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# $f$-Radical Extensions of Rings.

JEFFREY BERGEN - ANTONIO GIAMBRUNO (*)

SUMMARY - If $R$ is a ring with subring $A$ and if $f(x_1, \ldots, x_d)$ is a multilinear, homogeneous polynomial in $d$ non-commutating variables, we say that $R$ is an $f$-radical extension of $A$ if for every $r_1, \ldots, r_d \in R$ there is an integer $n = n(r_1, \ldots, r_d) \geqslant 1$ such that $f(r_1, \ldots, r_d)^n \in A$. With an addition technical hypothesis added we prove that if $R$ is prime with no non-zero nil left ideals and if $R$ is an $f$-radical extension of $A$ then: (1) if $A$ is a subdivision ring either $R = A$ or $f$ is power-central valued, (2) if $A$ has no non-zero nilpotent elements either $R$ is a domain or $f$ is power-central valued, and (3) if $\psi$ is an automorphism of $R$ which is the identity on $A$ then either $\psi$ is the identity on $R$ or $f$ is power-central valued.

Let $R$ be a ring and $f = f(x_1, \ldots, x_d)$ a multilinear, homogeneous polynomial in $d$ non-commuting variables. In [4] Herstein, Procesi, and Schacher examine the situation where $f$ is power-central valued, that is, for every $r_1, \ldots, r_d \in R$ there is an integer $n = n(r_1, \ldots, r_d) \geqslant 1$ such that $f(r_1, \ldots, r_d)^n$ is central. It follows from the work in [4] that if $R$ has no non-zero nil left ideals then $R$ must satisfy $S_{d+2}$, the standard identity in $d + 2$ variables, provided an additional technical hypothesis also holds.

In general, we say that $R$ is an $f$-radical extension of a subring $A$ if, for every $r_1, \ldots, r_d \in R$ there is an integer $n = n(r_1, \ldots, r_d) \geqslant 1$ such

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that $f(r_1, \dots, r_d)^n \in A$. The results in [4] can be viewed as the case where $R$ is $f$-radical over its center. In this paper, we will consider the situation where $R$ is $f$-radical over an arbitrary subring $A$ and we will be concerned with contrasting the structure of $R$ and $A$. More precisely, we are interested in seeing when certain properties of $A$ must also hold for $R$.

At this point, we introduce the following notation, which we will use throughout this paper:

(1) $R$ will be an associative ring with center $Z$.

(2) $f(x_1, \dots, x_d)$ will denote a multilinear, homogeneous polynomial in $d$ non-commuting variables. Therefore we will assume that $f$ is of the form $f = \alpha x_1 \dots x_d + \sum_{\pi \neq 1} \alpha_\pi x_{\pi(1)} \dots x_{\pi(d)}$, where $\pi \in S_d$, the symmetric group on $d$ letters and $\alpha, \alpha_\pi \in C$ where $C$ is some commutative ring with 1 such that $R$ is an algebra over $C$ and $\alpha R \neq 0$.

(3) $f(x_1, \dots, x_d)$ will often be abbreviated as $f$ or $f(x_i)$. Similarly, integers $n(r_1, \dots, r_d)$ may be abbreviated as n or $n(r_j)$.

(4) $T(R) = \{a \in R\colon af(r_i)^n = f(r_i)^n a,\ n = n(a, r_1, \dots, r_d) \geqslant 1$ for all $r_1, \dots, r_d \in R\}$.

(5) If $S$ is a subset of $R$, then $r(S) = \{x \in R\colon Sx = 0\}$.

In our study as in [4], one problem does arise. Suppose $R$ is a division ring of characteristic $p > 0$ and $f$ is a power-central valued, multilinear, homogeneous polynomial of degree $d$. The proof in [4] that $R$ satisfies $S_{d+2}$ uses the assumption that $f$ is not an identity for the $p \times p$ matrices of characteristic $p$. It is still an open question as to whether this hypothesis is necessary. However to apply the results in [4] to prime rings with no non-zero nil left ideals, one must use the hypothesis that if the characteristic of $R$ is $p > 0$ then $f$ is not an identity for the $p \times p$ matrices of characteristic $p$. As a result, we will also need this extra hypothesis in order to prove our main result.

