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A NOTE CONCERNING THE INTEGRALS INVOLVING THE DERIVATIVES OF LEGENDRE POLYNOMIALS

Nota () di* BLAGOJ SAZDOV POPOV *(a* Skopje)

Here is an alternative, shorter, proof of the result proved by S. K. Chatterjee¹).

From the well known formula

$$\int_{-1}^1 uv^{(n)} dx = \left[\sum_{k=1}^n (-1)^{k-1} u^{(k-1)} v^{(n-k)} \right]_{-1}^1 + (-1)^n \int_{-1}^1 v u^{(n)} dx,$$

it follows that

$$\begin{aligned} \int_{-1}^1 \frac{d^r P_m(x)}{dx^r} \frac{d^s P_n(x)}{dx^s} dx &= \left[\sum_{k=1}^r (-1)^{k-1} \frac{d^{s+k-1} P_n(x)}{dx^{s+k-1}} \frac{d^{r-k} P_m(x)}{dx^{r-k}} \right]_{-1}^1 + \\ &+ (-1)^r \int_{-1}^1 P_m(x) \frac{d^{r+s} P_n(x)}{dx^{r+s}} dx, \quad (1 \leq r \leq m, 1 \leq s \leq n). \end{aligned}$$

Since

$$\begin{aligned} \left| \frac{d^r P_n(x)}{dx^r} \right|_{x=1} &= P_n^{(r)}(1) = (-1)^{n-r} P_n^{(r)}(-1) = (2r-1)!! \binom{n+r}{n-r}, \\ (2r-1)!! &= 1.3 \dots (2r-1), \end{aligned}$$

(*) Pervenuta in Redazione il 10 febbraio 1959.

Indirizzo dell'A.: Istituto matematico, Università di Skopje (Jugoslavia).

¹) S. K. CHATTERJEE, *On certain definite Integrals involving Legendre's Polynomials*, Rend. Sem. Mat. Univ. Padova, vol. XXVII, 1957, pp. 144-148.

and

$$\int_{-1}^1 P_m(x) \frac{d^{r+s}P_n(x)}{dx^{r+s}} dx = 0, \quad m > n - r - s,$$

we have

$$\int_{-1}^1 \frac{d^r P_m(x)}{dx^r} \frac{d^s P_n(x)}{dx^s} dx = [1 + (-1)^{m+n-r-s}] 2^{1-s-r} \sum_{k=1}^r (-1)^{k-1} (r-k)! \\ \times (s+k-1)! \binom{n+s+k-1}{s+k-1} \binom{n}{s+k-1} \binom{m+r-k}{r-k} \binom{m}{r-k}, \\ (m-r \geq n-s).$$

The same method can be employed to obtain some other more general integral formulae for the product of the derivatives of Legendre polynomials.

We find

$$\int_{-1}^1 \frac{d^p P_l(x)}{dx^p} \frac{d^r P_m(x)}{dx^r} \frac{d^s P_n(x)}{dx^s} dx = \\ = K \sum_{k=1}^p \sum_{v=0}^{k-1} (-1)^{k-1} \binom{k-1}{v} (p-k)! (r+k-v-1)! (s+v)! \\ \times \binom{l+p-k}{p-k} \binom{l}{p-k} \binom{m+r+k-v-1}{r+k-v-1} \binom{m}{r+k-v-1} \binom{n+s+v}{s+v} \binom{n}{s+v} \\ K = [1 + (-1)^{m+n+l-p-r-s}] 2^{1-p-r-s}, \quad l-p \geq (m-r) + (n-s), \\ 1 \leq p \leq l, \quad 1 \leq r \leq m, \quad 1 \leq s \leq n.$$