Rendiconti del Seminario Matematico della Università di Padova

Umberto Marconi

On the uniform paracompactness

Rendiconti del Seminario Matematico della Università di Padova, tome 72 (1984), p. 319-328

<http://www.numdam.org/item?id=RSMUP_1984__72__319_0>

Rend. Sem. Mat. Univ. Padova, Vol. 72 (1984)

On the Uniform Paracompactness.

Umberto Marconi (*)

0. Introduction.

Uniform paracompactness was defined by M. D. Rice in [R] (this concept was actually used in a previous paper by H.H. Corson [C]). In [H_1], [F], Tamano's theorem on paracompactness has been given a uniform analogue.

Countable uniform paracompactness has been discussed in [H_1], [H_2]. In this work, we plan to discuss uniform μ-paracompactness. In § 1 definitions and basic properties are given.

In § 2 uniform analogues of Morita's product theorems for μ-paracompactness [M_1] are obtained; finally, in § 3 countable uniform paracompactness is discussed, obtaining an analogue of Dowker's theorem.

1. Definitions and basic properties.

We will denote by uX a uniform space, by X the associated topological space, by fX the finest uniform space on the topological space X. Furthermore puX will denote the paracompact reflection of uX and $p^{\mu}uX$ the coarsest uniform space for which the uniform maps from uX to metric spaces of density μ are uniform.

A filter base $\mathcal{F}$ of subsets of uX is said to be weakly Cauchy if for every uniform covering $\mathcal{U}$ of uX there exists an element $U \in \mathcal{U}$ such that $U \cap F \neq \emptyset$ for every $F \in \mathcal{F}$.

(*) Indirizzo dell'A.: Seminario Matematico, Via Belzoni 7, 35131 Padova.

Let $\mathcal{A}$ be a family of subsets of X; denote by $\mathcal{A}_f$ the family of all finite unions of elements of $\mathcal{A}$. A directed family is a family $\mathcal{A}$ such that $\mathcal{A} = \mathcal{A}_f$. A family $\mathcal{A}$ is said to be uniformly locally finite if there exists a uniform covering $\mathcal{U}$ such that every $U \in \mathcal{U}$ meets $\mathcal{A}$ only for a finite number of elements of $\mathcal{A}$.

Let μ be a cardinal number. Consider the following conditions on uX:

1) every weakly Cauchy filter base of cardinal $\leq \mu$ has a cluster point;
2) every directed open covering $\mathcal{A}$ of power $\leq \mu$ is uniform;
3) every open covering $\mathcal{A}$ of power $\leq \mu$ has an open uniformly locally finite refinement.

We have the following obvious implications: $1 \Leftrightarrow 2$ and $3 \Rightarrow 2$. Later we will prove that $2 \Rightarrow 3$.

DEFINITION 1. A uniform space uX is said to be uniformly μ-paracompact if it satisfies the above condition 1.

In [M_1] a topological space X is said to be μ-paracompact if every open covering of X of power $\leq \mu$ has an open locally finite refinement.

PROPOSITION 1. *If uX is uniformly μ-paracompact, then X is μ-paracompact.*

PROOF. Let $\mathcal{A}$ be an open covering of power $\leq \mu$; $\mathcal{A}_f$, being a uniform covering, has a locally finite open refinement $\mathcal{B}$. For every $B \in \mathcal{B}$, consider a finite subset $\mathcal{A}_B$ of $\mathcal{A}$ such that $B \subset \bigcup \mathcal{A}_B$. Then the open covering

$$\{B \cap A : B \in \mathcal{B}, A \in \mathcal{A}_B\}$$

is a locally finite open refinement of $\mathcal{A}$.

I don't know if a $T_{3\frac{1}{2}}$ μ-paracompact space is uniformly μ-paracompact in the fine uniformity. This occurs if X is a normal space.

PROPOSITION 2. *A normal space X is μ-paracompact if and only if fX is uniformly μ-paracompact.*

PROOF. If X is a normal μ-paracompact topological space then every open covering of power $\leq \mu$ is normal in the sense of Tukey ([M_1] th. 1.1).

REMARK 1. There exist countably uniformly paracompact spaces that fail to be normal. Let X be a countably compact $T_{3\frac{1}{2}}$ non normal space, for example $\omega_1 \times (\omega_1 + 1)$. If uX is a compatible uniform space, uX is countably uniformly paracompact.

For the proof of $2 \Rightarrow 3$, we need a lemma. (I am indebted to A. Hohti for a suggestion which led to this lemma).