At this point we can now state the main result of this paper.

THEOREM. Let $R$ be a prime ring with no non-zero nil left ideals and let $f(x_1, \dots, x_d)$ be a multilinear, homogeneous polynomial. Suppose $R$ is an $f$-radical extension of a subring $A$ where if char $R = p > 0$

assume $f$ is not an identity for the $p \times p$ matrices of characteristic $p$. Then:

(1) If $A$ is a subdivision ring either $R = A$ or $f$ is power-central valued.

(2) If $A$ has no non-zero nilpotent elements either $R$ is a domain or $f$ is power-central valued.

(3) If $\psi$ is an automorphism of $R$ which is the identity on $A$ either $\psi$ is the identity on $R$ or $f$ is power-central valued.

We note that if $f$ is power-central valued then $R$ is $f$-radical over every subring $A$ which contains $Z$, however in this case $R$ and $A$ need not have much in common. Therefore, in all three parts of our theorem, the general flavor of the results is that if $f$ is not power-central valued then $R$ and $A$ are similar.

*In all that follows*, unless stated otherwise, *we will assume* that $R$ is a prime ring with no non-zero nil left ideals and $R$ is $f$-radical over a subring $A$. Furthermore, assume that $f$ is not power-central valued and if $\operatorname{char} R = p > 0$, assume that $f$ is not an identity for the $p \times p$ matrices of characteristic $p$.

We now begin the work necessary to prove the first part of our theorem.

LEMMA 1. If $A$ is a subdivision ring of $R$ then $R$ is simple.

PROOF. Let $e$ be the unit element of $A$; therefore $e \in T(R)$. By a result of Felzenszwalb and Giambruno [1], $T(R) = Z$, thus $e$ is a central idempotent of $R$. Since $R$ is prime, $e$ is the unit element of $R$, hence every non-zero element of $A$ is invertible in $R$.

As a result, if $r_1, \ldots, r_d \in R$ either $f(r_i)$ is nilpotent or invertible. Now, let $I \neq 0$ be a proper ideal of $R$; if $s_1, \ldots, s_d \in I$ it follows that $f(s_i)$ must be nilpotent. Since $I$ is also a prime ring with no non-zero nil left ideals, by another result of Felzenszwalb and Giambruno [2], $f$ is a polynomial identity for $I$. Thus $f$ is also an identity for $R$ hence, in particular, $f$ is power-central valued, a contradiction. Therefore, $R$ is simple.

We proceed with

LEMMA 2. If $A$ is a subdivision ring then either $R$ is a division ring or $R$ satisfies a polynomial identity.

PROOF. Suppose $R$ is not a division ring and let $L$ be a proper left ideal; if $s_1, \ldots, s_d \in L$ then $f(s_i)$ is nilpotent. The ring $\bar{L} = L/L \cap r(L)$ is also prime with no non-zero nil left ideals and all the values of $f$ on $\bar{L}$ are nilpotent. By the result in [2], $f$ is an identity for $\bar{L}$, hence $x_{d+1} f(x_1, \ldots, x_d)$ is an identity for $L$.

Now let $M$ be a left ideal of $R$ maximal with respect to satisfying $x_{d+1} f(x_i)$. If $s \in R$, $Ms$ also satisfies a polynomial identity and, by a result of Rowen [6], $M + Ms$ also satisfies a polynomial identity. If $M + Ms$ is a proper left ideal for every $s \in S$, then $M + Ms$ will satisfy $x_{d+1} f(x_i)$. By the maximality of $M$, $M + Ms \subset M$, thus $Ms \subset M$ and so, $M = R$. On the other hand, if $M + Ms = R$, for some $s \in R$, then once again, $R$ satisfies a polynomial identity.

We can now state and prove the first part of our main theorem.

THEOREM 3. If $A$ is a subdivision ring of $R$ then $R = A$.

