RENDICONTI
del
SEMINARIO MATEMATICO
della
UNIVERSITÀ DI PADOVA

J. A. LÓPEZ MOLINA
M. J. RIVERA

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Rendiconti del Seminario Matematico della Università di Padova, tome 96 (1996), p. 105-113
<http://www.numdam.org/item?id=RSMUP_1996__96__105_0>

REND. SEM. MAT. UNIV. PADOVA, Vol. 96 (1996)

A Note on Natural Tensor Products Containing Complemented Copies of c_0.

J. A. LÓPEZ MOLINA(*) - M. J. RIVERA(*)(**)

ABSTRACT - Let H be a Fréchet lattice containing a positive sequence equivalent to the unit basis of c_0. We prove that the natural tensor product $H \widehat{\bigotimes_{\mu}} X$ contains a complemented subspace isomorphic to c_0 for every infinite dimensional Banach space X, which generalizes a previous result of Cembranos and Freniche.

0. - Introduction.

Cembranos [4] shows in 1984 that $\mathcal{C}(K, X)$ contains a complemented subspace isomorphic to c_0 if K is an infinite compact Hausdorff space and X an infinite dimensional Banach space. The theorem was extended in 1986 by E. Saab and P. Saab [14] who proved that the injective tensor product $X \widehat{\bigotimes_{\varepsilon}} Y$ of two infinite dimensional Banach spaces X and Y contains a complemented copy of c_0 if X or Y contains c_0, using a proof inspired by the Cembranos's one. However both results have been obtained also in 1984, indeed in a little more general version, in a paper of Freniche [8]. On the other hand, Emmanuele [7] showed in 1988 that if $(\Omega, \mathcal{A}, \nu)$ is a not purely atomic measure space and X is a Banach space containing c_0, the space $L^p(\mu, X)$, $1 \leq p < \infty$, of Bochner integrable X-valued functions, contains a complemented copy of c_0.

(*) Indirizzo degli AA.: E.T.S. Ingenieros Agrónomos, Camino de Vera, 46071 Valencia, Spain.
(**) Supported in part by the DGICYT, project PB91-0538.

The spaces used by Freniche, Cembranos and Emmanuele are particular cases of the Levin natural tensor product $H \widehat{\underset{\mu}{\otimes}} X$ of a Banach lattice H and a Banach space H (see the original paper [11] and also [5]). *The main purpose of this paper is to extend the above results to this more general setting.* Our method will be useful also to deal with the same problem in the case of the Saphar tensor product $X \widehat{\underset{d_\infty}{\otimes}} Y$ (see [15]).

The notation and terminology is standard (see [10] for general theory of locally convex spaces and [1] for Banach lattices). If H is a Fréchet space (resp. a Fréchet lattice), $(\|\cdot\|_s)_{s=1}^{\infty}$ will be an increasing fundamental system of continuous seminorms (resp. continuous lattice seminorms) in H and $U_s = \{x \in H \mid \|x\|_s \leq 1\}$. The usual Schauder basis of ℓ^1 and c_0 will be denoted by $(e_n)_{n=1}^{\infty}$. A sequence (h_i) in the Fréchet space H is said to be weakly absolutely summable if, for every $n \in \mathbb{N}$, the inequality

$$\varepsilon_n((h_i)) = \sup_{h' \in U_n^0} \sum_{i=1}^{\infty} |\langle h_i, h' \rangle| < \infty$$

holds. We denote by $\ell^1(H)$ the set of all weakly absolutely summable sequences in H and by $\ell_0^1(H)$ the set of all $(h_i) \in \ell^1(H)$ such that, for every $n \in \mathbb{N}$, we have

$$\lim_{k \to \infty} \varepsilon_n((0, 0, \ldots, 0, h_k, h_{k+1}, \ldots)) = 0 .$$

1. - Levin natural tensor products.

