);\, m} f(z))'}{R^{(\gamma_m),(\delta_m)}_{(\beta_m);\, m} f(z)} \right\} > 0 \Rightarrow R^{(\gamma_m),(\delta_m)}_{(\beta_m);\, m} f(z) \in S(n). \tag{33}$$

THEOREM 4. *Under the conditions stated in* (8), (9) *and* (10), *if the function* $f(z)$ *defined by* (1) *is such that* $f \in K(n)$ *and*

$$w(z) * \left\{\frac{1+abz}{1-bz} zf'(z)\right\} \neq 0 \qquad (z \in U - \{0\}), \tag{34}$$

for a and b on the unit circle, where $w(z)$ *is given by* (30), *then*

$$R^{(\gamma_m),(\delta_m)}_{(\beta_m);\, m} f(z) \in K(n) \qquad \textit{also}.$$

PROOF. From (4) and Theorem (3), it follows that

$$\begin{aligned} f(z) \in K(n) &\Leftrightarrow zf'(z) \in S(n) \\ &\Rightarrow R^{(\gamma_m),(\delta_m)}_{(\beta_m);\, m}\, zf'(z) \in S(n) \\ &\Leftrightarrow (w * zf'(z)) \in S(n) \\ &\Leftrightarrow z(w * f)'(z) \in S(n) \\ &\Leftrightarrow z(w * f)(z) \in K(n) \\ &\Leftrightarrow R^{(\gamma_m),(\delta_m)}_{(\beta_m);\, m} f(z) \in K(n), \end{aligned}$$

which proves Theorem 4.

Our next characterization property uses the following result:

LEMMA 5 (Ruscheweyh and Sheil-Small [11]). *Let $g(z)$ be convex and let $h(z)$ be starlike in U. Then for each function $F(z)$ analytic in U satisfies the inequality*:

$$\begin{cases} \operatorname{Re}\{F(z)\} > 0 \quad (z \in U), \\ \operatorname{Re}\left\{ \dfrac{(g * Fh)(z)}{(g * h)(z)} \right\} > 0 \quad (z \in U). \end{cases} \tag{35}$$

THEOREM 5. *Under the conditions* (8), (9) *and* (10),

$$f(z) \in S(n) \text{ and } w(z) \in K(n) \Rightarrow \{R^{(\gamma_m),(\delta_m)}_{(\beta_m);\, m} f(z)\} \in S(n),$$

where $w(z)$ is given by (30).

PROOF. The theorem which is based upon Lemma 5 above can be proved analogously to Theorem 3.

Lastly, we also have the following:

THEOREM 6. *Under the conditions* (8), (9) *and* (10),

$$f(z) \in K(n) \text{ and } w(z) \in K(n) \Rightarrow R^{(\gamma_m),(\delta_m)}_{(\beta_m);\, m} f(z) \in K(n),$$

where $w(z)$ is given by (30).

REMARK 3. It is observed that the function $w(z)$ defined by (30) can be expressed in terms of the generalized hypergeometric function

${}_{m+1}F_m$ [6, p. 43]:

$$(36) \quad w(z) = z + \prod_{j=1}^{m} \left\{ \frac{(\beta_j + \gamma_j + 1)_n}{(\beta_j + \gamma_j + \delta_j + 1)_n} \right\} \cdot$$

$$\cdot \; {}_{m+1}F_m \left[\begin{matrix} 1, & \beta_1 + \gamma_1 + n + 1, \ldots, \beta_m + \gamma_m + n + 1 & ; \\ & \beta_1 + \gamma_1 + \delta_1 + n + 1, \ldots, \beta_m + \gamma_m + \delta_m + n + 1 & ; \end{matrix} \; z \right],$$

which converges absolutely in U. Further more, the series ${}_{m+1}F_m$ in (36) converges also for $z = 1$ when $\sum_{j=1}^{m} \delta_j > 1$.

