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COMPACT OPERATORS ON ORLICZ SPACES

by J. J. Uhl, Jr. *)

Recently there has been some effort [2, 3, 7] devoted to representations of the general bounded linear operator in the Orlicz spaces L^{Φ} of Banach space valued functions. On the hand, it appears that comparatively little effort has been directed to obtaining information about special types of bounded linear operators on the Orlicz spaces. The purpose of this paper is to investigate some properties of compact linear operators defined on, or with values in, an Orlicz space.

In the first section, preliminary results concerning Orlicz spaces, whose underlying measure is possibly only finitely additive, will be given to establish the setting of the work which follows. The second section is concerned with the problem of characterizing the compact operators on or into a fairly general class of Orlicz spaces and investigating some of their properties- including their property of being limits of linear operators with a finite dimensional range. The results obtained in this analysis will then be applied, in section three, to existing representations of bounded linear operators on Orlicz spaces to obtain a characterization of operator valued set function which represent compact operators.

I. Some Preliminaries.

Throughout this paper, μ is a finitely additive non-negative extended real valued set function defined on a field Σ of subsets of a point set Ω.

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Further it is assumed that μ has the finite subset property; i.e. if $E \in \Sigma$ and $\mu(E) = \infty$, then there exists $E_0 \in \Sigma$, $E_0 \subset E$ such that $0 < \mu(E_0) < \infty$. $\mathfrak{X}$ and $\mathfrak{Y}$ are Banach spaces with conjugate spaces $\mathfrak{X}^*$ and $\mathfrak{Y}^*$ repectively. $B(\mathfrak{X}, \mathfrak{Y})$ is the Benach space of all bounded linear operators from $\mathfrak{X}$ to $\mathfrak{Y}$.

Φ is a continuous Young's function [9] with complementary function Ψ. L^Φ $(\Omega, \Sigma, \mu, \mathfrak{X})(=L^\Phi(\mathfrak{X}))$ is the linear space of all totally μ-measurable [4] $\mathfrak{X}$ valued functions f satisfying $\int_\Omega \Phi(\| f \| / k) d\mu \leq 1$ for some positive k, where the integration procedure here and throughout unless noted otherwise is that of [4]. Upon the identification of functions which differ on at most a μ-null set, $L^\Phi(\mathfrak{X})$ becomes a normed linear space under each of the equivalent norms N_Φ and $\|\cdot\|_\Phi$ defined for $f \in L^\Phi(\mathfrak{X})$ by

$$N_\Phi(f) = \inf \left\{ k > 0 : \int_\Omega \Phi(\| f \| / k) d\mu \leq 1 \right\}$$

and

$$\| f \|_\Phi = \sup \left\{ \int_\Omega \|f\| \, \| g \| \, d\mu : g \in L^\Psi(\mathfrak{X}^*), \; N_\Psi(g) \leq 1 \right\}$$

respectively. $M^\Phi(\mathfrak{X})$ denotes the closure of the subspace of $L^\Phi(\mathfrak{X})$ spanned by the simple functions. If Φ satisfies the Δ_2-condition ($\Phi(2x) \leq \leq K\Phi(x)$ for all x and some finite K), then $M^\Phi(\mathfrak{X}) = L^\Phi(\mathfrak{X})$ [7, 9].

A partition $\pi = \{E_n\}$ is a finite collection of disjoint Σ-sets, each of finite measure. The class of partitions II directed by the partial ordering $\pi_1 \leq \pi_2$ if each member of π_1 is the union of members of π_2.

LEMMA 1. *Let $\pi = \{E_n\}$ be a partition. If E_π is defined for $f \in M^\Phi(\mathfrak{X})$ by*

$$E_\pi(f) = \sum_\pi \frac{\int_{E_n} f d\mu}{\mu(E_n)} \chi_{E_n}$$

(χ_E is the characteristic or indicator function of E), then

a) $N_\Phi(E_\pi(f)) \leq N_\Phi(f)$,

and

b) $\lim_\pi N_\Phi(E_\pi(f)-f)=0$ for all $f \in M^\Phi(\mathcal{H})$.

PROOF. The proof of (*a*) which follows from the convexity of Φ can be constructed from [7, Thm II.5] and [7, Thm I.9] and will be omitted. (*b*) is an immediate consequence of [4, II.3.6] an the fact that the E_π are contractions. Q.E.D.