Let H be a Fréchet lattice and F a Fréchet space. The Levin (or natural) tensor product $H \widehat{\underset{\mu}{\otimes}} F$ is the completion of $H \underset{\mu}{\otimes} F$ where μ is the topology defined by the family of seminorms $\{\mu_s \mid s \in \mathbb{N}\}$ and

$$\mu_s(z) = \inf\left\{ \left\| \sum_{i=1}^{n} \|x_i\|_s h_i \right\|_s \;\middle|\; z = \sum_{i=1}^{n} h_i \otimes x_i \in H \otimes X,\ h_i \geq 0 \right\}.$$

This tensor product was introduced by Levin [11] in the case of a Banach lattice H and a Banach space G, although with a different equivalent definition, (see [9] for a proof of the equivalence) and by Chaney [5] and Lotz [13] with the present definition. The μ-topology verifies that $\varepsilon \leq \mu \leq \pi$ (ε and π are the injective and projective topologies of Grothendieck respectively) but μ is not a tensornorm since it does not verify in general the metric mapping property.

If $H = \mathcal{C}(K)$ or $H = L^{\infty}(\nu)$ then μ coincides with ε. If H is an $L^1(\mu)$ space, $\mu = \pi$ (see [5]).

The importance of the Levin tensor product lies in the fact that many usual function spaces can be represented by means of such a product. For instance, let $(\Omega, \mathcal{A}, \mu)$ be a complete σ-finite measure space and $\mathcal{M}(\Omega, \mathcal{A})$ be the set of classes, modulo equality almost everywhere, of measurable real functions on $(\Omega, \mathcal{A})$. A Köthe function space (or a Banach ideal function space) on $(\Omega, \mathcal{A}, \mu)$ will be a Banach space H which is an ideal in $\mathcal{M}(\Omega, \mathcal{A})$. Given a Banach space X, we define $H(X)$ as the set of strongly measurable functions $f\colon \Omega \to X$ such that $\|f(\cdot)\| \in H$ endowed with the norm $\|f\| = \| \|f(\cdot)\|_X \|_H$. It can be proved (see [11], [2], [3]) that, when H has an order continuous norm, we have $H(X) = H \widehat{\bigotimes_{\mu}} X$. In particular, the familiar Lebesgue-Bochner spaces $L^p(\mu. X)$, $1 \leqslant p < \infty$, are isometric to $L_p(\Omega) \widehat{\bigotimes_{\mu}} X$.

We shall need the following result about the representation of the elements of $H \widehat{\bigotimes_{\mu}} X$.

LEMMA 1. *Let H be a Fréchet lattice and F a Fréchet space. Every $z \in H \widehat{\bigotimes_{\mu}} F$ has a representation $z = \sum_{i=1}^{\infty} h_i \otimes x_i$ with $h_i \geqslant 0$, where $(x_i)_{i=1}^{\infty}$ is a bounded sequence in F and $(h_i)_{i=1}^{\infty} \in \ell_0^1(H)$. Moreover, if $n \in \mathbb{N}$, $\mu_n(z) \leqslant \left(\sup_{i \in \mathbb{N}} \|x_i\|_n\right) \varepsilon_n(\!(h_i)\!)$ for every such representation of z and $\mu_n(z) = \inf\left\{\left\| \sum_{i=1}^{\infty} \|x_i\|_n h_i \right\|_n\right\}$, where the infimun is taken over all representations of z as above.*

PROOF. If $z \in H \widehat{\bigotimes_{\mu}} F$, there exists a sequence $(z_n)_{n=0}^{\infty} \subset H \otimes F$ such that $\lim_{n \to \infty} \mu_k(z - z_n) = 0$ for each $k \in \mathbb{N}$. Choosing a subsequence if it is needed, we can suppose that $z = z_0 + \sum_{n=1}^{\infty} (z_n - z_{n-1})$ with