LEMMA 1. *If a covering $\mathcal{A}$ is locally finite, there exists an open covering $\mathcal{B}$, with $|\mathcal{B}| \leq |\mathcal{A}|$, such that every element of B meets only a finite number of elements of $\mathcal{A}$.*

PROOF. Let $\mathcal{V}$ be an open covering of X such that every member of $\mathcal{V}$ meets only a finite number of elements of $\mathcal{A}$. For every finite subset $\mathcal{F}$ of $\mathcal{A}$ put $V_{\mathcal{F}} = \cup \{V \in \mathcal{V}\colon V \cap A \neq \emptyset \text{ iff } A \in \mathcal{F}\}$. Then $\mathcal{B} = \{V_{\mathcal{F}}\colon \mathcal{F} \in \text{finite subsets of } \mathcal{A}\}$ satisfies the required properties.

PROPOSITION 3. *Condition 2 $\Rightarrow$ condition 3.*

PROOF. Let $\mathcal{A}$ be an open covering of X of power $\leq \mu$. By proposition 1 and theorem 1.4 ch. VIII of [Du], it has an open locally finite refinement $\mathcal{B}$, with $|\mathcal{B}| \leq |\mathcal{A}|$. By lemma 1, there exists an open covering $\mathcal{C}$, with $|\mathcal{C}| \leq \mu$, such that every member of $\mathcal{C}$ meets only a finite number of elements of $\mathcal{B}$. Since $\mathcal{C}_f$ is uniform, $\mathcal{B}$ is uniformly locally finite.

PROPOSITION 4. *If uX is uniformly μ-paracompact, every uniform covering of power $\leqq \mu$ belongs to $p^{\mu}uX$ and $p^{\mu}uX$ has a point finite base.*

PROOF. By [V] it sufficies to prove that every uniform covering of power $\leq \mu$ has a point finite uniform refinement. If $\mathcal{U}$ is a uniform covering of power $\leq \mu$, it has a uniform open refinement $\mathcal{V}$ of power $\leq \mu$ (argue as in the proof of th. 1.4 ch. VIII of [Du]). By uniform μ-paracompactness, $\mathcal{V}$ has an uniformly locally finite refinement. Therefore, by [Sm] th. 4.5, $\mathcal{V}$ has a locally finite uniform refinement.

From the above proposition we get the following

COROLLARY 1. *uX is uniformly μ-paracompact if and only if every directed open covering $\mathcal{A}$ of power $\leqq \mu$ belongs to $p^{\mu}uX$.*

2. Uniform products.

As for μ-paracompactness ([M_1] th. 2.1) uniform μ-paracompactness is preserved under products by compact spaces. When considered as uniform spaces, compact (T_2) spaces are of course equipped with their unique admissible uniformity.

THEOREM 1. *If a uniform space uX is uniformly μ-paracompact, and Y is compact, then $uX \times Y$ is uniformly μ-paracompact.*

PROOF. Let $\mathcal{F}$ be a weakly Cauchy filter base of power $\leq \mu$. Since the first projection $p_1: uX \times Y \to uX$ is a closed mapping, the filter

$$\mathcal{F}_1 = \{p_1(\overline{F}): F \in \mathcal{F}\}$$

is a weakly Cauchy filter base of closed sets, with $|\mathcal{F}_1| \leq \mu$. Therefore there exists a point $p \in uX$ such that $(\{p\} \times Y) \cap \overline{F} \neq \emptyset$ for every $F \in \mathcal{F}$. The compactness of Y ensures the existence of a point $y \in Y$ such that $(p, y) \in \overline{F}$ for every $F \in \mathcal{F}$.

As usual, let I denote the closed unit interval, D the discrete two-point space $\{0, 1\}$. It is well-known that a normal space X is μ-paracompact if and only if $X \times I^\mu$ is normal, or equivalently, $X \times D^\mu$ is normal ([M$_2$]; [D] for $\mu = \omega$). We plan to give analogous characterizations of uniform μ-paracompactness.

DEFINITION. *We say that a uniform space uX satisfies property P for a compact space Y if whenever A and B are disjoint closed sets of $X \times Y$, there exists a uniform covering $\mathcal{C}$ of $uX \times Y$ such that for every $T \in \mathcal{C}$ the sets $A \cap T$ and $B \cap T$ are far (uniformly separated) in $pfX \times Y$.*

REMARK 2. By compactness of Y the uniform covering $\mathcal{C}$ of the above definition may be assumed of the form:

$$\mathcal{C} = \{U \times Y: U \in \mathcal{U}\}$$

for a suitable uniform covering $\mathcal{U}$ of uX.