PROOF. By Theorem 1 of [3], if $R$ is a division ring then $R = A$. Therefore, by Lemmas 1 and 2, if $R \neq A$ then $R$ satisfies a polynomial identity and $R$ is the $n \times n$ matrices over a division ring where $n > 1$.

If $Z(A)$, the center of $A$, were finite than $A$ would be a field since $A$ is finite dimensional over its center. However, in this case, $A$ would then lie in the center of $R$, contradicting the assumption that $f$ is not power-central valued. Thus $Z(A)$ is infinite and, in particular, there exists a $0 \neq \alpha \in Z(A) \subset Z(R)$ such that $\alpha + 1 \neq 0$. Let $y = \alpha + e_{1n}$; then both $y$ and $1 + y$ are invertible, non-central elements of $R$.

Since $y \notin T(R)$ there exist $r_1, \ldots, r_d \in R$ such that $yf(r_i)^m \neq f(r_i)^m y$, for every positive integer $m$. Now let $m > 1$ be such that

$$f(r_i)^m, \qquad f(yr_i y^{-1})^m = yf(r_i)^m y^{-1},$$

and

$$f((1 + y) r_i (1 + y)^{-1})^m = (1 + y) f(r_i)^m (1 + y)^{-1}$$

all belong to $A$. Hence

(1) $yf(r_i)^m = ay$ and

(2) $(1 + y) f(r_i)^m = b(1 + y)$ for some $a, b \in A$.

Subtracting (1) from (2) yields $f(r_i)^m = b + (b - a) y$. If $b = a$ then

$yf(r_i)^m = f(r_i)^m y$, a contradiction. However, if $b \neq a$ then

$$y = (b-a)^{-1}(f(r_i)^m - b)\,, \qquad \text{hence } y \in A\,.$$

As a result, $e_{1n} = y - a \in A$, which is a contradiction, since $e_{1n}$ is nilpotent.

In light of Theorem 3, it is natural to wonder what can be said if we merely assume that $A$ is a domain. Clearly it no longer need be that $R = A$, however one might hope to prove that $R$ is also a domain. In fact, in order to prove that $R$ is a domain, we only need to assume that $A$ has no non-zero nilpotent elements. This is part two of our main theorem which we now record as

THEOREM 4. If $A$ has no non-zero nilpotent elements then $R$ is a domain.

PROOF. Suppose $R$ is not a domain; since $R$ is prime there is some $0 \neq t \in R$ such that $t^2 = 0$. If $r_1, \ldots, r_d \in R$ there is a positive integer $m$ such that $f(r_i t)^m$ and $f((1+t)r_i t(1-t))^m = (1+t)f(r_i t)^m(1-t)$ both belong to $A$. Therefore $tf(r_i t)^m \in A$ and, since $A$ has no non-zero nilpotent elements, $tf(r_i t)^m = 0$. As in the proof of Lemma 2, $f$ has only nilpotent values on $Rt/Rt \cap r(Rt)$, thus $f$ is an identity for $Rt/Rt \cap r(Rt)$. Thus $tf(x_1 t, \ldots, x_d t)$ is a generalized polynomial identity for $R$

Now, if $R$ satisfies a polynomial identity then $A$ is a semiprime P.I. ring and so, every non-zero ideal of $A$ intersects $Z(A)$, the center of $A$, non-trivially. However, $Z(A) \subset T(R) = Z(R)$ and $Z(R)$ is a domain, thus $A$ is prime and therefore is a domain. We can now localize $R$ and $A$ at the non-zero elements of $Z(A)$ to obtain rings $R_1$ and $A_1$ respectively. $R_1$ is an $f$-radical extension of the division ring $A_1$ and, by Theorem 2, $R_1 = A_1$. Hence $R$ is a domain.