$$\mu_n(z_n - z_{n-1}) < 2^{-2n}, \qquad n \in \mathbb{N}.$$

For every $n \in \mathbb{N}$ there exists a representation $z_n - z_{n-1} = \sum_{i=1}^{j(n)} h_{ni} \otimes x_{ni}$, such that $h_{ni} \geqslant 0$ and $\left\| \sum_{i=1}^{j(n)} \|x_{ni}\|_n h_{ni} \right\|_n < 2^{-2n}$. By the monotony of the lattice seminorms, z can be represented by a convergent series in $H \widehat{\bigotimes_{\mu}} F$, $z = \sum_{k=1}^{\infty} h_k \otimes x_k$ with every $h_k \geqslant 0$, $x_k \neq 0$ and $\sum_{k=1}^{\infty} \|x_k\|_n h_k$ converges in H for every $n \in \mathbb{N}$. We define a sequence $(\alpha_k)_{k=1}^{\infty}$ in $\mathbb{K}$ as follows: given $k \in \mathbb{N}$, there are $n, i \in \mathbb{N}$, $1 \leqslant i \leqslant j(n)$, such that $x_k = x_{ni}$.

We put $\alpha_k = \|x_{ni}\|_n$ if $\|x_{ni}\|_n \neq 0$ and $\alpha_k = 2^{-2n}\left(\left\|\sum_{i\in J_n} h_{ni}\right\|_n\right)^{-1}$, where $J_n = \{i \in \mathbb{N} \mid 1 \leqslant i \leqslant j(n) \text{ and } \|x_{ni}\| = 0\}$, if $\left\|\sum_{i\in J_n} h_{ni}\right\|_n \neq 0$; if $\|x_{ni}\|_n = 0$ and $\left\|\sum_{i\in J_n} h_{ni}\right\|_n = 0$, we put $\alpha_k = 1$. Then $z = \sum_{k=1}^{\infty} (\alpha_k h_k) \otimes (x_k/\alpha_k)$, is a representation of the announced type. In fact, it is clear that $(x_k/\alpha_k)_{k=1}^{\infty}$ is a bounded sequence in E. On the other hand, H' is a lattice with its canonical order. Denoting $V_n = \{h \in H \mid \|h\|_n \leqslant 1\}$, we have that V_n^0 is a solid set in H'. Then, if $k_n = 1 + \sum_{i=1}^{n-1} j(i)$, we have

$$\sup_{h'\in V_n^0} \sum_{k=k_n}^{\infty} \alpha_k |\langle h_k, h'\rangle| \leqslant 2\left(\sup_{h'\in V_n^0,\, h'\geqslant 0} \sum_{k=k_n}^{\infty} \alpha_k \langle h_k, h'\rangle\right) \leqslant$$

$$\leqslant 2\left\|\sum_{k=k_n}^{\infty} \alpha_k h_k\right\|_n \leqslant 2\sum_{k=n}^{\infty}\left\|\sum_{i=1}^{j(k)} \|x_{ki}\|_k h_{ki}\right\|_k < 4\sum_{k=n}^{\infty} 2^{-2k} \leqslant 2^{-2n-3},$$

and thus, $(\alpha_k h_k)_{k=1}^{\infty}$ is in $\ell_0^1(H)$.

Finally, we fix $n \in \mathbb{N}$. It is clear that $\mu_n(z) \leqslant \left\|\sum_{k=1}^{\infty} \|x_k\|_n h_k\right\|_n$ for every representation of z of the above type. Given one of them, for every $\varepsilon > 0$, there is a k_0 such for every $k \geqslant k_0$, $\left\|\sum_{j=1}^{\infty} \|x_j\|_n h_j\right\|_n < \varepsilon/3$. Putting $w_k = \sum_{j=1}^{k-1} h_j \otimes x_j$, we have $\mu_n(z - w_k) < \varepsilon/3$. Now it is possible to take a new representation of w_k, say $w_k = \sum_{j=1}^{t} \bar{h}_j \otimes \bar{x}_j$ with $\bar{h}_j \geqslant 0$, such that

$$\left\|\sum_{j=1}^{t} \|\bar{x}_j\|_n \bar{h}_j\right\|_n \leqslant$$

$$\leqslant \mu_n(w_k) + \in/3 \leqslant \mu_n(z - w_k) + \mu_n(z) + \varepsilon/3 \leqslant \mu_n(z) + 2\varepsilon/3.$$

Then $z = \sum_{j=1}^{t} \bar{h}_j \otimes \bar{x}_j + \sum_{j=k}^{\infty} h_j \otimes x_j$, so we get another representation $z = \sum_{i=1}^{\infty} h_i \otimes x_i$ of the above mentioned type, satisfying $\left\|\sum_{i=1}^{\infty} \|x_i\|_n h_i\right\|_n \leqslant \mu_n(z) + \varepsilon$. The remaining inequality follows easily. ■

2. Complemented copies of c_0 in Levin tensor products.

Next result generalizes the theorem of Cembranos and Freniche quoted in the introduction from the lattice point of view.

