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AN EXTENDED OPPORTUNITY-BASED AGE REPLACEMENT POLICY (*)

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Abstract. – *The present study proposes an extended opportunity-based age replacement policy where opportunities occur according to a Poisson process. When the age, x of the system satisfies $x < S$ for a prespecified value S, a corrective replacement is conducted if the objective system fails. In case x satisfies $S \leq x < T$ for another prespecified value T, we take an opportunity to preventively replace the system by a new one with probability p, and do not take the opportunity with probability $1 - p$. At the moment x reaches T, a preventive replacement is executed independently of opportunities. The long-term average cost of the proposed policy is formulated. The conditions under which optimal values for S and T exist for a prespecified value of T and S, respectively, are then clarified. Numerical examples are also presented to illustrate the theoretical underpinnings of the proposed replacement policy formulation.*

Keywords: Extended opportunity-based age replacement, renewal reward process, long-term average cost, optimal policy.

1. INTRODUCTION

Personal computers(PCs) have, in recent years, become an essential key component in an office as well as in a manufacturing system. They are used to preserve and update significant information on their hard disks. For this reason, the moment a PC fails, we may lose such significant information. It is most important to backup files on the hard disks periodically [8, 13, 14]. However, it is also important to make a schedule for their preventive replacements before failures and we focus on the preventive replacement for a PC in the present study.

On the other hand, technology associated with PCs have shown their remarkable development in the past two decades. New models of PCs have

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been released every half-year and therefore the prices of old models have become lower. New versions of operating systems and their related major application software have also been released frequently. Hard disks with larger volumes have also been developed. These factors have sometimes compelled us to replace our PCs by new ones even when they have not failed. This indicates that these factors can be regarded as opportunities to conduct preventive replacements of our PCs. Such a tendency can be considered to continue in the future as well.

Among several kinds of opportunistic maintenance policies [2, 5–7, 10, 17, 18], an opportunity age replacement policy by Dekker and Dijkstra [6] seems to be suitable to the above PC replacement problem. Under their policy, a corrective replacement is performed to the objective system whenever it fails, but the system is preventively replaced by taking an opportunity if its age exceeds a prespecified value. The disadvantage of their policy is that a preventive replacement is conducted only at an opportunity, and that we cannot replace our PCs by new ones until an opportunity occurs even when they have become old enough to replace.

In this study, we extend the model by Dekker and Dijkstra. Under the extended model (1) the system is replaced by a new one whenever it fails (2) when its age, x satisfies $x < S$ for a prespecified value of $S(> 0)$, no preventive replacement is performed regardless of opportunity occurrences (3) if x satisfies $S \leq x < T(S < T)$ for S and $T(0 \leq S < T)$, we take an opportunity to perform a preventive replacement with probability p, and do not with probability $1 - p$, and (4) at the moment x reaches T, the system is preventively replaced independently of opportunities. In the case of $S = T$, the proposed preventive replacement policy becomes identical to an age replacement policy [1].

The idea associated with (S, T) in the above is similar to that of (t, T)-policy proposed by Ohnishi *et al.* [9]. Under their policy, we conduct a minimal repair to the objective system when it fails if $x < t$. In the case of $t \leq x < T$, a corrective replacement is performed to the system at a failure. When x becomes equal to T, the system is preventively replaced. Onishi *et al.* provided a general formulation to such a problem based on the semi-Markov decision process to discuss its optimal policy. Nevertheless, the (t, T)-policy does not take an opportunistic replacement into account.

This study first formulates the long-term average cost of the extended opportunity-based age replacement policy. Secondly, the conditions under which an optimal value for S and T exists for a prespecified value of T

and S, respectively, are clarified. Numerical examples are also presented to illustrate the theoretical underpinnings of the proposed replacement policy formulation.

2. MODEL FORMULATION

Let us assume that opportunities occur according to a Poisson process with rate λ. If we focus on preventive replacements at opportunities by neglecting S and T constraints, the cumulative distribution function(*cdf*) of the time between consecutive preventive replacements due to opportunities is given by

$$G_p(t) = 1 - e^{-\int_0^t \lambda p dx} = 1 - e^{-\lambda pt}. \tag{1}$$

It should be noted that the *cdf* in equation (1) is that of an exponential distribution with parameter λp, and that it has a memoryless property. The derivation of this result can easily be obtained from Block *et al.* [3].