Therefore, we may now assume that $R$ satisfies a G.P.I. but not a P.I. By a theorem of Martindale [5], if $C$ is the extended center of $R$ then $S = RC$ is a primitive ring with minimal right ideal $eS$; moreover the commuting division ring $D = eSe$ is finite dimensional over $C$. Since $R$ does not satisfy a P.I., by a theorem of M. Smith [7], for any $n \geqslant 1$, $R$ contains a prime P.I. subring $R^{(n)}$ such that $R^{(n)}$ is isomorphic to the ring of $n \times n$ matrices over a subring $E$ of $D$ and $R^{(n)}$ satisfies no P.I. of degree less than $n$. Let $n > \frac{1}{2}(d+2)$; then by the work in [4], $f$ cannot be power-central valued on $R^{(n)}$. However,

$R^{(n)}$ is an $f$-radical extension of $R^{(n)} \cap A$, therefore $R^{(n)}$ is a domain contradicting the fact that $n > 1$.

We can now begin the series of reduction necessary to prove the third part of our main theorem. *In all that follows, we will assume* that $\psi$ is an automorphism of $R$ which is the identity on $A$.

LEMMA 5. If $a \in R$ is invertible or formally invertible then $\psi(a) = \beta a$, for some $\beta \in Z(R)$.

PROOF. If $r_1, \ldots, r_d \in R$, let $m \geqslant 1$ be such that $\psi(f(r_i)^m) = f(r_i)^m$ and $\psi(f(ar_i a^{-1})^m) = \psi(af(r_i)^m a^{-1}) = af(r_i)^m a^{-1}$. Thus

$$\psi(a) f(r_i)^m \psi(a)^{-1} = af(r_i)^m a^{-1}$$

and so,

$$a^{-1}\psi(a) f(r_i)^m = f(r_i)^m a^{-1}\psi(a) .$$

Therefore $a^{-1}\psi(a) = \beta \in T(R) = Z(R)$, hence $\psi(a) = \beta a$.

We continue with

LEMMA 6. If $J(R)$, the Jacobson radical of $R$, is non-zero then $\psi = 1$.

PROOF. If $r \in J(R)$ then $1 + r$ is formally invertible. By Lemma 5, $\psi(1 + r) = \beta(1 + r)$, for some $\beta \in Z(R)$, which yields $\psi(r) = (\beta - 1) + \beta r$ and, finally $r\psi(r) = \psi(r)r$. It is well-known and easy to prove that if $r\psi(r) = \psi(r)r$, for all $r$ in a non-zero ideal of a prime ring $R$, then either $R$ is commutative or $\psi = 1$. Since $f$ is not power-central valued, $\psi = 1$.

We now proceed to the hardest part of the theorem, the case where $R$ is primitive.

LEMMA 7. If $R$ is primitive then $\psi = 1$.

PROOF. Let $V$ be a faithful, irreducible right $R$ module with commuting ring $D$. If $t \in R$, $t^2 = 0$ then, by Lemma 5, $\psi(1 + t) = \beta(1 + t)$, resulting in $\psi(t) = (\beta - 1) + \beta t$. Squaring both sides of this equation yields $0 = (\beta - 1)^2 + 2\beta(\beta - 1)t$ and multiplication by $t$ gives us $0 = (\beta - 1)^2 t$, hence $\beta = 1$. Thus, $\psi(t) = t$.

Now, suppose $V$ is finite dimensional over $D$, therefore $R = D_n$, for some integer $n \geqslant 1$. If $n = 1$ then by Theorem 1 of [3] or by Theorem 3, $R = A$ and clearly $\psi = 1$. On the other hand, if $n > 1$ then the subring of $R$ generated by all the elements of square zero is all of $R$. However, by the argument in the previous paragraph, $\psi$ fixes all elements of sqare zero, so again $\psi = 1$. Therefore, we may now assume that $V$ is infinite dimensional over $D$. Now, since $R$ is primitive, we can write $V = vR$ for some $v \neq 0 \in V$ and, so, we can define an action of $C$ on $V$ as follows: if $w = vr \in V$ and $c \in C$, $wc = vcr$. In this way it is easy to check that $C$ can be identified with a subring of the center of the commuting ring $D$.