For ease of reference, we shall now introduce a space of set functions which will play a major role in the theorems which follow.

DEFINITION 2. [7] Let $\Sigma_0 \subset \Sigma$ be the ring of sets of finite μ-measure. $V^\Phi(\mathcal{H})$ is the space of all finitely additive μ-continuous $\mathcal{H}$ valued functions F defined of Σ_0 which satisfy

$$N_\Phi(F) = \inf \left\{ k>0 : \sup_\pi \sum_\pi \Phi\left(\frac{\| F(E_n) \|}{k\mu(E_n)} \right) \mu(E_n) \leq 1 \right\}$$

is finite.

According to [7, Thm I.16], N_Φ furnishes a norm for $V^\Phi(\mathcal{H})$ under which $V^\Phi(\mathcal{H})$ becomes a Banach space. Of crucial importance in the later work is the following theorem which is proved in [7, Section V].

THEOREM 3. *Let Φ be continuous. The conjugate space to $M^\Phi(\mathcal{H})$, $(M^\Phi(\mathcal{H}))^*$ is equivalent to $V^\Psi(\mathcal{H}^*)$. If $l \in M^\Phi(\mathcal{H})^*$, there exists a unique $G \in V^\Psi(\mathcal{H}^*)$ such that*

$$l(f) = \int_\Omega f dG, \quad f \in M^\Phi(\mathcal{H}),$$

where the integral is that of [1]. *Conversely each $G \in V^\Psi(\mathcal{H}^*)$ defines a member of $M^\Phi(\mathcal{H})^*$ through the above formula. Moreover, if $M^\Phi(\mathcal{H})$ is normed with $\| \cdot \|_\Phi$, the induced norm of l above is $N_\Psi(G)$.*

This section will be terminated with a theorem essentially due to Vala [8]. It is given here its present form for use later and because of its possible independent interest.

THEOREM 4. *Let $\{t_\tau, \tau \in I\}$ be a net of compact linear operators (i.e. operators which map bounded sets into relatively compact sets) mapping $\mathfrak{X}$ into $\mathfrak{Y}$. If*

(i) $\lim_\tau t_\tau(x) = t(x)$ exists for all $x \in \mathfrak{X}$,

and

(ii) there exists a compact operator $s \in B(\mathfrak{X}, \mathfrak{Y})$ such that $\| t_\tau(x) \| \leq \| s(x) \|$ for all $x \in \mathfrak{X}$, $\tau \in I$, then t is compact and $\lim_\tau \| t_\tau - - t \| = 0$ in the uniform operator topology.

PROOF. Suppose conditions (i) and (ii) are satisfied and $\varepsilon > 0$ is given. Since s is compact, there exitst a finite covering $\{A_n\}$ of the unit ball U of $\mathfrak{X}$ such that $x_1, x_2 \in A_n$ implies $\| s(x_1) - s(x_2) \| < \varepsilon/3$. Now choose an element $x_n \in A_n$. By (i) and (ii), there exists for each x_n an index $\tau_n \in I$ such that $\tau', \tau \geq \tau_n$ implies $\| t_{\tau'}(x_n) - t_\tau(x_n) \| < \varepsilon/3$. Since I is directed, there exists $\tau_0 \in I$ such that $\tau_0 \geq \tau_n$ for all τ_n defined above. Thus, if $x \in U$ is arbitrary and A_n is chosen such that $x \in A_n$, then for $\tau, \tau' \geq \tau_0$, we have

$$\| t_{\tau'}(x) - t_\tau(x) \| \leq \| t_{\tau'}(x) - t_{\tau'}(x_n) \|$$

$$+ \| t_{\tau'}(x_n) - t_\tau(x_n) \| + \| t_\tau(x_n) - t_\tau(x) \|$$

$$\leq 2 \| s(x) - (x_n) \| + \| t_{\tau'}(x_n) - t_\tau(x_n) \|,$$

by condition (ii),

$$< 2\varepsilon/3 + \varepsilon/3 = \varepsilon,$$

by the choice of A_n and τ_0. Thus, for $\tau, \tau' \geq \tau_0$, and $x \in U$,

$$\| t_{\tau'}(x) - t_\tau(x) \| < \varepsilon.$$

It follows that $\lim_{\tau, \tau'} \| t_\tau - t_{\tau'} \| = 0$, and that

$$\lim_\tau \| t_\tau - t \| = 0. \qquad \text{Q.E.D.}$$

II. Compact operators on and into Orlicz spaces.

This section is concerned with the characterization of compact linear operators which map an Orlicz space into a range space $\mathcal{Y}$ or are defined on $\mathcal{Y}$ with range in an Orlicz space. The characterization obtained will then be applied to the problem of approximating these compact operators by bounded linear operators with a finite dimensional range. The following theorem is the main result of this paper.

