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ESTIMATING THE VALUE OF CONTRACTS A SIMULATION STUDY (1)

by Charles S. TAPIERO (*) and Philippe de LOZE (**)

Abstract. — *Contracts are bindinq bi-lateral agreements by which agreed on exchange terms between two firms are used as substitutes for current market mechanisms as a medium of economic exchange. A model describing the probabilistic evolution of prices and demands, contract clauses and the value of contracts is simulated. As an example, five contract terms are considered and the simulated probability distributions for the value of each of these contracts is given. They are then compared in terms of their expected returns, variance and stochastic dominance.*

1. INTRODUCTION

A contract is a binding bi-lateral agreement by which agreed on exchange terms between two firms are used as substitute for current market mechanisms as a medium of economic exchange. This may involve determination of future prices, delivery rates, and a set of clauses intended to protect each party against possible failure by the other party in fulfilling the terms of the contract. The essential advantage resulting from negotiating a contract is *to reduce*, for both parties, *the uncertainty concerning future exchange* and operational conditions. For example, a manufacturing firm in need of raw materials will be eager to secure stable long term sources of supplies. To reduce or remove the uncertainty concerning market supply conditions and to insure the regular delivery of raw materials, the firm seeks binding agreements. For example, agreements would substitute certain delivery dates for uncertain ones.

As the uncertainty grows for a firm concerning the future of prices and delivery rates, the more important and necessary is a binding agreement.

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In 1972, the Commonwealth Company [7] signed a contract with the Algerian Government for the delivery of crude over the next twenty years. The terms of the contract, well above current market prices, were justified in terms of the expectations of future market conditions and uncertainties concerning *availability* of future energy supplies. Naturally, the Commonwealth decision was not a simple one and had economic as well as political overtones. The current and recent energy crises have justified the terms of the contract — a standard OPEC contract. It does not provide any price breaks for crude but it ensures a higher priority in the delivery of crude. This is particularly important for the refinery of Commonwealth in Puerto Rico, which does not have an independant source of supply.

The decision processes by which contract terms are reached are complex and difficult. They involve bargaining and negotiations by two, often conflicting sides who have only partial information concerning the evolution of the market. A mathematical model, yielding optimum contract terms, involve the solution of stochastic games [9, 10] or stochastic differential games [2]. Even if we were able to simplify the problem faced by firms in determining contract terms and obtain a mathematical model, it is unlikely at this stage that an analytical or numerical solution could yield normative contracts. An alternative method, based on the simulation of future market conditions could also be used to determine the probability distribution of the value of the contract. Building a computer program which simulates alternative contract terms may thus provide an efficient tool for testing and evaluating on-line, the economic and risk implications of particular contracts. The purpose of this paper is to demonstrate by means of a simulation study how the probability distribution of the value of contract terms for one party may be determined. Given a set of probability distributions, managers may then select a particular contract on the basis of its expected value, probability and costs of ruin, and criteria based on stochastic dominance [4].

2. THE MODEL

We consider, for simplification purposes, the contract terms concerning a single product firm. A « portfolio » of commodities would render the formulation of the problem more complex without altering the basic framework defined. The value of the contract is given as the difference between the total aggregated costs when entering a contractual agreement and the costs incurred otherwise (that is without any contract). Since these costs are a function of future and uncertain demand conditions, the value of a contract will, consequently, be a random variable. This random variable has an empirical probability distribution characterizing the risk properties of the contract. To determine this empirical probability distribution we simulate the evolution of market conditions described by stochastic processes. For demonstration purposes, the model we construct takes the point of view of a manufacturer's acquisition needs for raw materials.

We define the following variables :

$\tilde{p}_t$ = Market price at time t.

$\tilde{d}_t$ = Firm's demand for the product at time t.

P_t^* = Contracted price at time t.

D_t^* = Contracted delivery rate at time t.

$\tilde{x}_t$ = Inventory on hand at time t.

$\tilde{\beta}_t$ = An index of a supplier's default, describing the probability that a contractual agreement concerning future supplies at time t is not met.

T = Contract period.

