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ω_μ-Additive Topological Spaces.

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0. Introduction.

In [S] R. Sikorski has introduced ω_μ-additive spaces, strengthening the axioms of Kuratowski for a topological space: namely for every cardinal number ω_μ he requires that the intersection of less than ω_μ many open sets is still an open set. This leads us to consider many properties which generalize the usual ones such as ω_μ-compactness, ω_μ-metrizability, locally ω_μ covers, and to study the related problems.

In this theory some results have had formulations quite similar to the classical ones, some results cannot have an analogous statement (e.g. the Tychonoff product theorem and the matter discussed in the remark of the second section); finally some other questions have been studied but have unsatisfactory solutions, partly because the considered context was not the right (and natural) one.

In the first section we point out that the « ω_μ-product topology » is the right topology in the product of ω_μ-additive spaces, and this remark enables us to improve a theorem of Yasui and give to it the most desirable formulation (theorem 1.1).

Although we have claimed that ω_μ-compactness is not productive, we show (theorem 2.3) that it is finitely productive: with this tool we get that ω_μ-compact spaces admit a unique ω_μ-additive uniformity (theorem 2.4), obtaining a significant improvement of a result of Reichel. By the way, notice that « ω_1-compact » means « Lindelöf »

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and consider the statement: «*the product of two Lindelöf spaces is Lindelöf*»; everybody knows that it is false in the general case, but the theorem 2.3 says that it is true in the category of ω_1-additive spaces (that is, P-spaces).

As regards the results which do not have analogues in the ω_μ-additive spaces, we observe that the reason for this pathological behaviour lies chiefly in the existence of singular cardinals (see again the remark of the second section): and this obstacle clearly cannot be overcome.

Going on, we state and prove a theorem (3.4) whose main result is that what we could define as a kind of «ω_μ-paracompactness» is really nothing but paracompactness.

In the last paragraph we remark that ω_μ-compact topologies are minimal among the ω_μ-additive (Hausdorff) ones and formulate some results concerning with the minimality of ω_μ-additive topologies.

1. Topological and uniform ω_μ-products.

Every topological space is assumed to be Hausdorff.

Throughout the paper ω_μ will denote an initial ordinal, X a Hausdorff topological space, $\boldsymbol{U}$ a uniformity given as a family of entourages of the diagonal.

We say that X is ω_μ-additive or ω_μ-topological if the intersection of less than ω_μ many open sets is an open set; $\boldsymbol{U}$ is an ω_μ-uniformity and $(X, \boldsymbol{U})$ is an ω_μ-uniform space if the intersection of less than ω_μ many entourages is an entourage.

Clearly if $\omega_\mu > \omega_\nu$, «ω_μ-topological (uniform)» implies «ω_ν-topological (uniform)»; and it is easy to show that if ω_μ is a singular ordinal, then «ω_μ-topological (uniform)» implies «$\omega_{\mu+1}$-topological (uniform)»: hence dealing with these spaces we may assume that ω_μ is a regular (initial) ordinal: we shall make this assumption from now on.

Denote by $\mathfrak{T}$, $\mathfrak{U}$ respectively the categories of topological and uniform spaces and by $\mathfrak{T}_\mu$ ($\mathfrak{U}_\mu$) the full subcategory of $\mathfrak{T}$ ($\mathfrak{U}$) consisting of ω_μ-topological (-uniform) spaces; clearly $\mathfrak{T}_0 = \mathfrak{T}$, $\mathfrak{U}_0 = \mathfrak{U}$. $\mathfrak{T}_\mu$ and $\mathfrak{U}_\mu$ are closed under arbitrary products, coproducts and subspaces: however, while coproducts and subspaces coincide in $\mathfrak{T}_\mu$ and $\mathfrak{T}$ ($\mathfrak{U}_\mu$ and $\mathfrak{U}$), we have a different behaviour of products: that is quite natural since the usual product fails to be ω_μ-additive for $\mu \neq 0$. The ω_μ-topological product of spaces X_s, $s \in S$, is denoted by $\mu \prod_{s\in S} X_s$ and defined as the Cartesian product of the X_s equipped with the topology generated by the subsets of the form: $\{(x_s)_{s\in S}: x_s \in V_s,\ V_s$ open in X_s, $V_s \neq X_s$ for less than ω_μ many indexes$\}$. Similarly if $\boldsymbol{U}_s$ is an ω_μ-uniformity on X_s, define the ω_μ-uniform product as the Cartesian product equipped with the uniformity generated by the entourages $\{((x_s), (y_s)): (x_s, y_s) \in U_s \in \boldsymbol{U}_s,\ U_s \neq X_s \times X_s$ for less than ω_μ many indexes$\}$.

