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A NOTE ON JACOBI POLYNOMIALS

*Nota ** di PRABHAKAR RAGHUNATH KHANDEKAR (a Bhopal)

1. In this note the result of [1, p 236] has been generalized. The results are believed to be new.

2. Formulae required in the proof:

We have [2, p 263]

$$(2.1) \quad \left(\frac{d}{dk}\right)^k P_n^{(\alpha, \beta)}(x) = \frac{(1 + \alpha + \beta + n)_k}{2^k} P_{n-k}^{(\alpha+k, \beta+k)}(x)$$

and [3, p 75]

$$(2.2) \quad P_n^{(\alpha, \beta)}(z) = \frac{\Gamma(1 + \alpha + n)}{\Gamma(1 + \alpha + \beta + n)} \sum_{t=0}^n \frac{(2t + \alpha' + \beta' + 1) \Gamma(t + \alpha' + \beta' + 1)}{(n-t)! \Gamma(1 + \alpha + t)} \times \\ \times \frac{\Gamma(1 + \alpha + \beta + n + t)}{\Gamma(2t + \alpha' + \beta' + 2)} P_t^{(\alpha', \beta')}(x) u^t \\ {}_2F_1 \left[\begin{matrix} -n + t, 1 + \alpha + \beta + n + t, 1 + \alpha' + t; u \\ 1 + \alpha + t, 2t + \alpha' + \beta' + 2 \end{matrix} \right]$$

where $z = 1 - (1 - x)u$.

*) Pervenuta in redazione il 13 agosto 1962.

Indirizzo dell'A.: Motilal Vigyan Mahavidyalaya, Bhopal (India).

3. (2.1) and (2.2) will give

$$(3.1) \quad \left(\frac{d}{dx}\right)^k P_n^{(\alpha, \beta)}(z) = \frac{\Gamma(1 + \alpha + n)}{2^k \Gamma(1 + \alpha + \beta + n)} \\ \times \sum_{t=0}^{n-k} \frac{(2t + \alpha' + \beta' + 1)\Gamma(t + \alpha' + \beta' + 1)\Gamma(1 + \alpha + \beta + n + k + t)}{(n - k - t)! \Gamma(1 + \alpha + k + t)\Gamma(2t + \alpha' + \beta' + 2)} \\ \times P_t^{(\alpha', \beta')}(x) u^{t+k} {}_2F_2 \left[\begin{matrix} -n + k + t, 1 + \alpha + \beta + n + k + t, 1 + \alpha' + t; u \\ 1 + \alpha + k + t, 2t + \alpha' + \beta' + 2 \end{matrix} \right]$$

u being treated as constant and $k \leq n$.

Now (3.1) with the orthogonal property of Jacobi Polynomials will give

$$(3.2) \quad \int_{-1}^{+1} (1-x)^{\alpha'} (1+x)^{\beta'} \left[\left(\frac{d}{dx}\right)^k P_n^{(\alpha, \beta)} \{1 - (1-x)u\} \right] \\ \cdot \left[\left(\frac{d}{dx}\right)^s P_m^{(\rho, \sigma)} \{1 - (1-x)v\} dx \right] \\ = \frac{\Gamma(1 + \alpha + n)\Gamma(1 + \rho + m)2^{\alpha' + \beta' + 1 - k - s} u^k v^s}{\Gamma(1 + \alpha + \beta + n)\Gamma(1 + \rho + \sigma + m)} \times \sum_{t=0}^{\min(n-k, m-s)} \\ \cdot \frac{\Gamma(t + \alpha' + \beta' + 1)\Gamma(1 + \alpha + \beta + n + k + t)\Gamma(1 + \rho + \sigma + m + s + t)(uv)^t}{(n - k - t)!(m - s - t)!\Gamma(1 + \alpha + k + t)\Gamma(1 + \rho + s + t)\Gamma(2t + \alpha' + \beta' + 2)} \\ \times \frac{\Gamma(1 + \alpha' + t)\Gamma(1 + \beta' + t)}{\Gamma(2t + \alpha' + \beta' + 1)t!} \\ \cdot {}_2F_2 \left[\begin{matrix} -n + k + t, 1 + \alpha + \beta + n + k + t, 1 + \alpha' + t; u \\ 1 + \alpha + k + t, 2t + \alpha' + \beta' + 2 \end{matrix} \right] \\ \times {}_2F_2 \left[\begin{matrix} -m + s + t, 1 + \rho + \sigma + m + s + t, 1 + \alpha' + t; v \\ 1 + \rho + s + t, 2t + \alpha' + \beta' + 2 \end{matrix} \right]$$

Real values of $\alpha, \alpha', \beta, \beta', \rho, \sigma$ are > -1 , $k \leq n$, $s \leq m$ and u and v are constants.

When $v = 1 = v$ and $\alpha, \beta, \alpha', \beta'$ are all equal to zero, (3.2) reduces to the result [1, p 236].

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REFERENCES

1. CHATTERJEA S. K., *Rend. Sem. Mat. Padova*, 30 (1960), pagg. 232-236.
2. RAINVILLE E. D., *Special functions*, 1960.
3. W. A. AL-SALAM., *Port. Math.*, 15-Fasc., 3. 1960, pagg. 73-77.