THEOREM 5. *Let $\mathcal{X}$ be reflexive, Φ be continuous. If Ψ obeys the Δ_2-condition ($\Psi(2x)\leq K\Psi(x)$), then $t\in B(M^\Phi(\mathcal{X}), \mathcal{Y})$ is compact if and only if*

(*i*) *For each $E\in\Sigma_0$, the operator $T(E):\mathcal{X}\to\mathcal{Y}$ defined by $T(E)[x]=t(x\chi_E)$ is a compact linear operator,*

and

(*ii*) $\lim_{\pi} \| t\cdot E_\pi - t \| = 0$ *in the uniform operator tolology.*

PROOF. (Sufficiency). First we shall show that condition (*i*) implies $t\cdot E_\pi$ is compact for each partition π. For, if $\pi=\{E_n\}$ is a partition, then

(*)
$$t\cdot E_\pi(f)=t\left[\sum_\pi \frac{\int_{E_n} f d\mu}{\mu(E_n)}\chi_{E_n}\right]$$
$$=\sum_\pi \frac{t\left(\int_{E_n} f d\mu \chi_{E_n}\right)}{\mu(E_n)}=\sum_\pi \frac{T(E_n)\left[\int_{E_n} f d\mu\right]}{\mu(E_n)},$$

by the definition of $T(E_n)$. By condition (i), $T(E_n)$ is compact for each $E_n\in\pi$. Hence $T(E_n)$ takes boundes sets in $\mathcal{X}$ into conditionally compact sets in $\mathcal{Y}$. Now, if $\| f \|_\Phi\leq 1$, then

$$\left\| \int_{E_n} f d\mu \right\| \leq \int_\Omega \| f \| \chi_{E_n} d\mu \leq \| \chi_{E_n} \|_\Psi$$

by the Hölder inequality [7, 9]. Hence $\left\{\int_{E_n} f d\mu : \|f\|_\Psi \leq 1\right\}$ is a bounded set in $\mathcal{X}$ for each E_n, and hence

$$\left\{ T(E_n)\left[\int_{E_n} f d\mu\right] : \|f\|_\Phi \leq 1 \right\}$$

is conditionally compact in $\mathcal{Y}$. From this and (*) above, we infer immediately that $t \cdot E_\pi$ is compact. To complete the proof of the sufficiency, note that by condition (ii), $\lim_\pi \|t \cdot E_\pi - t\| = 0$. Thus t, as the operator limit of compact operators, is itself compact.

(Necessity). Suppose $t \in B(M^\Phi(\mathcal{X}), \mathcal{Y})$ is compact and for $E \in \Sigma_0$ consider $S = \{T(E)[x] : \|x\| \leq 1\} = \{t(x\chi_E\} : \|x\| \leq 1\}$. Since $\{x\chi_E : \|x\| \leq 1\}$ is a bounded set in $M^\Phi(\mathcal{X})$ and t is compact, it follows that S is conditionally compact in $\mathcal{Y}$. This proves the necessity of condition (i).

To establish (ii), let $y^* \in \mathcal{Y}^*$ be arbitrary. Then $y^* t \in M^\Phi(\mathcal{X})^*$ and according to theorem 3 there exists a unique $G(=G_y) \in \vee \Psi(\mathcal{X}^*)$ such that for $f \in M^\Phi(\mathcal{X})$,

$$y^* t(f) = \int_\Omega f dG.$$

Now, for the same $y^* \in \mathcal{Y}^*$, consider

$$y^* t \cdot E_\pi(f) = \int_\Omega E_\pi(f) dG = \int_\Omega \left[\sum_\pi \frac{\int_{E_n} f d\mu}{\mu(E_n)} \chi_{E_n} \right] dG$$

$$= \sum_\pi \frac{G(E_n)\left[\int_{E_n} f d\mu\right]}{\mu(E_n)} = \int_\Omega f dG_\pi ,$$

where

$$G_\pi(E) = \sum_\pi \frac{G(E_n)}{\mu(E_n)} \mu(E_n \cap E) \text{ for } E \in \Sigma_0 .$$