It turns out easily that the topology (uniformity) introduced above is the weakest ω_μ-topology (uniformity) such that all the projections are continuous (uniformly continuous); moreover if X_s is an ω_μ-topological space and its topology is induced by the uniformity $\boldsymbol{U}_s$, then the topology of $\mu \prod\limits_{s\in S} X_s$ is induced by the uniformity of the ω_μ-uniform product. These considerations show that the concept of ω_μ-topological (uniform) product (introduced in [MS]) arises quite naturally in the category $\mathfrak{T}_\mu$ ($\mathfrak{U}_\mu$). Furthermore we recall that, for $\mu \neq 0$, ω_μ-additive regular spaces are 0-dimensional [E]: therefore such a space of weight $\mathfrak{m}$ can be embedded in $\{0, 1\}^{\mathfrak{m}}$: observe then that the usual diagonal embedding is still an embedding equipping $\{0, 1\}^{\mathfrak{m}}$ with the ω_μ-product topology. Finally in this section we consider ω_μ-metrizable spaces: it is well known that these spaces are exactly the (ω_μ-additive) ones which admit a uniformity with a totally ordered (for the reversed inclusion) basis whose cofinality is ω_μ; Yasui in [Y] obtained a partial result on the ω_μ-metrizability of a product of ω_μ-metrizable spaces: we extend his result and show that the analogy with the usual metric case is complete.

1.1. THEOREM. Let $\{X_\alpha: \alpha < \omega_\nu\}$ be a family of ω_μ-metrizable spaces each of them containing at least two points; then $\mu \prod\limits_{\alpha<\omega_\nu} X_\alpha$ is ω_μ-metrizable if and only if $\omega_\nu \leqslant \omega_\mu$.

PROOF. Necessity: suppose $\omega_\nu > \omega_\mu$ and choose a point $x \in \mu \prod\limits_{\alpha<\omega_\nu} X_\alpha$; since in the ω_μ-metrizable spaces the weight at any point is less than or equal to ω_μ, it is enough to show that the weight at x is greater than ω_μ: in fact if we take a family of power less than or equal to ω_μ of basic open neighbourhoods of x, then the collection of the indexes α for which the projection p_α of some neighbourhood is a proper subset of X_α has cardinality less than or equal to ω_μ.

Sufficiency: clearly it is enough to prove the result for $\omega_\mu = \omega_\nu$. Every X_α admits a compatible uniformity with a totally ordered basis of entourages, say $\{V_{\alpha,\lambda}: \lambda < \omega_\mu,\ \lambda_1 < \lambda_2$ implies $V_{\alpha,\lambda_1} \supseteq V_{\alpha,\lambda_2}\}$. Consider the sets:

$$W_\xi = \{((x_\alpha)_{\alpha<\omega_\mu}, (y_\alpha)_{\alpha<\omega_\mu}): (x_\alpha, y_\alpha) \in V_{\alpha,\xi},\ \forall \alpha < \xi\};$$

using the fact that ω_μ is regular observe that the family $\{W_\xi : \xi < \omega_\mu\}$ is a basis for the ω_μ-uniform product which, as we have remarked above, is admissible for $\mu \prod_{\alpha<\omega_\mu} X_\alpha$. ■

2. ω_μ-compact spaces.

DEFINITION. We say that a space is ω_μ-compact if it is ω_μ-additive and every open cover has a subcover of power less than ω_μ.