THEOREM 2. *Let uX be a normal uniform space. The following conditions are equivalent:*

1) uX *is a uniformly μ-paracompact space;*

2) $p^\mu uX$ *satisfies property P for every compact space of weight μ;*

3) uX *satisfies property P for I^μ,*

4) uX *satisfies property P for D^μ.*

PROOF. $1 \Rightarrow 2$. Let Y be a compact space of weight μ. Let $\mathcal{B}$ be a directed basis of power μ for the open sets of Y. Let $\Delta = \{d_\alpha: \alpha \in \mu\}$ be a basis of power $\leq \mu$ consisting of continuous pseudometrics of Y, such that every open covering of Y has d_α-Lebesgue number 1, for some $\alpha \in \mu$. Denote by Γ the set of all triples (d_α, H, K) with $d_\alpha(H, K) \geqslant 1$, $d_\alpha \in \Delta$, $H, K \in \mathcal{B}$; of course $|\Gamma| \leq \mu$. Let A, B be closed and disjoint subsets of $X \times Y$. For every $x \in X$ put

$$A[x] = \{y \in Y: (x, y) \in A\}, \qquad B[x] = \{y \in Y: (x, y) \in B\}.$$

For every $\gamma \in \Gamma$, let

$$V_\gamma = \{x \in X: A[x] \subset H, B[x] \subset K, \text{ where } \gamma = (d_\alpha, H, K)\}.$$

From the compactness of Y follows that the family $\mathcal{V} = \{V_\gamma: \gamma \in \Gamma\}$ is an open covering of uX of power at most μ.

Therefore there exists a uniform covering $\mathcal{U}$ of closed sets such that every $U \in \mathcal{U}$ is contained in a finite union of elements of $\mathcal{V}$, that is $U \subset \bigcup_{\gamma \in F_U} V_\gamma$ for a suitable finite subset F_U of Γ. For every $U \in \mathcal{U}$ the open covering of U

$$\{V_{\gamma_1} \cap U, \ldots, V_{\gamma_n} \cap U: \gamma_i \in F_U\}$$

is induced by the finite open covering of X:

$$\mathcal{V}_U = \{X \setminus U, V_{\gamma_1}, \ldots, V_{\gamma_n}: \gamma_i \in F_U\}.$$

By the normality of X, $\mathcal{V}_U$ is a uniform covering of the space pfX.

By corollary 1, covering $\mathcal{U}$ may be taken belonging to $p^\mu uX$. Let

$$\mathcal{T} = \{U \times Y: U \in \mathcal{U}\}.$$

For every $V_\gamma \in \mathcal{V}$, there exist a pseudometric $d_\gamma \in \Delta$ and two open sets of Y, say H_γ, K_γ, such that for every $x \in V_\gamma$ we have $A[x] \subset H_\gamma$, $B[x] \subset K_\gamma$ and $d_\gamma(H_\gamma, K_\gamma) \geq 1$. If $F_U = \{\gamma_1, \ldots, \gamma_n\}$ let $d_U = d_{\gamma_1} \vee \ldots \vee d_{\gamma_n}$. Let σ_U be an admissible pseudometric of pfX such that the covering $\mathcal{V}_U$ has σ_U-Lebesgue number 1. Let $T \in \mathcal{T}$, $T = U \times Y$ for some $U \in \mathcal{U}$.

Let $(x_1, y_1) \in A \cap T$ and $(x_2, y_2) \in B \cap T$. If $\sigma_U(x_1, x_2) \leq 1$ there exists some $\gamma_i \in F_U$ such that $x_1, x_2 \in V_{\gamma_i}$ and therefore $y_1 \in A[x_1] \subset H_{\gamma_i}$,

$y_2 \in B[x_2] \subset K_{\gamma_i}$. Therefore $d_U(y_1, y_2) \geq d_{\gamma_i}(y_1, y_2) \geq d_{\gamma_i}(H, K) \geq 1$. Thus $(\sigma_U \times d_U)\big((x_1, y_1), (x_2, y_2)\big) \geq 1$ and therefore $A \cap T$ and $B \cap T$ are separated by a uniform covering of $pfX \times Y$.

$2 \Rightarrow 3$. Obvious.