THEOREM 2. *Let H be a Fréchet lattice which contains a positive sequence equivalent to the unit basis of c_0. Then for every infinite dimensional Banach space E, $H \widehat{\bigotimes}_{\mu} E$ contains a complemented subspace isomorphic to c_0.*

PROOF. Let $(b_n)_{n=1}^{\infty}$ be a sequence in H^+ equivalent to the standard basis $(e_n)_{n=1}^{\infty}$ of c_0 by an isomorphism $\Phi: c_0 \to H$ such that $\Phi(e_n) = b_n$. As c_0 is isomorphic to a subspace of H, ℓ^1 is isomorphic to a quotient of H'_β. The sequence $(e_n)_{n=1}^{\infty}$ is also bounded in ℓ^1 and verifies that $\langle e_m, e_n \rangle = \delta_{nm}$. As the quotients of the DF spaces lift bounded sets [10, 12.4.8], there exist a bounded sequence $(b'_n)_{n=1}^{\infty}$ in H'_β such that $\langle b'_m, b_n \rangle = \delta_{nm}$. Since $(b'_n)_{n=1}^{\infty}$ is equicontinuous, there is $k \in \mathbb{N}$ such that, $(b'_n)_{n=1}^{\infty} \subset U_k^0$ holds. By the theorem of Josefson-Nissenzweig, there is a $\sigma(E', E)$ null sequence $(a'_n)_{n=1}^{\infty}$ in E' such that $\|a'_n\| = 1$, $\forall n \in \mathbb{N}$. An application of the principle of local reflexivity gives us a sequence $(a_n)_{n=1}^{\infty}$ in E such that $\langle a'_n, a_n \rangle = 1$ and $\|a_n\| \leq 2$, for every $n \in \mathbb{N}$.

We define $Q: c_0 \to H \widehat{\bigotimes}_{\mu} E$ such that $Q((\xi_i)) = \sum_{i=1}^{\infty} \xi_i b_i \otimes a_i$, for every $(\xi_i) \in c_0$. The map Q is well defined: $(b_i)_{i=1}^{\infty}$ being equivalent to $(e_i)_{i=1}^{\infty}$, we have that $(\xi_i b_i) \in \ell_0^1(H)$. Moreover, Q is linear and continuous: in fact, given $s \in \mathbb{N}$

$$\mu_s(Q((\xi_i))) \leq \sup_{x' \in U_s^0} \sum_{i=1}^{\infty} |\langle \xi_i \Phi(e_i), x' \rangle| \|a_i\| \leq$$

$$\leq 2 \sup_{i \in \mathbb{N}} |\xi_i| \sup_{x' \in U_s^0} \sum_{i=1}^{\infty} |\langle e_i, \Phi'(x') \rangle| \leq 2 \sup_{i \in \mathbb{N}} |\xi_i| \sup_{x' \in U_s^0} \|\Phi'(x')\|_{l^1} .$$

Then we define $T: H \widehat{\bigotimes}_{\mu} E \to c_0$ in the following way. By Lemma 1, every $H \widehat{\bigotimes}_{\mu} E$ can be represented as $z = \sum_{i=1}^{\infty} u_i \otimes v_i$ where, for each $n \in \mathbb{N}$

$$\lim_{m \to \infty} \sup_{h' \in U_n^0} \sum_{i=m}^{\infty} |\langle h', u_i \rangle| = 0 . \tag{1}$$

and $\sup_{i \in \mathbb{N}} \|v_i\| \leqslant M$ for some $M > 0$. Then we put

$$T(z) = (\langle z, b_n' \otimes a_n' \rangle)_{n=1}^{\infty} = \left(\sum_{i=1}^{\infty} \langle u_i, b_n' \rangle \langle v_i, a_n' \rangle \right)_{n=1}^{\infty} .$$