A « $\sim$ » over a variable denotes random variables whose evolution is given by a stochastic process. P_t^* and D_t^* are the contract terms. That is D_t^* is a contracted supply at a price P_t^* at time t. The contract prices may be conditional on market fluctuations as is the case of OPEC contracts. We have assumed, in this paper, a set of agreed on future prices, thus the contract completely substitutes the market as a medium of economic exchange. When scheduled supplies D_t^* are subject to default we have assumed that there is a cost associated to that default and simulated the probability of default by $\tilde{\beta}_t$. When $\tilde{\beta}_t = 1$ the supply equals the schedule D_t^*. When $\tilde{\beta}_t < 1$, the supply equals $\tilde{\beta}_t D_t^*$ and therefore the supplier incurs a fixed penalty cost plus a cost proportional to $(1 - \tilde{\beta}_t)D_t^*$. The relationship between prices $\tilde{p}_t$ and demands $\tilde{d}_t$ describes the market in which the firm is operating. A statistical dependence between $\tilde{p}_t$ and $\tilde{d}_t$ will, for example, be indicative of monopolistic or oligopolistic practices, while the statistical independence of prices and demands will presume a competitive market. The model adopted to describe the probabilistic evolution of prices and demands is based on an empirical model of industry trends as well as a firm's market share. Fore example, the industry demand y_t may be forecasted by an exponential smoothing model :

$$y_t = \alpha y_{t-1} + \varepsilon_t$$

while a firm's demand, d_t function of the market share m_t may be given by :

$$d_t = m_t y_t \varepsilon'_t$$

or

$$\log d_t = \log m_t + \log y_t + \varepsilon''_t$$

where both ε_t and ε''_t are zero-mean uncorrelated normal disturbances. For simplification, we shall assume that prices and demands are given by equations (2.1) and (2.2) respectively.

$$\tilde{p}_t = p_t^* + \varepsilon_t \tag{2.1}$$

$$\tilde{d}_t = d_t^* + \xi_t \tag{2.2}$$

where p_t^* and d_t^* are forecasted prices and demands, and where ε_t and ξ_t are two normal uncorrelated zero-mean random variables with known variances θ_t^2 and σ_t^2 respectively. The variables p_t^* and d_t^* may, for example, be determined through an econometric forecasting model, while θ_t^2 and σ_t^2 characterize the uncertainty associated with the forecasted values p_t^* and d_t^* at time t. Assuming that this uncertainty grows exponentially, θ_t, σ_t can be simply written as :

$$\begin{aligned} &\theta_t = \theta_0 e^{r_1 t} \\ &\theta_0 > 0 \quad , \quad r_1 > 0 \end{aligned} \tag{2.3}$$

$$\begin{aligned} &\sigma_t = \sigma_0 e^{r_2 t} \\ &\sigma_0 > 0 \quad , \quad r_2 > 0 \end{aligned} \tag{2.4}$$

where θ_0^2 and σ_0^2 are the initial variances while $r_1/2$ and $r_2/2$ are the exponential rates of growth for these variances. If r_1 and r_2 are small, for example, the firm will have greater confidence in the quality of future forecasts. Of course, when r_1 and r_2 are large, the opposite holds true. It x_t is a random process given by

$$x_{t+1} = \alpha_t x_t + \varepsilon_t$$

where ε_t is a zero-mean uncorrelated normal disturbance with known variance θ_t^2, it can be shown that the mean and variance of x_t; u_t and σ_t^2 evolve according to :

$$u_{t+1} = \alpha_t u_t$$

$$1/\sigma_{t+1}^2 = 1/\sigma_t^2 + 1/\theta_t^2$$

In continuous time this will indicate an exponential growth of the variance σ_t^2.

$$\mathrm{d}\sigma^2/\mathrm{d}t = 2\alpha_t \sigma_t^2 + \theta_t^2$$

For the mean-variance evolution of stochastic processes see for example, Aoki [1] and Sage and Melsa [6].