Another application of theorem 3 and the introduction of the adjoint operators t^* and $(t\cdot E_\pi)^*$ of t and $t\cdot E_\pi$ respectively yeld

$$\| t^*(y^*) \| = \| y^*t \| = N_\Psi(G), \text{ and}$$

$$\| (t\cdot E_\pi)(y^*) \| = \| y^*t\cdot E_\pi \| = N_\Phi(G_\pi).$$

But according to [7, Thm I.9], $N_\Psi)G_\pi) \leq N_\Psi(G)$. Whence $\| (t\cdot E_\pi)^*(y^*) \| \leq \| t^*(y^*) \|$. Moreover, by [7, Cor. IV.7], the hypothesis of the theorem ensures $\lim_\pi N_\Psi(G_\pi - G) = 0$. Thus $\lim_\pi \| (t\cdot E_\pi)^*(y^*) - - t^*(y^*) \| = 0$. Further note that since t is compact so are the $t\cdot E_\pi$, and Schauder's theorem guarantees the compactness of t^* and $(t\cdot E_\pi)^*$. Hence the hypothesis of theorem 4 is satisfied by the net $\{(t\cdot E_\pi)^*, \pi\in\Pi\}$ and the bounding compact operator t^*. Using theorem 4, we have

$$0 = \lim_\pi \| (t\cdot E_\pi)^* - t^* \| = \lim_\pi \| t\cdot E_\pi - t \|. \qquad \text{Q.E.D.}$$

Focusing our attention on the problem of approximating compact operators by bounded linear operators with a finite dimensional range, we have the following.

COROLLARY 6. *Under the hypothesis of theorem 5 and with the further assumption that $\mathcal{X}$ is the scalar field, each compact member of $B(M^\Phi(\mathcal{X}), \mathcal{Y})$ is the limit, in the uniform operator topology, of a net of bounded linear operators whose range is finite dimensional.*

PROOF. By theorem 5, $\lim_\pi \| t\cdot E_\pi - t \| = 0$ for each compact t in $B(M^\Phi(\mathcal{X}), \mathcal{Y})$. But, since $\mathcal{X}$ is the scalar field the range of each $t\cdot E_\pi$ is contained in the span of $\{T(E_n) : E_n \in \pi\}$ which is a finite dimensional subspace of $\mathcal{Y}$. Q.E.D.

It can be shown that if $\mathcal{X}$ and $\mathcal{Y}$ are Banach spaces, such that any compact member of $B(\mathcal{X}, \mathcal{Y})$ can be approximated in the uniform operator topology by bounded linear operators whise ranges are finite dimensional, then corollary 6 remains true for compact members of $B(M^\Phi(\mathcal{X}), \mathcal{Y})$. The details are omitted here.

REMARK. The hypothesis of theorem 5 (and hence corollary 6) can be weakened slightly. As the proof. shows, the Δ_2-condition was needed for Ψ only to permit the conclusion $\lim_\pi N_\Psi(G_\pi - G) = 0$. If we specify instead that $\lim_\pi N_\Psi(G_\pi - G) = 0$ for all $G \in V^\Psi(\mathcal{X}^*)$, then theorem 5 remains true. Thus, in addition to all the $L^p(\mathcal{X})(1 < p < \infty)$ spaces where $\mathcal{X}$ is a reflexive Banach space, the conclusion of theorem 5 holds for $L^1(\mathcal{X})$ when $\mathcal{X}$ is the scalar field and the underlying measure space is finite and countably additive.

The hypothesis of theorem 5 cannot be weakened further than the slight generalization indicated above. For, if (i) and (ii) of theorem 5 provide a necessary condition for an operator in $B(M^\Phi(\mathcal{X}), \mathcal{Y})$ to be compact, then these conditions must be necessary when $\mathcal{X}$ is the scalar field. This implies $\lim_\pi \| lE_\pi - l \| = 0$ for all $l \in M^\Phi(\mathcal{X})^*$. This and theorem 3 imply $\lim_\pi N_\Psi(G_\pi - G) = 0$ for all $G \in V^\Psi(\mathcal{X}^*)$.