$3 \Rightarrow 4$. Obvious, because D^μ is a closed subspace of I^μ.

Before proving that 4) $\Rightarrow$ 1) we need the following:

LEMMA 2. *Let B a subset of $X \times Y$ and Y_0 a subset of Y. If B and $X \times Y_0$ are separated by the covering $\mathcal{U} \times \mathcal{V}$, then they are separated by the covering $\{X\} \times \mathcal{V}$.*

PROOF. Let $\mathcal{U} \times \mathcal{V} = \{U_\alpha \times V_\beta \colon U_\alpha \in \mathcal{U}, V_\beta \in \mathcal{V}\}$. Thus

$$\mathrm{St}\,(X \times Y_0, \mathcal{U} \times \mathcal{V}) = X \times \mathrm{St}\,(Y_0, \mathcal{V}) = \mathrm{St}\,(X \times Y_0, \{X\} \times \mathcal{V}) ,$$

$$\mathrm{St}\,(B, \{X\} \times \mathcal{V}) = X \times (\cup \{V_\beta \colon B \cap (X \times V_\beta) \neq \emptyset\}) .$$

If the stars meet, there exist $V_\beta, V_{\beta'} \in \mathcal{V}$, $V_\beta \cap V_{\beta'} \neq \emptyset$, such that $V_{\beta'} \cap Y_0 \neq \emptyset$ and $(X \times V_\beta) \cap B \neq \emptyset$.

Let $y \in V_\beta \cap V_{\beta'}$ and let $x \in X$ such that $(\{x\} \times V_\beta) \cap B \neq \emptyset$. Then

$$(x, y) \in \mathrm{St}\,(B, \mathcal{U} \times \mathcal{V}) \cap \mathrm{St}\,(X \times Y_0, \mathcal{U} \times \mathcal{V}) ,$$

against the hypothesis.

PROOF OF $4 \Rightarrow 1$. For every $\alpha \in \mu$, let $p_\alpha \colon D^\mu \to D$ the projection on the α-th coordinate.

Let 0 be the point of null coordinates.

If $\mathcal{A} = \{A_\alpha \colon \alpha \in \mu\}$ is an open covering of uX of power μ, the open set

$$\Omega = \bigcup_{\alpha \in \mu} A_\alpha \times p_\alpha^{-1}(0)$$

is a neighborhood of $X_0 = X \times \{0\}$.

Therefore there exists a uniform covering $\mathcal{U}$ of uX such that for every $U \in \mathcal{U}$ the sets $X_0 \cap (U \times Y)$ and $(X \times Y \setminus \Omega) \cap (U \times Y)$ are far in $pfX \times Y$. By lemma 2 there exists an open covering $\mathcal{V}_U$ of D^μ such that

$$\mathrm{St}\,(U \times \{0\}, U \times \mathcal{V}_U) \subset \bigcup_{\alpha \in \mu} A_\alpha \times p_\alpha^{-1}(0) .$$

Let F be a finite subset of μ such that

$$\bigcap_{\alpha\in F} p_\alpha^{-1}(0) \subset \mathrm{St}\,(0, \mathcal{V}_U)\,.$$

Let $y \in D^\mu$ such that $p_\alpha(y) = 0$ exactly for $\alpha \in F$. If $x \in U$, then $(x, y) \in \bigcup_{\alpha\in F} A_\alpha \times p_\alpha^{-1}(0)$ and therefore $x \in \bigcup_{\alpha\in F} A_\alpha$.

Then the directed covering $\mathcal{A}_f$ is uniform and the proof is complete.

From the proof of the above theorem we can deduce the following result.

COROLLARY 2. *A normal uniform space uX is uniformly μ-paracompact if and only if for a suitable (and thus for every) compact space Y of weight μ and for a suitable (and thus for every) compact subspace Y_0 of Y the following condition is satisfied: for every closed subspace K of $X\times Y$ disjoint from $X_0 = X\times Y_0$, there exists an open covering of the form $\mathcal{W} = \{U_\alpha \times V_\beta^\alpha\}$, where $\{U_\alpha\}$ is a uniform covering of uX and, for each α, $\{V_\beta^\alpha\}$ is a uniform covering of Y, such that*

$$K \cap \mathrm{St}\,(X_0, \mathcal{W}) = \emptyset\,.$$

Covering $\{U_\alpha\}$ may be taken belonging to $p^\mu uX$.

PROOF. Sufficiency is proved in the same way as the implication $4 \Rightarrow 1$ of theorem 2.