Several classes of characterization properties for various fractional integral operators of certain analytic functions can be derived from the results given in this paper. For instance, by noting the connection ([12, p. 142]) that

$$I_{(1,1);2}^{(0,\eta-\beta),(-\beta,\alpha+\beta)} f(z) = I_{0,z}^{\alpha,\beta,\gamma} f(z),$$

where $I_{0,z}^{\alpha,\beta,\eta}$ denotes the Saigo operator (see [9]), the results of the paper [7] would follow from the corresponding results presented in this paper.

Acknowledgement. The work of the first author was supported by Department of Science and Technology, Govt. of India, under Grant No. DST/MS/PM-001/93.

REFERENCES

[1] B. L. J. BRAAKSMA, *Asymptotic expansions and analytic continuations for a class of Barnes integrals*, Compos. Math., **15** (1964), pp. 239-341.
[2] C. FOX, *The G- and H-functions as symmetrical Fourier kernels*, Trans. Amer. Math. Soc., **98** (1961), pp. 395-429.
[3] S. L. KALLA - V. S. KIRYAKOVA, *An H-function generalized fractional calculus based upon compositions of Erdélyi-Kober operators in L_p*, Math. Japon., **35** (1990), pp. 1151-1171.
[4] V. S. KIRYAKOVA, *Fractional integration operators involving Fox's $H_{m,m}^{m,0}$-function*, C. R. Bulg. Acad. Sci., **41** (11) (1988), pp. 11-14.
[5] V. S. KIRYAKOVA, *Generalized $H_{m,m}^{m,0}$-function fractional integration operators in some classes of analytic functions*, Math. Vesnik, **40** (1988), pp. 259-266.

[6] A. M. MATHAI - R. K. SAXENA, *The H-Function with Applications in Statistics and other Disciplines*, Halsted Press, New York, London, Sydney, Toronto (1978).

[7] S. OWA - M. SAIGO - H. M. SRIVASTAVA, *Some characterization theorems for starlike and convex functions involving a certain fractional integral operator*, J. Math. Anal. Appl., **140** (2) (1989), pp. 419-426.

[8] A. P. PRUDNIKOV - JU. A. BRYCHKOV - O. I. MARICHEV, *Integrals and Series*, Vol. 3, *More Special Functions*, Gordon and Breach, New York, Philadelphia, London, Paris, Montreux, Tokyo, Melbourne (1990).

[9] R. K. RAINA - M. SAIGO, *Fractional calculus operators involving Fox's H-function on spaces $F_{p,\mu}$ and $F'_{p,\mu}$*, in *Advances in Fractional Calculus* (R. N. KALIA, Editor), Global Publ., Sauk Rapids (1993), pp. 219-229.

[10] R. K. RAINA - M. BOLIA, *On distortion theorems involving generalized fractional calculus operators*, Tamkang J. Math., **27** (3) (1996), pp. 233-241.

[11] S. RUSCHEWEYH - T. SHEIL-SMALL, *Hadamard products of schlicht functions and the Pólya-Schoenberg conjecture*, Comment Math. Helv., **48** (1973), pp. 119-135.

[12] M. SAIGO - R. K. RAINA - A. A. KILBAS, *On generalized fractional calculus operators and their compositions with axisymmetric differential operators of the potential theory on spaces $F_{p,\mu}$ and $F'_{p,\mu}$*, Fukuoka Univ. Sci. Reports, **23** (1993), pp. 133-154.

[13] H. SILVERMAN, *Univalent functions with negative coefficients*, Proc. Amer. Math. Soc., **51** (1975), pp. 109-116.

[14] H. M. SRIVASTAVA - K. C. GUPTA - S. P. GOYAL, *The H-Functions of One and Two Variables with Applications*, South Asian Publ., New Delhi, Madras (1982).

[15] H. M. SRIVASTAVA - M. SAIGO - S. OWA, *A class of distortion theorems involving certain operators of fractional calculus*, J. Math. Anal. Appl., **131** (2) (1988), pp. 412-420.

Manoscritto pervenuto in redazione il 28 giugno 1995.