Next we shall turn our attention to the case where t is a compact operator with its range in $M^\Phi(\mathcal{X})$.

THEOREM 7. *Let Φ be continuous and $\mathcal{X}$ be the scalar field. An operator t in $B(\mathcal{Y}, M^\Phi(\mathcal{X}))$ is compact if and only if* $\lim \| E_\pi t - t \| = 0$ *in the uniform operator topology. Consequently each compact member of $B(\mathcal{Y}, M^\Phi(\mathcal{X}))$ is the limit in the uniform operator topology of bounded linear operators whose ranges are finite dimensional.*

PROOF. (Sufficiency). Suppose $\lim_\pi \| E_\pi t - t \| = 0$ and consider for $y \in \mathcal{Y}$

$$E_\pi t(y) = \sum_\pi \frac{\int_{E_n} t(y) d\mu}{\mu(E_\pi)} \chi_{E_n}.$$

Since $\mathcal{X}$ is the scalar field the range of $E_\pi t$ is contained in the span of $\{\chi_{E_n} : E_\pi \in \pi\}$ which is finite dimensional.

(Necessity). Let $t \in B(\mathcal{Y}, M^\Phi(\mathcal{X}))$ be compact. According to lemma 1,

$$N_\Phi(E_\pi t(y)) \leq N_\Phi(t(y)),$$

$$\text{and } \lim_\pi N_\Phi(E_\pi t(y) - t(y)) = 0 \text{ for all } y \in \mathcal{Y}.$$

Thus the hypothesis of theorem 4 is satisfied by the net $\{E_\pi t, \pi\in\Pi\}$ and the dominating compact operator t. By theorem 4, $\lim_\pi \| E_\pi t - t \| = 0$.
Q.E.D.

Combining theorems 5 and 7 is a result on quasitriangular operators.

DEFINITION 8. A member t of $B(\mathcal{X}, \mathcal{X})(=B(\mathcal{X}))$ is called quasi-triangular if there exists an increasing net $\{\mathcal{X}_\tau, \tau\in I\}(\mathcal{X}_{\tau_1}\subseteq\mathcal{X}_{\tau_2}$ if $\tau_1\leq\tau_2)$ of finite dimensional subspaces of $\mathcal{X}$ and projections E_τ of $\mathcal{X}$ onto $\mathcal{X}_\tau$ such that $\| E_\tau \|\leq 1$ and $\lim_\tau \| tE_\tau - E_\tau t E_\tau \| = 0$.

THEOREM 9. *Let $\mathcal{X}$ be the scalar field, Φ be continuous and Ψ obey the Δ_2-condition (or, more generally, suppose $\lim_\pi N_\Psi(G_\pi - G) = 0$ for all $G\in V^\Psi(\mathcal{X})$. Then every compact member of $B(M^\Phi(\mathcal{X}))$ is quasi-triangular.*

PROOF. Consider the net of projections $\{E_\pi, \pi\in\Pi\}$. Then $\{E_\pi(M^\Phi(\mathcal{X})), \pi\in\Pi\}$ is an increasing net of finite dimensional subspaces of $M^\Phi(\mathcal{X})$ and $\| E_\pi \| \leq 1$. Moreover,

$$\begin{aligned}
&\lim_\pi \| tE_\pi - E_\pi t E_\pi \| \\
&\leq \lim_\pi \| tE_\pi - t \| + \lim_\pi \| t - E_\pi t \| + \lim_\pi \| E_\pi t - E_\pi t E_\pi \| \\
&\leq 2 \lim_\pi \| E_\pi t - t \| + \lim_\pi \| t - E_\pi t \|,
\end{aligned}$$

since $\| E_\pi \| = 1$,

$= 0$ by theorems 5 and 7. Q.E.D.