Necessity: the same theorem ensures the existence of a uniform covering $\mathcal{U}$ of $p^\mu uX$ and, for every $U \in \mathcal{U}$, of an open covering $\mathcal{V}_U$ of Y such that

$$K \cap \mathrm{St}\,(U\times Y_0,\; U\times \mathcal{V}_U) = \emptyset$$

for every $U \in \mathcal{U}$.

We claim that $K \cap \mathrm{St}\,(X_0, \mathcal{W}) = \emptyset$, where $\mathcal{W} = \{U\times V\colon U \in \mathcal{U}, V \in \mathcal{V}_U\}$. In fact, if $(x, y) \in \mathrm{St}\,(X_0, \mathcal{W})$, there exist $U \in \mathcal{U}$ and $V \in \mathcal{V}_U$ such that $V \cap Y_0 \neq \emptyset$ and $(x, y) \in U\times V$.

Therefore $(x, y) \in \mathrm{St}\,(U\times Y_0,\; U\times \mathcal{V}_U)$ and so $(x, y) \notin K$.

Recall that if uX and vY are two uniform spaces, the semiuniform product $uX * vY$ is the uniform space whose uniform coverings are those coverings which are normal with respect to the coverings considered in the above corollary 2.

If $p^\mu uX$ admits a point finite basis, the coverings of this form, $\{U_\alpha \times V_\beta^\alpha\}$, are a basis for the uniform coverings of $p^\mu uX * vY$ (see [F]). Therefore, by proposition 4, theorem 2 may be stated in a much nicer form, which generalizes results found in [H_1], [F].

THEOREM 3. *Let uX be a normal uniform space. The following conditions are equivalent*:

1) *uX is uniformly μ-paracompact*;
2) *For every compact space Y of weight μ and for every closed subspace $Y_0 \subset Y$, whenever A and $X \times Y_0$ are disjoint closed subsets of $X \times Y$, they are uniformly separated in $uX * Y$,*
3) *If A a closed subspace of $X \times I^\mu(X \times D^\mu)$ disjoint from $X \times \{0\}$, A and $X \times \{0\}$ are uniformly separated in $uX * I^\mu(uX * D^\mu)$.*

PROOF. It sufficies to prove $1 \Rightarrow 2$. By corollary 2, there exists a covering $\mathcal{W} = \{U_\alpha \times V_\beta^\alpha\}$, where

$\{U_\alpha\}$ is a uniform covering of $p^\mu uX$ and, for each α,

$\{V_\beta^\alpha\}$ is a uniform covering of Y, such that

$$A \cap \mathrm{St}\,(X \times Y_0, \mathcal{W}) = \emptyset .$$

If $\mathcal{S}$ is a uniform star refinement of $\mathcal{W}$, we have $\mathrm{St}\,(A, \mathcal{S}) \cap \cap \mathrm{St}(X \times Y_0, \mathcal{S}) = \emptyset$.

3. Countable uniform paracompactness.

The characterization of uniform countable paracompactness has a form which is more expressive than the general case. Equivalence $1 \Leftrightarrow 2$ of the following theorem is a uniform analogue of Dowker's theorem in [D].

THEOREM 4. *Let uX be a normal uniform space. The following conditions are equivalent*:

1) *uX is countably uniformly paracompact*;
2) *For every closed subset B of $uX * I$ disjoint from $X_0 = X \times \{0\}$, B and X_0 are uniformly separated in $uX * I$.*

3) *X is countably paracompact and for every zero set Z of βX disjoint from X there exists a uniform covering $\mathcal{U}$ of uX such that $Z \cap \mathrm{cl}_{\beta X} U = \emptyset$ for every $U \in \mathcal{U}$.*

PROOF. $1 \Rightarrow 2$. Follows from theorem 3. $2 \Rightarrow 1$. Let $f: D^\omega \to I$ be the map $f(t) = \sum_{i=0}^{\infty} (t_i/2^{i+1})$, where $t = (t_i)_{i\in\omega}$.

Consider the map $\tilde{f}: X \times D^\omega \to X \times I$ so defined: $\tilde{f}(x, t) = (x, f(t))$. $\tilde{f}$ is continuous and closed and furthermore if A is a closed subset of $X \times D^\omega$ disjoint from $X_0 = X \times \{0\}$, $\tilde{f}(A)$ is disjoint from $\tilde{f}(X_0) = X \times \{0\}$.