Let $F(t)$ and $\overline{F}(t)$ express the *cdf* and the survivor function of the failure distribution of the system, respectively. It is convenient to introduce $f(t)$ and $r(T)$ which respectively signify the probability density function(*pdf*) and the failure rate associated with $F(t)$.

Let $C(S,T)$ signify the long-term average cost per unit time over an infinite time operation. Then we have, from the renewal reward theory [11, 12],

$$C(S,T) = \frac{B(S,T)}{A(S,T)}, \quad 0 \le S \le T, \tag{2}$$

where $A(S,T)$ and $B(S,T)$ express the expected cycle length and the long-term average cost per cycle, respectively, and one cycle corresponds to the time between consecutive replacements.

Let c_1 and $c_2(< c_1)$ respectively denote the cost for a corrective and a preventive replacement. Then, $A(S,T)$ and $B(S,T)$ are given by

$$\begin{aligned} A(S,T) &= \int_0^S tf(t)dt + \int_S^T t\left[e^{-\lambda p(t-S)} f(t) + \lambda p e^{-\lambda p(t-S)} \overline{F}(t)\right] dt \\ &\quad + T e^{-\lambda p(T-S)} \overline{F}(T) \\ &= \int_0^S \overline{F}(t)dt + \int_S^T e^{-\lambda p(t-S)} \overline{F}(t)dt, \end{aligned} \tag{3}$$

$$
\begin{aligned}
B(S,T) &= c_1 F(S) + c_1 \int_S^T e^{-\lambda p(t-S)} f(t)dt \\
&\quad + c_2 \int_S^T \lambda p e^{-\lambda p(t-S)} \overline{F}(t)dt + c_2 e^{-\lambda p(t-S)} \overline{F}(T) \\
&= (c_1 - c_2)\left[F(S) + \int_S^T e^{-\lambda p(t-S)} f(t)dt\right] + c_2. \qquad (4)
\end{aligned}
$$

In the above, we have formulated the long-term average cost, $C(S,T)$. When we deal with a PC replacement problem, we need not necessarily minimize $C(S,T)$ with respect to S and T simultaneously. In some cases, we may seek for an optimal T^* for a given S since the value of S will be prespecified because of depreciation of PCs. In other cases, the value of T may represent the period of durability of PCs and we need an optimal S^* for a given T. For these reasons, we consider to minimize $C(S,T)$ in relation to S and T for a specified value of T and S, respectively in the following.

3. OPTIMAL POLICY

3.1. Optimal S

This subsection examines the existence of an optimal S that minimizes $C(S,T)$ for a fixed value of T.

We first have

$$
C(T,T) = \frac{(c_1 - c_2)F(T) + c_2}{\int_0^T \overline{F}(t)dt}, \qquad (5)
$$

which is the long-term average cost of the age replacement policy. We also have

$$
C(0,T) = \frac{(c_1 - c_2)\int_0^T e^{-\lambda pt} f(t)dt + c_2}{\int_0^T e^{-\lambda pt}\overline{F}(t)dt}. \qquad (6)
$$

To obtain an optimal S^* which minimizes $C(S,T)$ in equation (2), we differentiate $C(S,T)$ with respect to S. By letting $\partial C(S,T)/\partial S \geq 0$, we have

$$
\beta(S,T)\int_0^S \overline{F}(t)dt - F(S) \geq \frac{c_2}{c_1 - c_2}, \qquad (7)
$$

where

$$
\beta(S,T) = \frac{\int_S^T e^{-\lambda pt} f(t)dt}{\int_S^T e^{-\lambda pt}\overline{F}(t)dt}. \qquad (8)
$$

We here notice that if $r(t) \equiv f(t)/\overline{F}(t)$ is strictly increasing in t, then $\beta(S,T)$ is also strictly increasing in S and that

$$r(S) < \beta(S,T) < r(T), \quad 0 \leq S < T. \tag{9}$$

In the following we assume that $r(t)$ strictly increases with t.