The map T is well defined: it is easy to see that $b_n' \otimes a_n' \in \left(H \widehat{\otimes}_{\mu} E \right)'$; on the other hand, since $(u)_{i=1}^{\infty}$ is bounded in H, we have $M_k := \sup_{i \in \mathbb{N}} \|u_i\|_k < \infty$; as $b_n' \in U_k^0$ for all $n \in \mathbb{N}$, from (1), given $\varepsilon > 0$, there is $h \in \mathbb{N}$ such that, for all $n \in \mathbb{N}$

$$\left| \sum_{i=1}^{\infty} \langle u_i, b_n' \rangle \langle v_i, a_n' \rangle \right| \leqslant \sum_{i=1}^{h} |\langle u_i, b_n' \rangle \langle v_i, a_n' \rangle| +$$

$$+ M \sum_{i=h+1}^{\infty} |\langle u_i, b_n' \rangle| \leqslant M_k \sum_{i=1}^{h} |\langle v_i, a_n' \rangle| + \frac{\varepsilon}{2}$$

and since $(a_n')_{n=1}^{\infty}$ is $\sigma(E', E)$ null, there is n_0 such that for every $n \geqslant n_0$

$$\left| \sum_{i=1}^{\infty} \langle u_i, b_n' \rangle \langle v_i, a_n' \rangle \right| \leqslant M_k \sum_{i=1}^{h} |\langle v_i, a_n' \rangle| + \frac{\varepsilon}{2} \leqslant \varepsilon ,$$

and hence $T(z) \in c_0$.

Moreover T is continuous. In fact, by Lemma 1, for every $z \in H \otimes_{\mu} E$, given $\varepsilon > 0$, there exist a representation $z = \sum_{i=1}^{\infty} h_i \otimes x_i$ with $h_i \geqslant 0$, such that

$$\mu_k(z) + \frac{\varepsilon}{2} > \sup_{h' \in U_k^0} \left| \sum_{i=1}^{\infty} \|x_i\| \langle h', h_i \rangle \right| .$$

As U_k^0 is a solid set we have $b_n' = (b_n')^+ - (b_n')^-$ with $(b_n')^+, (b_n')^- \in U_k^0$ for each $n \in \mathbb{N}$. Then we obtain

$$\|T(z)\| = \sup_{n \in \mathbb{N}} \left| \sum_{i=1}^{\infty} \langle h_i, b_n' \rangle \langle x_i, a_n' \rangle \right| \leqslant$$

$$\leqslant 2 \sup_{n \in \mathbb{N}} \sup_{h' \in U_k^0} \left| \sum_{i=1}^{\infty} \langle h_i, h' \rangle \|x_i\| \right| \leqslant 2\mu_k(z) + \varepsilon ,$$

and hence $\|T(z)\| \leqslant 2\mu_k(z)$.

It is easy to see that $TQ(\langle\!\langle \xi_i \rangle\!\rangle) = (\xi_i)$ for every $(\xi_i) \in c_0$, and then $Q(c_0) = QT\left(H \widehat{\otimes}_{\mu} E\right)$ is a complemented of $H \widehat{\otimes}_{\mu} E$ isopmorphic to c_0. ■