Since $\tilde{f}(A)$ and $\tilde{f}(X_0)$ are separated in $uX * I$, A and $X \times \{0\}$ are separated in $uX * D^\omega$. The conclusion follows from the implication $3 \Rightarrow 1$ of theorem 3.

$1 \Rightarrow 3$. Obviously X is countably paracompact (proposition 1).

Let Z be a zero set of $\beta X \setminus X$. $Z = Z(f)$ for same $f \in C(\beta X)$, $f \geq 0$.

Let $A_n = \{x \in X: f(x) > 1/(n+1)\}$. The countable open covering of uX, $\mathcal{A} = \{A_n: n \in \omega\}$ is uniform because $\mathcal{A} = \mathcal{A}_f$. Furthermore, for every $n \in \omega$, $Z \cap \mathrm{cl}_{\beta X} A_n = \emptyset$.

$3 \Rightarrow 1$. Let $\mathcal{A} = \{A_n: n \in \omega\}$ be a directed open covering of uX. From the countable paracompactness of the normal space X, there exists a countable open covering of cozero sets, $\{\mathrm{coz}\,(f_n): n \in \omega\}$ where each f_n is continuous and bounded, such that $\mathrm{coz}\,(f_n) \subset A_n$ for every $n \in \omega$; we may also assume that $\mathrm{coz}\,(f_n) \subset \mathrm{coz}\,(f_{n+1})$, for every $n \in \omega$. Let $Z = \bigcap_{n\in\omega} Z(f_n^\beta)$, where f_n^β denotes the extension of f_n to βX. There exists a uniform covering $\mathcal{U}$ of uX such that $Z \cap \mathrm{cl}_{\beta X}\,(U) = \emptyset$ for every $U \in \mathcal{U}$. Therefore, by the compactness of βX, for every $U \in \mathcal{U}$ there exists an index $n_U \in \omega$ such that $Z(f_{n_U}) \cap U = \emptyset$; thus $\mathcal{A}$ is uniform.

REFERENCES

[C] H. CORSON, *The determination of paracompactness by uniformities*, Amer. J. Math., **80** (1958), pp. 185-190.

[D] C. H. DOWKER, *On countably paracompact spaces*, Canad. J. of Math., **3** (1951), pp. 219-224.

[Du] J. DUGUNDIJ: *Topology*, Allyn and Bacon, Inc. (1966).

[F] J. FRIED - Z. FROLIK, *A characterization of uniform paracompactness* (1982), to appear.

[Ha] A. HAYES, *Uniformities with totally ordered basis have paracompact topologies*, Proc. Cambr. Phil. Soc., **74** (1973), pp. 67-68.

[H_1] A. HOHTI, *On uniform paracompactness*, Ann. Acad. Sc. Fenn., ser. A, Math. Diss., **36** (1981).

[H_2] A. HOHTI, *A theorem on uniform paracompactness* (1981), to appear.

[I] J. R. ISBELL, *Uniform spaces*, Math. Surveys n. 12, Amer. Math. Soc., Providence, Rhod Island (1964).

[K] A. KUCIA, *On coverings of a uniformity*, Coll. Math., **27** (1973), pp. 73-74.

[M_1] K. MORITA, Paracompactness and product spaces, *Fund. Math.*, **50** (1962), pp. 223-236.

[M_2] K. MORITA, *Note on paracompactness*, Proc. Jap. Acad. **37** (1961), pp. 1-3.

[M_3] K. MORITA, *Cech cohomology and covering dimension for topological spaces*, Fund. Math., **87** (1975), pp. 31-52.

[P] J. PELANT, *Cardinal reflections and point character of uniformities-counterexamples*, Seminar Uniform spaces (1973-74), Prague, pp. 49-158.

[R] M. D. RICE, *A note on uniform paracompactness*, Proc. Amer. Math. Soc. 62.2 (1977), pp. 359-362.

[Sc] E. V. SCEPIN, *On a problem of Isbell*, Soviet Math. Dokl., **16** (1975), pp. 685-687.

[Sm] SMITH, *Refinements of Lebesgue covers*, Fund. Math., **70** (1971), pp. 1-6.

[T] H. TAMANO, *On paracompactness*, Pacific J. Math., **10** (1960), pp. 1043-1047.

[V] G. VIDOSSICH, *A note on cardinal reflections in the category of uniform spaces*, Proc. Amer. M. S., **23** (1969), pp. 55-58.

Manoscritto pervenuto in redazione in forma riveduta il 3 Ottobre 1983.