Suppose $v_0$, $v_1$ are linearly indepenent elements of $V$; let $v_2, \ldots$ $\ldots, v_d \in V$, be such that $v_0, \ldots, v_d$ are all linearly independent. Now by Jacobson density, let $r_1, \ldots, r_d \in R$ such that $v_0 r_1 = v_2$, $v_i r_i = v_{i+1}$ for $1 \leqslant i \leqslant d-1$, $v_d r_d = v_0$, and $v_i r_j = 0$ otherwise. Since

$$f(x_i) = \alpha x_1 \ldots x_d + \sum_{\pi \neq 1} \alpha_\pi x_{\pi(1)} \ldots x_{\pi(d)} ,$$

we have $v_1 f(r_i) = \alpha v_0$ and $v_0 f(r_i) = \alpha v_0$, thus $v_1 f(r_i)^m = \alpha^m v_0$, for any integer $m \geqslant 1$. If we let $m$ be such that $f(r_i)^m = a \in A$, we have $v_1 a = \alpha^m v_0$ and therefore $A$ acts both faithfully and irreducibly on $V$. As a result, we may now assume that both $R$ and $A$ act densely on $V$.

Let $0 \neq u \in V$ and suppose $uz$ and $u\psi(z)$ are linearly dependent, for all $z \in R$. Now, let $x, y \in R$ such that $ux$ and $uy$ are linearly independent then

$$u\psi(x) = \lambda_x ux , \quad u\psi(y) = \lambda_y uy , \quad \text{and} \quad u\psi(x+y) = \lambda_{x+y} u(x+y) ,$$

where $\lambda_x, \lambda_y, \lambda_{x+y} \in D$. Therefore $\lambda_{x+y} ux + \lambda_{x+y} uy = \lambda_x ux + \lambda_y uy$, thus $\lambda_x = \lambda_y$. As a result, $u\psi(z) = \lambda uz$, where $\lambda$ does not depend on $z$. However, if we let $z \in A$ such that $uz \neq 0$ we obtain $uz = u\psi(z) = \lambda uz$, hence $\lambda = 1$. By this argument, if $uz$ and $u\psi(z)$ are linearly dependent for all $u \in V$ and $z \in R$, then $\psi = 1$. We may therefore assume that there exists a $u \in V$ and $z \in R$ such that $uz$ and $u\psi(z)$ are linearly independent.

Let $v_1, \ldots, v_{d-1} \in V$ be such that $u, v_1, \ldots, v_{d-1}$ are linearly independent; by the density of $A$, let $r_2, \ldots, r_d \in A$ be such that $v_i r_{i+1} = v_{i+1}$ for $1 \leqslant i \leqslant d-2$, $v_{d-1} r_d = u$, $ur_j = 0$ for $2 \leqslant j \leqslant d$, and $v_i r_j = 0$ otherwise. In addition, let $a \in A$ such that $u\psi(z)a = v_1$, $uza = 0$ and let $r_1 = za$. Therefore, $uf(r_1, \ldots, r_d) = 0$ and $uf(\psi(r_1), r_2, \ldots, r_d) = \alpha u$. Hence, for any integer $m \geqslant 1$, $uf(r_i)^m = 0$ whereas $u\psi(f(r_i))^m = \alpha^m u$. However, there exists some $m \geqslant 1$ such that $\psi(f(r_i))^m = f(r_i)^m$, a contradiction, thereby proving the lemma.

We now have all the pieces necessary to prove the third part of our main theorem which we record as

THEOREM 8. If $\psi$ is an automorphism of $R$ which is the identity on $A$, then $\psi$ is the identity on $R$.