Let $Q_1(S|T)$ denote the left-hand-side of inequality (7). Then $Q_1(S|T)$ is increasing in S and we have

$$Q_1(0|T) = 0, \tag{10}$$

$$Q_1(T|T) = r(T)\int_0^T \overline{F}(t)dt - F(T). \tag{11}$$

When the equation $Q_1(T|T) = c_2/(c_1 - c_2)$ has a solution, let us denote by $\tilde{T}$ its solution. We also let $\mu = \int_0^\infty \overline{F}(t)dt$. It is then noted that if $r(\infty) \equiv \lim_{t\to+\infty} r(t) > c_1/[\mu(c_1-c_2)]$, then $\tilde{T}$ is finite and exists uniquely.

From the above analysis, we have the following optimal policy:

(1) if $r(\infty) \leq c_1/[\mu(c_1 - c_2)]$, then the equation $Q_1(T|T) = c_2/(c_1 - c_2)$ does not have any solutions. In this case, we have $S^* = T$ and the long-term average cost is given by equation (5).

(2) If $r(\infty) > c_1/[\mu(c_1 - c_2)]$, then the equation $Q_1(T|T) = c_2/(c_1 - c_2)$ have a unique finite solution, $\tilde{T}$.

   (i) If $T > \tilde{T}$, there exists a unique and finite S^* $(0 < S^* < T)$ which satisfies $Q_1(S|T) = c_2/(c_1 - c_2)$. The resulting long-term average cost is then given by

$$C(S^*,T) = (c_1 - c_2)\frac{\int_{S^*}^T e^{-\lambda pt} f(t)dt}{\int_{S^*}^T e^{-\lambda pt}\overline{F}(t)dt}. \tag{12}$$

   (ii) If $T \leq \tilde{T}$, then we have $S^* = T$ and the long-term average cost is given by equation (5).

We also have that S^* is a decreasing function of T, since $\partial Q_1(S|T)/\partial S > 0$.

3.2. Optimal T

In this subsection, we seek an optimal T which minimizes $C(S,T)$ for a fixed value of S.

By taking the partial differential of $C(S,T)$ in reference to T, we notice that $\partial C(S,T)/\partial T \geq 0$ agrees with

$$r(T)\left[\int_0^S \overline{F}(t)dt + \int_S^T e^{-\lambda p(t-S)}\overline{F}(t)dt\right] - \left[F(S) + \int_S^T e^{-\lambda p(t-S)} f(t)dt\right] \geq \frac{c_2}{c_1 - c_2}. \quad (13)$$

Let $Q_2(T|S)$ express the left-hand-side of inequality (13). Then $Q_2(T|S)$ increases with T, and we have

$$Q_2(S|S) = r(S)\int_0^S \overline{F}(t)dt - F(S). \quad (14)$$

It should be noted in equation (14) that $Q_2(S|S) = Q_1(S|S)$. Hence, if $r(\infty) > c_1/[\mu(c_1 - c_2)]$, then $Q(S|S) = c_2/(c_1 - c_2)$ has a unique finite solution, $\tilde{S}$.

The similar analysis to that in Section 3.1 yields the following optimal policy:

(1) if $r(\infty) \leq c_1/[\mu(c_1 - c_2)]$, then the equation $Q_2(S|S) = c_2/(c_1 - c_2)$ does not have any solutions. In this case, $T^* = +\infty$.

(2) If $r(\infty) > c_1/[\mu(c_1 - c_2)]$, then the equation $Q_2(S|S) = c_2/(c_1 - c_2)$ has a unique finite solution $\tilde{S}$.

   (i) If $S < \tilde{S}$, then there exists a unique and finite T^* which satisfies $Q_2(T|S) = c_2/(c_1 - c_2)$. The long-term average cost when $T = T^*$ becomes

$$C(S,T^*) = (c_1 - c_2)r(T^*). \quad (15)$$

   (ii) If $S \geq \tilde{S}$, then we have $T^* = S$ and the corresponding long-term average cost is given by replacing T by S in equation (5).

In addition, it can be noted that T^* is a decreasing function of S since $\partial Q_2(T|S)/\partial S > 0$.

In the above, we have clarified the conditions under which a unique finite optimal value of T and S exists for a fixed value of S and T, respectively.