Trivially, taking the complements, one sees that an ω_μ-additive space is ω_μ-compact if and only if every family of closed sets such that each intersection of less than ω_μ many elements is non-empty, has non-empty intersection.

These spaces have been studied by several authors (see e.g. [MS], [R], [S], [ST], [Y]) and were introduced by R. Sikorski who used for them the term « ω_μ-bicompact ».

In the following proposition we collect some folkloristic properties of ω_μ-compact spaces.

2.1. PROPOSITION:

i) closed subspaces of ω_μ-compact spaces are ω_μ-compact;

ii) ω_μ-compact subspaces of ω_μ-additive spaces are closed;

iii) ω_μ-additive continuous images of ω_μ-compact spaces are ω_μ-compact;

iv) ω_μ-compact spaces are normal.

The proof is trivial, using ω_μ-additivity. ■

Clearly a discrete space of power less than ω_μ is ω_μ-compact; more generally the coproduct of less than ω_μ many ω_μ-compact spaces is ω_μ-compact too; moreover we get an example of a non discrete ω_μ-compact space as follows: denote by $X \cup \{\infty\}$ a space where X is a discrete space of power greater than or equal to ω_μ (always assumed regular) and ∞ is a point whose neighbourhoods are the sets which contain ∞ and have complements of power less than ω_μ.

REMARK. Given an ω_μ-additive space X, each of the following conditions implies the next one:

i) every subset of X of power greater than or equal to ω_μ has a complete accumulation point;

ii) X is ω_μ-compact;

iii) every subset of X whose power is a regular cardinal number greater than or equal to ω_μ has a complete accumulation point.

One can show i) $\Rightarrow$ ii) using the argument of [K], Problem 5 I, with the suitable modifications; the proof of ii) $\Rightarrow$ iii) is standard.

It is clear that here and in other cases the difference of the behaviour between compact and ω_μ-compact spaces depends on the existence of singular initial ordinal numbers. We can give an example for which ii) $\Rightarrow$ i) fails to be true.

For $n \in \mathbb{N}$, let X_n be the discrete space of power ω_n, $X_n^+ = X_n \cup \{p_n\}$ where the neighbourhoods of the point p_n are the sets: $\{p_n\} \cup V$ where $V \subseteq X_n$, $X_n \setminus V$ is countable; $X = \bigoplus X_n^+$. X is ω_1-compact since it is the coproduct of a countable family of ω_1-compact spaces, and clearly its power is ω_ω; nevertheless the set of all the points of X has no complete accumulation point since, as X_n^+ is an open set in X for every $n \in \mathbb{N}$, every point has a neighbourhood of power less than ω_ω. ■

We say that a filter $\mathcal{F}$ is an ω_μ-filter if the intersection of less than ω_μ many members of $\mathcal{F}$ belongs to $\mathcal{F}$. The ω_μ-compact spaces can be characterized in terms of ω_μ-filters.

2.2. Proposition. Let X be an ω_μ-additive space. The following are equivalent:

i) X is ω_μ-compact;

ii) every ω_μ-filter has a cluster point;

iii) every ω_μ-filter is contained in a convergent filter.

Proof. i) $\Rightarrow$ ii): by definitions.

ii) $\Rightarrow$ i): using the regularity of ω_μ, one can show that a family of closed sets such that each intersection of less than ω_μ many sets is non-empty, can be enlarged to an ω_μ-filter, hence the conclusion follows.

ii) $\Rightarrow$ iii): standard using ω_μ-additivity.

iii) $\Rightarrow$ ii): trivial. ■

We say that an ω_μ-filter on X is an ω_μ-ultrafilter if there is no ω_μ-filter which contains it properly; we show that an ω_μ-ultrafilter $\mathcal{F}$ is an ultrafilter: otherwise there would exist $A \subseteq X$, $A \notin \mathcal{F}$ such that

$A \cap F \neq \emptyset$, $\forall F \in \mathcal{F}$, and $A \cup \mathcal{F}$ could be embedded in an ω_μ-filter containing $\mathcal{F}$ properly. Moreover observe that if X has non-measurable power and $\mu \neq 0$, there is no free ω_μ-ultrafilter, hence ω_μ-ultrafilters cannot give any information about ω_μ-compactness.

