RENDICONTI
del
SEMINARIO MATEMATICO
della
UNIVERSITÀ DI PADOVA

PRABHAKAR RAGHUNATH KHANDEKAR

A note on Jacobi polynomials

Rendiconti del Seminario Matematico della Università di Padova, tome 33 (1963), p. 170-172

<http://www.numdam.org/item?id=RSMUP_1963__33__170_0>

A NOTE ON JACOBI POLYNOMIALS

Nota *) *di* PRABHAKAR RAGHUNATH KHANDEKAR (*a Bhopal*)

1. In this note the result of [1, p 236] has been generalized. The results are believed to be new.

2. Formulae required in the proof:

We have [2, p 263]

$$(2.1)\qquad \left(\frac{d}{dk}\right)^{k} P_n^{(\alpha,\beta)}(x) = \frac{(1+\alpha+\beta+n)_k}{2^k} P_{n-k}^{(\alpha+k,\beta+k)}(x)$$

and [3, p 75]

$$(2.2)\quad P_n^{(\alpha,\beta)}(z) = \frac{\Gamma(1+\alpha+n)}{\Gamma(1+\alpha+\beta+n)} \sum_{t=0}^{n} \frac{(2t+\alpha'+\beta'+1)\Gamma(t+\alpha'+\beta'+1)}{(n-t)!\,\Gamma(1+\alpha+t)} \times$$

$$\times \frac{\Gamma(1+\alpha+\beta+n+t)}{\Gamma(2t+\alpha'+\beta'+2)} P_t^{(\alpha',\beta')}(x) u^t$$

$${}_3F_2 \cdot \left[\begin{matrix} -n+t,\ 1+\alpha+\beta+n+t, 1+\alpha'+t;\ u \\ 1+\alpha+t,\ 2t+\alpha'+\beta'+2 \end{matrix}\right]$$

where $z = 1 - (1-x)u$.

---

*) Pervenuta in redazione il 13 agosto 1962.

Indirizzo dell'A.: Motilal Vigyan Mahavidyalaya, Bhopal (India).

3. (2.1) and (2.2) will give

$$(3.1)\qquad \left(\frac{d}{dx}\right)^k P_n^{(\alpha,\beta)}(z) = \frac{\Gamma(1+\alpha+n)}{2^k\Gamma(1+\alpha+\beta+n)}$$

$$\times \sum_{t=0}^{n-k} \frac{(2t+\alpha'+\beta'+1)\Gamma(t+\alpha'+\beta'+1)\Gamma(1+\alpha+\beta+n+k+t)}{(n-k-t)\,!\,\Gamma(1+\alpha+k+t)\Gamma(2t+\alpha'+\beta'+2)}$$

$$\times P_t^{(\alpha',\beta')}(x)u^{t+k}{}_3F_2\left[\begin{matrix}-n+k+t, 1+\alpha+\beta+n+k+t, 1+\alpha'+t; u\\ 1+\alpha+k+t, 2t+\alpha'+\beta'+2\end{matrix}\right]$$

u being treated as constant and $k \leqslant n$.

Now (3.1) with the orthogonal property of Jacobi Polynomials will give

$$(3.2)\qquad \int_{-1}^{+1}(1-x)^{\alpha'}(1+x)^{\beta'}\left[\left(\frac{d}{dx}\right)^k P_n^{(\alpha,\beta)}\{1-(1-x)u\}\right]\cdot$$

$$\cdot\left[\left(\frac{d}{dx}\right)^s P_m^{(\varrho,\sigma)}\{1-(1-x)v\}\,dx\right]$$

$$= \frac{\Gamma(1+\alpha+n)\Gamma(1+\varrho+m)2^{\alpha'+\beta'+1-k-s}u^k v^s}{\Gamma(1+\alpha+\beta+n)\Gamma(1+\varrho+\sigma+m)} \times \sum_{t=0}^{\min(n-k,m-s)}\cdot$$

$$\cdot\frac{\Gamma(t+\alpha'+\beta'+1)\Gamma(1+\alpha+\beta+n+k+t)\Gamma(1+\varrho+\sigma+m+s+t)(uv)^t}{(n-k-t)!(m-s-t)!\Gamma(1+\alpha+k+t)\Gamma(1+\varrho+s+t)\Gamma(2t+\alpha'+\beta'+2)}$$

$$\times\frac{\Gamma(1+\alpha'+t)\Gamma(1+\beta'+t)}{\Gamma(2t+\alpha'+\beta'+1)t\,!}\cdot$$

$$\cdot{}_3F_2\left[\begin{matrix}-n+k+t, 1+\alpha+\beta+n+k+t, 1+\alpha'+t; u\\ 1+\alpha+k+t, 2t+\alpha'+\beta'+2\end{matrix}\right]$$

$$\times{}_3F_2\left[\begin{matrix}-m+s+t, 1+\varrho+\sigma+m+s+t, 1+\alpha'+t; v\\ 1+\varrho+s+t, 2t+\alpha'+\beta'+2\end{matrix}\right]$$

Real values of α, α', β, β', ϱ, σ are > -1, $k \leqslant n$, $s \leqslant m$ and u and v are constants.

When $v = 1 = v$ and α, β, α', β' are all equal to zero, (3.2) reduces to the result [1, p 236].

I am thankful to Dr. B. R. Bhonsle for his guidance in the preparation of this note.

REFERENCES

1. CHATTERJEA S. K., *Rend. Sem. Mat. Padova*, 30 (1960), pagg. 232-236.
2. RAINVILLE E. D., *Special functions*, 1960.
3. W. A. AL-SALAM., *Port. Math.*, 15-Fasc., 3. 1960, pagg. 73-77.