III. Operator valued set functions which represent compact operators.

In [2, 3, and 7], representations of the general bounded linear operator on $M^\Phi(\mathcal{X})$ are given. In each case, the representation of $t\in B(M^\Phi(\mathcal{X}), \mathcal{Y})$ takes the form

$$t(f) = \int_\Omega f dH, \text{ for } f\in M^\Phi(\mathcal{X}),$$

where H is some $B(\mathfrak{X}, \mathfrak{Y})$-valued additive set function and the integral is that of Bartle [1]. The purpose of this section is to characterize those $B(\mathfrak{X}, \mathfrak{Y})$ valued finitely additive set functions which qualify to represent compact members of $B(M^{\Phi}(\mathfrak{X}), \mathfrak{Y}))$.

To be more precise, let $W^{\Psi}(B(\mathfrak{X}, \mathfrak{Y}))$ be the space of all μ-continuous finitely additive $B(\mathfrak{X}, \mathfrak{Y})$-valued set functions H defined on Σ_0 (the ring of sets of finite μ-measure) which satisfy

$$(i)\ y^*H \in V^{\Psi}(\mathfrak{X}^*) \text{ for all } y^* \in \mathfrak{Y}^*,$$

and

$$(ii)\ \| H \|_{W^{\Psi}} = \sup_{\|y^*\| \leq 1} N_{\Psi}(y^*H) < \infty .$$

Then, according to [7. Cor V. 9], if Φ is continuous $B(M^{\Phi}(\mathfrak{X}), \mathfrak{Y}) \cong W^{\Psi}(B(\mathfrak{X}, \mathfrak{Y}))$ with $t \in B(M^{\Phi}(\mathfrak{X}), \mathfrak{Y}))$ having the representation $t(f) = \int_{\Omega} f dH$ for all $f \in M^{\Phi}(\mathfrak{X})$ and some $H \in W^{\Psi}(B(\mathfrak{X}, \mathfrak{Y}))$. The same result says that $\| t \| = \| H \|_{W^{\Psi}}$ where $\| t \|$ is the operator norm induced on t by $\| \ \|_{\Phi}$ on $M^{\Phi}(\mathfrak{X})$. The following result characterizes those membres of $W^{\Psi}(\mathfrak{X}, \mathfrak{Y}))$ which represent compact operators on the $M^{\Phi}(\mathfrak{X})$ spaces under consideration.

THEOREM 10. *Let Φ be continuous, Ψ obey the Δ_2-condition, and $\mathfrak{X}$ be reflexive. If $t \in B(M^{\Phi}(\mathfrak{X}), \mathfrak{Y})$ is represented by $H \in W^{\Psi}(B(\mathfrak{X}, \mathfrak{Y}))$ (i.e. $t(f) = \int_{\Omega} f dH$) then t is compact of and only if*

$$(i)\ H(E) \in B(\mathfrak{X}, \mathfrak{Y}) \textit{ is compact for each } E \in \Sigma_0$$

and

$$(ii)\ \lim_{\pi} \| H - H_{\pi} \|_{W^{\Psi}} = 0, \textit{ where for each partition}$$

$$H_{\pi}(E) = \sum_{\pi} \frac{H(E_n)}{\mu(E_n)} \mu(E_n \cap E),\ E \in \Sigma_0 .$$

PROOF. Let $t(f) = \int_{\Omega} f dH$ for all $f \in M^{\Phi}(\mathfrak{X})$. By setting $f = x\chi_E$ for some $x \in \mathfrak{X}$ and $E \in \Sigma_0$, it is not difficult to see that $H(E)[x] = t(x\chi_E)$. In addition,

$$tE_{\pi}(f) = \int_{\Omega} E_{\pi}(f) dH$$

$$= \sum_{\pi} \frac{H(E_n)\left[\int_{E_n} f d\mu\right]}{\mu(E_n)} = \int_{\Omega} f dH_\pi .$$

Thus H_π represents tE_π. Hence, by theorem 5, t is compact if and only if $T(E)(=H(E))$ is compact and $\lim_\pi \| t - tE_\pi \| = 0$ which, in turn, is true if and only if $H(E)$ is compact and $\lim_\pi \| H - H_\pi \|_{W^\Psi} = 0$. Q.E.D.

Finally we note that all of the considerations of this paper (with certain straightforward modifications) remain true for the spaces of set fuctions $V^\Phi(\mathcal{H})$ studied in [7]. ($M^\Phi(\mathcal{H})$ is replaced by $S^\Phi(\mathcal{H})$ and $E_\pi(G) = G_\pi$. Precise statements of these results in the $S^\Phi(\mathcal{H})$ context are omitted here.

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