4. NUMERICAL EXAMPLES

This section presents numerical examples to illustrate the theoretical underpinnings of the proposed preventive replacement policy when the underlying failure distribution is a Gamma distribution with shape parameter 2.

The *cdf* and the *pdf* of such a Gamma distribution are written as

$$F(t) = 1 - (1 + \alpha t)e^{-\alpha t}, \tag{16}$$
$$f(t) = \alpha^2 t e^{-\alpha t}, \quad t > 0. \tag{17}$$

Under the Gamma failure time distribution whose *cdf* and *pdf* are given by equations (16) and (17), respectively, we have

$$\begin{aligned} A(S,T) &= \frac{2}{\alpha}\left(1 - e^{-\alpha S}\right) - S e^{-\alpha S} \\ &+ \frac{1}{\alpha + \lambda p}\left[e^{-\alpha S} - e^{-(\alpha+\lambda p)T + \lambda p S}\right] \\ &+ \frac{\alpha e^{\lambda p S}}{(\alpha + \lambda p)^2}\Big\{[1 + (\alpha + \lambda p)S]e^{-(\alpha+\lambda p)S} \\ &\qquad - [1 + (\alpha + \lambda p)T]e^{-(\alpha+\lambda p)T}\Big\}, \end{aligned} \tag{18}$$

$$\begin{aligned} B(S,T) &= (c_1 - c_2)\Bigg\{1 - e^{-\lambda p(T-S)} - \frac{\lambda p}{\alpha + \lambda p}\left[e^{-\alpha S} - e^{-(\alpha+\lambda p)T + \lambda p S}\right] \\ &- \frac{\alpha \lambda p}{(\alpha + \lambda p)^2}\Bigg[[1 + (\alpha + \lambda p)S]e^{-\alpha S} \\ &\qquad - [1 + (\alpha + \lambda p)T]e^{-(\alpha+\lambda p)T + \lambda p S}\Bigg] \\ &+ \left[1 - (1 + \alpha T)e^{-\alpha T}\right]e^{-\lambda p(T-S)}\Bigg\} + c_2. \end{aligned} \tag{19}$$

In addition, the equation $Q_1(y|y) = Q_2(y|y) = c_2/(c_1 - c_2)$ becomes

$$\frac{\alpha y - \left(1 - e^{-\alpha y}\right)}{1 + \alpha y} = \frac{c_2}{c_1 - c_2}. \tag{20}$$

The left-hand-side of equation (20) is strictly increasing in y from 0 to 1. Hence, note that if $c_1 > 2c_2$ then finite $\tilde{T}$ and $\tilde{S}$ exist uniquely.

In the following, we consider the case $\alpha = 2/3$, *i.e.*, the mean time to a system failure is three (*e.g.*, years). Figure 1 shows the optimal values of S

in the case of $T = 4$ (*e.g.*, years), while Figure 2 reveals the optimal values of T when $S = 1$ (*e.g.*, years). Both Figures 1 and 2 deal with cases with $c_2 = 1$, $p = 0.2, 0.5, 0.8$, $\lambda = 4, 5, 6$ and $c_1 = 3.5, 4, 4.5$. Table 1 indicates the solution, y^*, of the equation $Q_1(y|y) = c_2/(c_1 - c_2)$.

TABLE 1
Solution of $Q_1(y|y) = c_2/(c_1 - c_2)$.

c_1	3.5	4.0	4.5
y^*	3.205	2.603	2.223

It is observed in Figure 1 that S^* decreases with increasing c_1 and increases with λ and p. In Figure 2, the optimal value of T has the same behavior as S^*. These can intuitively be explained.