It is known that ω_μ-compactness is not productive in the category $\mathfrak{T}_\mu$: indeed in this category the product of ω_μ many compact spaces need not be even ω_μ-compact (see [MS]). However we can provide a result in the positive direction:

2.3. THEOREM. If X and Y are ω_μ-compact spaces, then $X \times Y$ is ω_μ-compact.

PROOF. Let $\mathcal{W}$ be a cover of $X \times Y$, $\mathcal{V}$ a basic cover which refines $\mathcal{W}$ and denote by p_1, p_2 the canonical projections onto X and Y respectively. For every $x \in X$ take $\{p_2(V)\colon V \in \mathcal{V},\ p_1(V) \ni x\}$: this is an open cover of Y and let $\mathcal{V}_{S(x),x} = \{V_{s,x}\colon s \in S(x)\}$ be a subcover of it, indexed in a set $S(x)$ of power less than ω_μ; now for every $V_{s,x}$ choose an open set $U_{s,x}$ containing x and such that $U_{s,x} \times V_{s,x}$ belongs to $\mathcal{V}$: $U_x = \bigcap_{s \in S(x)} U_{s,x}$ is still an open set containing x since X is ω_μ-additive, hence $\{U_x\colon x \in X\}$ is an open cover of X and has a subcover $\{U_x\colon x \in X'\}$ indexed in a subset X' of X of power less than ω_μ. Finally it is easy to show that $\{U_x \times V_{s,x}\colon x \in X',\ s \in S(x)\}$ is an open cover which refines $\mathcal{V}$ and its power is less than ω_μ since ω_μ is regular. ■

It is well known (see [I], VII.30) that a completely regular space is ω_μ-additive if and only if it admits an ω_μ-uniformity (however notice that our definitions are a little more general than the ones used by Isbell). With the following theorem we provide a result which generalizes in the most natural direction a fundamental theorem in the theory of compact spaces; by the way we observe that it contains as a particular case the theorem 5.2 of [R].

2.4. THEOREM. An ω_μ-compact space X admits exactly one ω_μ-uniformity.

PROOF. By Proposition 2.1 iv), X is completely regular, hence it admits an ω_μ-uniformity. Now let $\boldsymbol{U}$ be a compatible ω_μ-uniformity, V an open subset of $X \times X$ containing the diagonal Δ; since $\Delta = \cap \{U\colon U \in \boldsymbol{U}\}$ and $X \times X$ is ω_μ-compact by the theorem 2.3, there exists a family $\{U_\alpha\colon \alpha < \omega_\nu\}$, $\omega_\nu < \omega_\mu$, such that $\bigcap_{\alpha < \omega_\nu} U_\alpha \subseteq V$: therefore V belongs to $\boldsymbol{U}$ and $\boldsymbol{U}$ is the fine uniformity. ■

3. Paracompactness.

DEFINITION. A family $\mathcal{A}$ of subsets of a space X is said to be locally ω_μ if for every $x \in X$ there exists a neighbourhood V of x such that the family $\{A \in \mathcal{A}: A \cap V \neq \emptyset\}$ has power less than ω_μ (hence « locally ω_0 » means « locally finite »).

3.1. LEMMA. Let X be an ω_μ-additive space, $\mathcal{A}$ a locally ω_μ family of subsets of X; then, denoting by $\operatorname{cl} A$ the closure of the set A, we have:

$$\cup \{\operatorname{cl} A: A \in \mathcal{A}\} = \operatorname{cl} \cup \{A: A \in \mathcal{A}\}.$$

The proof is plain using ω_μ-additivity. ■

3.2. LEMMA. Let X be an ω_μ-additive regular space, $\mu \neq 0$, such that every open cover is refined by a locally ω_μ open cover; then X is normal.