REMARK 3. If K is an infinite compact Hausdorff space, $\mathcal{C}(K)$ contains a subspace isomorphic to c_0 by a positive isometry, and this fact is implicit in the Cembranos's proof and more explicit in the Freniche's one. To see that, take a sequence $(G_n)_{n=1}^{\infty}$ of open non empty pairwise disjoint sets in K, and a sequence $(t_n)_{n=1}^{\infty}$, $t_n \in G_n$, $\forall n \in \mathbb{N}$. The Urysohn lemma gives us a sequence $(b_n)_{n=1}^{\infty} \subset \mathcal{C}(K)^+$ such that $b_n: K \to \to [0, 1]$ with $b_n(t_n) = 1$ and $b_n(t) = 0$ if $t \in K \backslash G_n$. Let F be the linear span of $\{b_n \mid n \in \mathbb{N}\}$. The map $Q: F \to c_0$ such that $Q(b_n) = e_n$ is an isometry since if $f = \sum_{i=1}^{s} \lambda_i b_{n_i} \in C(K)$, then $\|f\| = \sup_{i \in \mathbb{N}} |\lambda_i| = \|Q(f)\|$, and it is clearly positive. Moreover, if we take $b_n' = \delta_{t_n}$ (the Dirac measure at t_n), choosing (a_n) as in Theorem 2, we get an easy representation of the projection P from $\mathcal{C}(K, X) = \mathcal{C}(K) \hat{\otimes}_{\varepsilon} X$ onto the closure of the linear span of $\{b_n(\cdot)\, a_n \mid n \in \mathbb{N}\}$.

EXAMPLE 4. If $M(t)$ is an Orlicz function which does not satisfies the Δ_2 condition at 0, the Orlicz sequence space h_M has a sublattice order isomorphic to c_0 (see [3] and [12]) and ℓ^{∞} is not a subspace of h_M. By Theorem 2, for every infinite dimensional Banach space X, c_0 is a complemented subspace of $h_M \hat{\otimes}_{\mu} X$.

COROLLARY 5. *Let $(\Omega, \mathcal{A}, \mu)$ be a complete σ-finite measure space such that $L^{\infty}(\mu)$ is infinite dimensional. Let X be a Banach space which does not contain c_0. Then $L^{\infty}(\mu) \hat{\otimes}_{\mu} X$ is not complemented in $L^{\infty}(\mu, X)$.*

PROOF. By Theorem 2, c_0 is complemented in $L^{\infty}(\mu) \hat{\otimes}_{\mu} X$. By [6], if c_0 were complemented in $L^{\infty}(\mu, X)$, c_0 would be a subspace of X. Then the result follows. ■

We note also, for the sake of completeness, the alternative result:

THEOREM 6. *If H is an order continuous norm Köthe function space, $H(X) = H \hat{\otimes}_{\mu} X$ contains a complemented copy of c_0 if X has a copy of c_0.*

PROOF. It is essentially the same one of Emmanuele for $L^p(\mu, X)$ in [7]. ■

Our method can be easily applied also in the context of the d_{∞}-tensor product of Saphar (see [15] for definitions and details):

THEOREM 7. *Let F be a Fréchet space containing a subspace isomorphic to c_0. Then for every infinite dimensional Banach space X, $F \widehat{\bigotimes}_{d_\infty} X$ contains a complemented subspace isomorphic to c_0.*

PROOF. It is similar to the proof of Theorem 2, since the topology of $F \widehat{\bigotimes}_{d_\infty} X$ is determined by the seminorms given, for each $z \in F \widehat{\bigotimes}_{d_\infty} X$, by

$$d_{\infty s}(z) = \inf\left\{\left(\sup_{i \in \mathbb{N}} \|x_i\|\right) \varepsilon_s((y_i))\right\} \qquad s \in \mathbb{N},$$

where the infimum is taken over all representations $z = \sum_{i=1}^{\infty} y_i \otimes x_i$, such that $\sup_{i \in \mathbb{N}} \|x_i\| < \infty$ and $(y_i) \in \ell_0^1(F)$. ■

A Grothendieck space is a Banach space X such that $\sigma(X', X)$ and $\sigma(X', X'')$ null sequences in X' are the same. In consequence a Grothendieck space can not contain a complemented copy of c_0. Then we have:

COROLLARY 8. Let *H, G be Banach lattices such that H contains a positive sequence equivalent to the standard basis of c_0 and G is a Köthe function space with order continuous norm. Let X and Y be infinite dimensional Banach spaces such that Y contains a subspace isomorphic to c_0. Then neither $G(Y)$, $H \widehat{\bigotimes}_{\mu} X$ nor $Y \widehat{\bigotimes}_{d_\infty} X$ are Grothendieck spaces.*

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Manoscritto pervenuto in redazione il 7 febbraio 1995.