PROOF. By Lemma 6, if $J(R)\neq 0$ then $\psi = 1$, thus it is enough to handle the case where $R$ is semisimple. We can extend the action of $\psi$ to $C$ and then can let $\bar{f}$ denote the polynomial

$$\bar{f}(x_1) = x_i \dots x_d + \sum_{\pi\neq 1} \psi(\alpha_\pi) x_{\pi(1)} \dots x_{\pi(d)} .$$

If $P$ is a primitive ideal of $R$ and $s_1, \dots, s_d \in P$ then $\bar{f}(\psi(s_1)) = \psi(f(s_i))$, hence $\bar{f}(\psi(s_i))^m = \psi(f(s_i))^m = f(s_i)^m \in P$, for some $m\geqslant 1$. Since $\bar{f}$ is nil valued on the ring $\psi(P) + P/P$, the the result in [2], $\bar{f}$ is an identity for $\Psi(P) + P/P$. Thus if $\psi(P)\not\subset P$ then $\psi(P) + P/P$ is a non-zero ideal of the primitive ring $R/P$, hence $R/P$ also satisfies $\bar{f}$ and clearly $\bar{f}$ is now an identity for $R/P$.

We now partition the primitive ideals of $R$ into three sets;

$B_1 = \{P\colon \psi(P)\not\subset P\}$ ,

$B_2 = \{P\colon \psi(P)\subset P$ and $f$ is power-central valued on $R/P\}$ ,

$B_3 = \{P\colon \psi(P)\subset P$ and $f$ is not power-central valued on $R/P\}$ .

In addition, let $I_i = \bigcap_{P\in B_i} P$, for $i = 1, 2, 3$.

Since $R$ is semisimple, $I_1 I_2 I_3 \subset I_1 \cap I_2 \cap I_3 = 0$; however, by the primeness of $R$, at least one of $I_1$, $I_2$, or $I_3$ is zero. If $I_1 = 0$ then $R$ is a subdirect sum of rings satisfying $f$, hence $f$ is an identity for $R$, a contradiction. If $I_2 = 0$ then, by the work in [4], $R$ is a subdirect sum of rings satisfying $S_{d+2}$, the standard identity in $d+2$ variables. Since $R$ satisfies a polynomial identity, we can let $R_Z$ denote the localization of $R$ at the non-zero elements of $Z(R)$. In particular, we can extend the action of $\psi$ to $R_Z$ and $R_Z$ is primitive. $R$ does not satisfy $f$, therefore there exist $r_i\in R$ such that $f(r_i)$ is not nilpotent [2]. If $0\neq\alpha\in Z(R)$ there is an $m\geqslant 1$ such that $\alpha^m f(r_i)^m = f(\alpha r_1, r_2, \dots, r_d)^m$ and $f(r_i)^m$ belong to $A$. Hence $\alpha^m f(r_i)^m = \psi(\alpha^m f(r_i)^m) = \psi(\alpha^m) f(r_i)^m$, thus $\psi(\alpha^m) = \alpha^m$. As a result, if $s_i\in R_Z$ there is an $n\geqslant 1$ such that $\psi(f(s_i)^n) = f(s_i)^n$, therefore, by lemma 7, either $f$ is power-central on on $R_Z$ or $\psi = 1$ on $R_Z$. However, $f$ is not power-central on $R$, hence $\psi = 1$ on $R_Z$, so certainly $\psi = 1$ on $R$.

Finally, if $I_3 = 0$ then $R$ is a subdirect sum of primitive rings on which $\psi$ induces an automorphism $\bar{\psi}$ satisfying all the hypotheses of Lemma 7. Since $\bar{\psi}$ is the identity on $R/P$, for each $P \in B_3$, $\psi$ is the identity on $R$. This concludes the proof of the theorem.

By combining Theorems 3, 4, and 8 we obtain our main result which we mentioned at the outset of the paper:

THEOREM. Let $R$ be a prime ring with no non-zero nil left ideals and let $f(x_1, \ldots, x_d)$ be a multilinear, homogeneous polynomial. Suppose $R$ is an $f$-radical extension of a subring $A$ where if $\operatorname{char} R = p > 0$ assume $f$ is not an identity for the $p \times p$ matrices of characteristic $p$. Then:

(1) If $A$ is a subdivision ring either $R = A$ or $f$ is power-central valued.

(2) If $A$ has no non-zero nilpotent elements either $R$ is a domain or $f$ is power-central valued.

(3) If $\psi$ is an automorphism of $R$ which is the identity on $A$ either $\psi$ is the identity on $R$ or $f$ is power-central valued.

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