PROOF. Let A, B be disjoint closed subsets of X; for every $x \in B$ there exists a clopen set V_x which contains x and does not meet A. Let $\mathcal{U}$ be a locally ω_μ open refinement of the cover $\{X \setminus B\} \cup \{V_x: x \in B\}$. Put $\mathcal{U}^* = \{U \in \mathcal{U}: U \subseteq V_x$ for some $x \in B\}$: we have $A \cap \operatorname{cl} U = \emptyset$ for every $U \in \mathcal{U}^*$ and $B \subseteq \cup \{U \in \mathcal{U}^*\}$; finally $\operatorname{cl} \cup \{U: U \in \mathcal{U}^*\} = \cup \{\operatorname{cl} U: U \in \mathcal{U}^*\}$ does not meet A. ■

3.3. LEMMA. Let X be an ω_μ-additive normal space, $\mu \neq 0$. For every pair of disjoint closed sets A, B there exists a clopen which contains A and does not meet B.

PROOF. Inductively choose a sequence of open sets A_n such that:

$$A \subseteq A_1 \subseteq \operatorname{cl} A_1 \subseteq A_2 \subseteq \ldots \subseteq A_n \subseteq \operatorname{cl} A_n \subseteq A_{n+1} \subseteq \ldots \subseteq X \setminus B$$

Then $\bigcup_{n \in \mathbb{N}} A_n = \bigcup_{n \in \mathbb{N}} \operatorname{cl} A_n$ which is closed by ω_μ-additivity. ■

3.4. THEOREM. Let X be an ω_μ-additive regular space, $\mu \neq 0$. The following are equivalent:

i) every open cover is refined by a locally ω_μ open cover;

ii) every open cover is refined by a locally ω_μ arbitrary cover;

iii) every open cover is refined by a locally ω_μ closed cover;

iv) every open cover is refined by a locally ω_μ clopen cover;

v) X is ultraparacompact;

vi) X is paracompact.

PROOF. Clearly part of the statement and of the proof of this theorem follows the theorem 5.1.4 of [E].

i) $\Rightarrow$ ii): trivial.

ii) $\Rightarrow$ iii): let $\mathcal{U}$ be an open cover, $\mathcal{V}$ an open cover such that $\{\mathrm{cl}\, V\colon V \in \mathcal{V}\}$ is a refinement of $\mathcal{U}$, $\mathcal{W}$ a locally ω_μ cover which refines $\mathcal{V}$: then the cover $\{\mathrm{cl}\, W\colon W \in \mathcal{W}\}$ is a locally ω_μ refinement of $\mathcal{U}$.

iii) $\Rightarrow$ iv): let $\mathcal{U}$ be an open cover, $\mathcal{A}$ a locally ω_μ closed cover which refines $\mathcal{U}$; for every $x \in X$ let $V(x)$ be a neighbourhood of x which meets less than ω_μ many elements of $\mathcal{A}$, and take a locally ω_μ closed cover $\mathcal{V}$ which refines $\{V(x)\colon x \in X\}$. For every $A \in \mathcal{A}$ choose $U_A \in \mathcal{U}$ such that $U_A \supseteq A$ and put: $W_A = U_A \setminus \bigcup \{V\colon V \in \mathcal{V},\ V \cap A = \emptyset\}$. Clearly W_A is open by lemma 3.1 hence $\{W_A\colon A \in \mathcal{A}\}$ is an open cover (since $W_A \supseteq A$); we show that this cover is locally ω_μ: in fact if $V \in \mathcal{V}$ does not meet A, then V does not meet W_A; moreover every $V \in \mathcal{V}$ is contained in a certain $V(x)$ and so it meets less than ω_μ many elements (of $\mathcal{A}$ hence) of $\{W_A\colon A \in \mathcal{A}\}$. Now for every $x \in X$ choose a neighbourhood $I(x)$ of x such that $I(x)$ meets less than ω_μ many elements $V \in \mathcal{V}$: by the regularity of ω_μ and the previous observations we get: $I(x) \cap W_A \neq \emptyset$ holds for less than ω_μ many W_A. Finally by lemmas 3.2 and 3.3, for every $A \in \mathcal{A}$ take a clopen set W'_A such that $A \subseteq W'_A \subseteq W_A$: clearly $\{W'_A\colon A \in \mathcal{A}\}$ is a locally ω_μ clopen cover which refines $\mathcal{U}$.