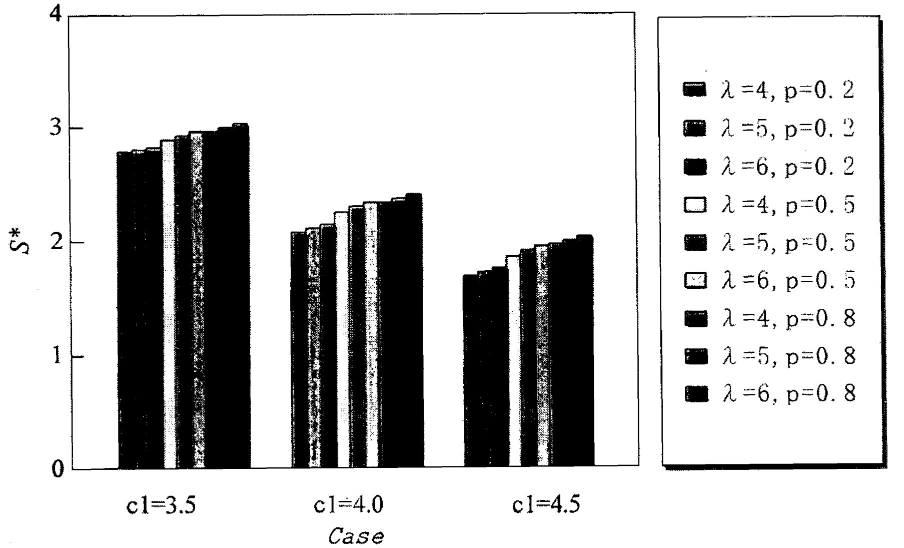

Figure 1. – Optimal S.

5. CONCLUDING REMARKS

This study proposed an extended opportunity-based age replacement policy, which will be effective to the replacement problems associated with personal computers. Under the proposed policy, a corrective replacement is performed whenever the system fails. When the age, x of the system

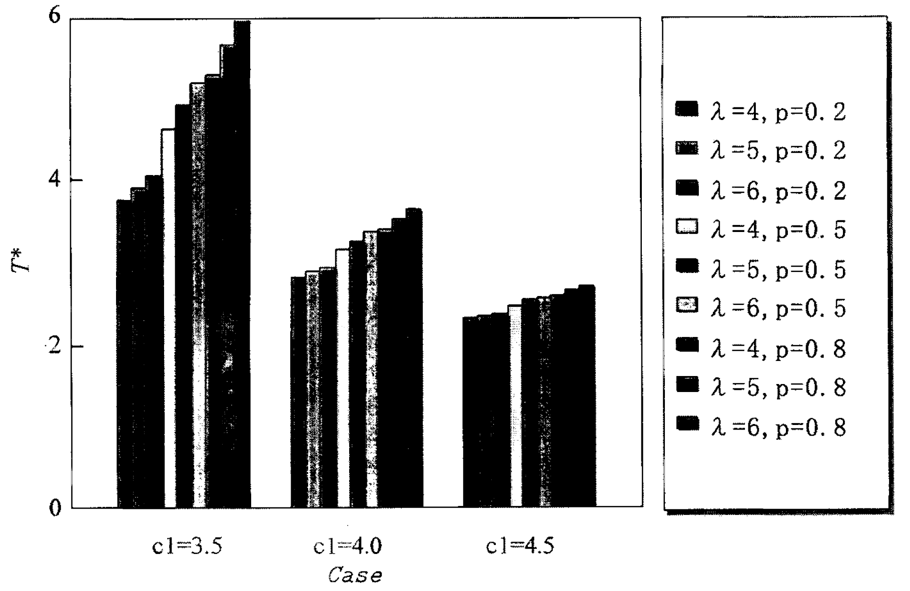

Figure 2. – Optimal T.

satisfies $x < S$ for a prespecified value S, no preventive replacement is conducted. If x satisfies $S \leq x < T$ for another prespecified value T, we take an opportunity, *i.e.*, preventively replace the system by a new one. When x reaches T, a preventive replacement is executed regardless of the occurrences of opportunities.

The long-term average cost of the proposed policy was first formulated. Secondly we clarified the conditions under which a unique finite optimal solution of S and T exists for a prespecified value of T and S, respectively. Numerical examples were also presented to illustrate the proposed policy where the underlying failure time distribution of the system is a Gamma distribution.

In a practical situation, an opportunity may occur for the first time just before the scheduled preventive replacement time T. In such a case, we may pass up the opportunity to conduct a preventive replacement at T. The replacement strategies based on the similar idea to this were discussed in [4, 15, 16]. The extended opportunity-based age replacement policy incorporated with such a concept is currently under investigation. It is also under investigation to introduce the concept of minimal repairs into the proposed model, particularly when $x \leq S$.

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