RENDICONTI
del
SEMINARIO MATEMATICO
della
UNIVERSITÀ DI PADOVA

C. MENINI

Errata-Corrige : “Linearly compact rings and strongly quasi-injective modules”

Rendiconti del Seminario Matematico della Università di Padova, tome 69 (1983), p. 305-306

<http://www.numdam.org/item?id=RSMUP_1983__69__305_0>

Rend. Sem. Mat. Univ. Padova, 69 (1983)

ERRATA - CORRIGE

« Linearly Compact Rings and Strongly Quasi-Injective Modules ».

C. MENINI (*)

Introduction.

I) Statement (*d*) of THE MAIN THEOREM of [*M*] must be changed in the following form:

« (*d*) *Let* ${}_RU$ *be the minimal cogenerator of* $\mathcal{C}_{\mathcal{F}}$, $T = \mathrm{End}({}_RU)$. *Then* ${}_RU_T$ *is faithfully balanced and the module* U_T *is s.q.i.* »

Consequently the last assertion of the same theorem has to be erased.

The proof of $(a) \Rightarrow (d)$ of this theorem is now the following:

« Let us remark, first of all, that in view of Lemma 5, R is linearly compact in its U-topology. Thus, since ${}_RU$ is a selfcogenerator, by Corollary 7.4 [2], $R = \mathrm{End}(U_T)$ and hence ${}_RU_T$ is faithfully balanced. Moreover, since we already proved that $(a) \Rightarrow (b)$, U_T is q.i.. Thus it is enough to prove that R separates points and submodules of U_T. Let $L \leqslant U_T$ and let $x \in U_T$. Assume that $\mathrm{Ann}_R(x) \geqslant \mathrm{Ann}_R(L)$. Since R is linearly compact in the U-topology, Rx is linearly compact discrete. Thus, since $Rx \leqslant {}_RU$, Rx is finitely embedded. Hence, by Lemma 8, there is a finite subset $\{y_1, \dots, y_n\} \subseteq L$ such that $\mathrm{Ann}_R(x) \geqslant \bigcap_{j=1}^{n} \mathrm{Ann}_R(y_j)$. Let $S_1, \dots, S_m \in S_{\mathcal{F}}$ such that the R-submodule of ${}_RU$ spanned by x and y_j's, $j = 1, \dots, n$, is contained in ${}_RM = \bigoplus_{i=1}^{m} t_{\mathcal{F}}(E(S_i)) \leqslant {}_RU$. Since

(*) Indirizzo dell'A.: Istituto di Matematica dell'Università di Ferrara, Via Machiavelli 35, 44100 Ferrara (Italy).

${}_RM$ is quasi-injective, by the proof of Lemma 7, there exists a morphism $f\colon {}_RM^n \to {}_RM$ such that $x = \big((y_1, \dots, y_n)\big)f$. Since ${}_RM$ is a direct summand of ${}_RU$, f extends to a morphism $g\colon {}_RU^n \to {}_RU$. Thus there exist $t_1, \dots, t_n \in T$ such that $x = \sum_{i=1}^{n} t_i y_i$ and hence $x \in L$. »

II) The last assertions of Theorem 10, concerning the explicit form of ${}_RK$ and K_A are false, while equivalence of statements (*a*), (*b*) and (*c*) is true and is also true that if (*a*), (*b*), (*c*) hold then A is linearly compact in its K-topology.

The proof of (*c*) $\Rightarrow$ (*b*) rules as follows.

« Let $x \in K$. Rx is linearly compact discrete and hence $\mathrm{Soc}(Rx)$ is a direct sum of a finite number of left simple R-modules $S_1, \dots, S_n$. By hypothesis, $\mathrm{Soc}({}_RK)$ is essential in ${}_RK$. Hence $\mathrm{Soc}(Rx)$ is essential in Rx. It follows that

$$(1) \qquad Rx \leqslant \bigoplus_{1=i}^{n} t_{\mathcal{F}}\big(E(S_i)\big)\,.$$

Let us prove that R separates points and submodules of K_A. Let $L \leqslant K_A$ and let $x \in K$. Assume that $\mathrm{Ann}_R(x) \geqslant \mathrm{Ann}_R(L)$. Note that, by (1), $R/\mathrm{Ann}_R(x) \simeq Rx$ is finitely embedded. Hence, by Lemma 8, there is a finite subset $F \subseteq L$ such that $\mathrm{Ann}_R(x) \geqslant \bigcap_{l \in F} \mathrm{Ann}_R(l)$. Thus, by Lemma 7, x belongs to the submodule of K_A spanned by F and hence $x \in L$.

III) Finally, the first part of the proof of Theorem 14 is modified as follows.

« PROOF. Let $\mathcal{F}$ be the filter of open left ideals of τ and let U_R be the minimal injective cogenerator of $\mathcal{C}_{\mathcal{F}}$: ${}_RU = t_{\mathcal{F}}\Big(E\big(\bigoplus_{S \in \mathcal{S}_{\mathcal{F}}} S\big)\Big)$. Set $A = \mathrm{End}({}_RU)$. By theorem 10 and Lemma 6, ${}_RU_A$ is faithfully balanced and both the modules ${}_RU$ and U_A are s.q.i. »

At this point the remaining part of the proof works.

Manoscritto pervenuto in redazione il 7 dicembre 1982.

REFERENCES

[M] C. MENINI, *Linearly compact rings and strongly quasi-injective modules.*