iv) $\Rightarrow$ v): let $\mathcal{U}$ be an open cover, $\mathcal{V}$ a locally ω_μ clopen cover which refines $\mathcal{U}$. For every $x \in X$ put $A_x = \bigcap \{V\colon V \in \mathcal{V},\ V \ni x\}$, $B_x = \bigcup \{V\colon V \in \mathcal{V},\ V \not\ni x\}$, $W_x = A_x \setminus B_x$; since $x \in V$ holds for less than ω_μ many $V \in \mathcal{V}$, A_x is open; furthermore B_x is closed by lemma 3.1 so that W_x is open (trivially it is closed and contains x). Now observe that if a point y belongs to W_x, then $V \in \mathcal{V}$ contains x if and only if it contains y: hence $W_x = W_y$ and conclude that the sets W_x form a partition of clopen sets which refines $\mathcal{U}$.

v) $\Rightarrow$ vi) $\Rightarrow$ i): trivial. ■

A standard argument (see again theorem 5.1.4 [E]) shows that the six conditions of theorem 3.4 are equivalent to:

vii) every open cover of X is refined by a cover which is union of ω_μ many locally ω_μ families of open sets.

Using this condition we can prove the following corollary which contains the third statement of VII.31 [I] and as a particular case that every ω_μ-compact space is ultraparacompact, for $\mu \neq 0$.

3.5. COROLLARY. For $\mu \neq 0$, every regular ω_μ-additive space X such that every open cover has a subcover of power at most ω_μ is ultraparacompact. ■

Moreover using an ω_μ-metrization theorem due to Wang Shu Tang ([W], theorem 6) we can obtain as a corollary that, for $\mu \neq 0$, every ω_μ-metrizable space is ultraparacompact ([H]).

4. Minimal ω_μ-additive topologies.

By means of proposition 2.1 we get the following theorem:

4.1. THEOREM. Let $f\colon X \to Y$ be a continuous injective function, X ω_μ-compact, Y ω_μ-additive; then f is a homeomorphism onto its image. Hence the ω_μ-compact topologies are minimal among the ω_μ-additive ones. ■

In view of this theorem, we shall conclude with some considerations about the minimal objects in the category $\mathfrak{T}_\mu$; we shall omit the details since the proofs in this case and in the usual topological spaces proceed at the same rate, naturally without losing sight of ω_μ-additivity. For references and definitions related to this matter, see [E], [PT].

4.2. LEMMA. Given an ω_μ-additive space, the topology consisting of all open domains ([E]) is still ω_μ-additive. ■

DEFINITIONS. We say that an ω_μ-additive space X is ω_μ-absolutely closed if it is a closed subset of every ω_μ-additive (Hausdorff) space containing it. A proximate cover is a family of subsets of X whose union is dense in X.

4.3. THEOREM. An ω_μ-additive space X is ω_μ-absolutely closed if and only if every open cover has a proximate subcover of power less than ω_μ. ■

4.4. PROPOSITION:

i) closed domains of ω_μ-absolutely closed spaces are ω_μ-absolutely closed;
ii) continuous ω_μ-additive images of ω_μ-absolutely closed spaces are ω_μ-absolutely closed.

PROOF. Use theorem 4.3. ■

4.5. THEOREM:

i) let $f: X \to Y$ be a continuous injective function, X ω_μ-absolutely closed and semiregular, Y ω_μ-additive; then f is a homeomorphism onto its image.
ii) a topology on X is minimal among the ω_μ-additive ones if and only if it is ω_μ-absolutely closed and semiregular.

PROOF. Use proposition 4.4 to prove i); use i) and lemma 4.2 to prove ii). ■

Finally, for ω_μ-additive regular spaces, we can state the following theorem which summarizes the previous results:

4.6. THEOREM. Let X be an ω_μ-additive regular space. The following are equivalent:

i) X is ω_μ-compact;
ii) the topology of X is minimal with respect to the ω_μ-additive ones;
iii) X is ω_μ-absolutely closed;
iv) every open cover of X has a proximate subcover of power less than ω_μ. ■

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