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ON THE STRUCTURE OF CLIFFORD ALGEBRAS

*Nota *) di GUEORGUIY V. BABIKOV (a Sverdlovsk)*

The principal result of the paper by G. B. Rizza published in this journal ¹⁾ may be defined as following:

« Clifford algebra C_n of index n over the field of real numbers is direct sum of 2^{n-2} fields of quaternions. »

This result is wrong. The mistake may be found in the inattentive application of the method of mathematical induction.

After the above given result has been considered and proved for $n = 3$ the process of proovement for $n > 3$ follows in such a way.

Let $x = \alpha_1 + \alpha_2 i_n \in C_n$ and $\alpha_1, \alpha_2 \in C_{n-1}$.

Suppose that $C_{n-1} = C_{\frac{1}{2}}^{(1)} + C_{\frac{1}{2}}^{(2)} + \dots + C_{\frac{1}{2}}^{(2^{n-2})}$ where $C_{\frac{1}{2}}^{(k)}$ is isomorphic to a field of quaternions.

Then $x = \sum_{\frac{1}{2}}^{2^{n-2}} [\alpha_1^{(k)} + \alpha_2^{(k)} i_n]$ and the sets $\{\alpha_1^{(k)} + \alpha_2^{(k)} i_n\}$ are isomorphic to C_2 when $\alpha_1^{(k)}$ and $\alpha_2^{(k)}$ run through all possible real quaternions. This last statement is wrong for $n \geq 4$. If a set $\{\alpha_1^{(k)} + \alpha_2^{(k)} i_n\}$ is isomorphic to $C_2 = C_{\frac{1}{2}}^{(1)} + C_{\frac{1}{2}}^{(2)}$, then i_n and base units of $C_{\frac{1}{2}}^{(k)}$ have specific forms depending on base units of $C_{\frac{1}{2}}^{(1)}$ and $C_{\frac{1}{2}}^{(2)}$.

The result by Rizza disproves also the following contradictory example.

*) Pervenuta in redazione il 5 gennaio 1963.

Indirizzo dell'A.: Pedagogical institute, Sverdlovsk (U.R.S.S.)

Suppose we have algebra C_n ($n > 3$) over the field of real numbers. Let us take an element $x = i_1 + i_2 i_3 i_4$.

Let $x = q_1 \dot{+} q_2 \dot{+} \dots \dot{+} q_{2^{n-2}}$, where q_k belongs to some field of quaternions $C_4^{(k)}$.

Then $x^2 = q_1^2 \dot{+} q_2^2 \dot{+} \dots \dot{+} q_{2^{n-2}}^2 = (i_1 + i_2 i_3 i_4)^2 = 0$. As the above sum is direct, we have $q_1^2 = q_2^2 = \dots = q_{2^{n-2}}^2 = 0$. Consequently $q_1 = q_2 = \dots = q_{2^{n-2}} = 0$, i.e. $x = 0$.

We come to a contradiction.

The structure of Clifford algebras defined by the following theorem ²⁾:

« Clifford algebras over the arbitrary field F of characteristic $\neq 2$ under different n (with exactness to isomorphism) are exhausted by the following list:

- 1) C_{8m} is algebra of matrices of power 2^{4m} over F .
- 2) C_{8m+1} is direct product $A \times K$, where A is algebra of matrices of power 2^{4m} over F and K is algebra of complex numbers over F .
- 3) C_{8m+2} is direct product $A \times Q$, where A is algebra of matrices of power 2^{4m} over F and Q is algebra of quaternions over F .
- 4) C_{8m+3} is algebra $Q \times A \dot{+} Q \times B$, where A and B are algebras of matrices of power 2^{4m} over F and Q is algebra of quaternions over F .
- 5) C_{8m+4} is direct product $A \times Q$, where A is algebra of matrices of power 2^{4m+1} over F and Q is algebra of quaternions over F .
- 6) C_{8m+5} is direct product $A \times K$, where A is algebra of matrices of power 2^{4m+2} over F and K is algebra of complex numbers over F .
- 7) C_{8m+6} is algebra of matrices of power 2^{4m+3} over F .
- 8) C_{8m+7} is direct sum $A \dot{+} B$, where A and B are algebras of matrices of power 2^{4m+3} over F . »

Relying upon this it's possible to solve the question about the distribution of divisors of zero in Clifford algebras.

- [1] RIZZA G. B.: *Sulla struttura delle algebre di Clifford*. Rend. sem. mat. Univ. Padova, 1954, 23, N. 1, 91-99.
- [2] BABIKOV G. V.: *Predstavleniya algebr Clifforda nad proizvol'nimi polynami*. Uchenie zapiski Uralskogo Universiteta, 1960, Vip. 23 N. 2. tetr. 3. s. 19-26