The inventory equation describing the accumulation of materials at time t is given here by

$$\begin{aligned} \tilde{x}_t &= \text{Max}\,\{\,0, \tilde{x}_{t-1} + \tilde{\beta}_t D_t^* - \tilde{d}_t\,\} \\ \tilde{u}_t &= -\,\text{Min}\,\{\,0, \tilde{x}_{t-1} + \tilde{\beta}_t D_t^* - \tilde{d}_t\,\} \end{aligned} \tag{2.5}$$

where $\tilde{u}_t$ is the quantity bought on the market at a price $\tilde{p}_t$ used to complement the required demand. To determine this equation we make the following assumptions; $\tilde{\beta}_t D_t^*$ is the actual quantity delivered at time t. Thus, if

$$\tilde{d}_t \leqslant \tilde{x}_{t-1} + \tilde{\beta}_t D_t^*$$

then

$$\tilde{x}_t = \tilde{x}_{t-1} + \tilde{\beta}_t D_t^* - \tilde{d}_t$$

Where

$$\tilde{d}_t > \tilde{x}_{t-1} + \tilde{\beta}_t D_t^*$$

that is, if the quantity required by the firm is greater than the actual deliveries and inventories, then the inventory in the next period $\tilde{x}_t$ equals zero and the necessary complement $\tilde{u}_t$ is :

$$\tilde{u}_t = \tilde{d}_t - (\tilde{x}_{t-1} + \tilde{\beta}_t D_t^*)$$

$$\tilde{x}_t = 0 \quad , \quad \tilde{u}_t > 0$$

The two cases above are briefly summarized by equation (2.5). Equations (2.1)-(2.5) describe the market process, the firm's needs as well as inventory and external acquisitions under contract terms $\{ P_t^*, D_t^*, t \varepsilon (0, T) \}$. A simulation of (2.1)-(2.5) will describe the evolution of each of these variables. To evaluate the importance of contracts we whall, as indicated earlier, compare the costs incurred with contractual agreements and the costs incurred without contractual agreements. When the firm is not subject to a contract it acquires the quantity $\tilde{d}_t$ at time t at a price $\tilde{p}_t$. If we further include a fixed ordering cost $k(\tilde{d}_t)$ we obtain :

$$\tilde{C}_t^1 = \tilde{d}_t \tilde{p}_t + k(\tilde{d}_t)$$
$$k(\tilde{d}_t) = \begin{cases} k \text{ if } \tilde{d}_t > 0 \\ 0 \text{ otherwise} \end{cases} \tag{2.6}$$

When the firm operates under the contract terms $\{ P_t^*, D_t^*, t \in (0, T) \}$, the cost is :

$$\tilde{C}_t^2 = \tilde{\beta}_t D_t^* P_t^* + \tilde{u}_t \tilde{p}_t + f(\tilde{x}_t) - g(\tilde{\beta}_t) + k(\tilde{u}_t)$$
$$k(\tilde{u}_t) = \begin{cases} k \text{ if } \tilde{u}_t > 0 \\ 0 \text{ otherwise} \end{cases} \tag{2.7}$$

where $\tilde{\beta}_t D_t^* P_t^*$ the cost under contracted price P_t^* and $\tilde{u}_t \tilde{p}_t$ is the cost of acquiring the necessary complement $\tilde{u}_t$ at market prices $\tilde{p}_t$. $f(\tilde{x}_t)$ is the inventory cost which is assumed to be proportional to the value of the inventory on hand. Or

$$f(\tilde{x}_t) = a_0 \tilde{x}_t \tilde{p}_t$$

Finally $g(\tilde{\beta}_t)$ is a penalty cost to the supplier and is a function of the quantity $(1 - \tilde{\beta}_t)D_t^*$ which was not supplied;

$$g(\tilde{\beta}_t) = a_1\tilde{\beta}_t + b_1(1 - \tilde{\beta}_t)D_t^*P_t^*$$

Recapitulating, the cost C_t^2 incurred under the contractual agreement is :

$$\tilde{C}_t^2 = \tilde{\beta}_t D_t^* P_t^* + \tilde{u}_t\tilde{p}_t + a_0\tilde{x}_t\tilde{p}_t + k(\tilde{u}_t) - a_1(\tilde{\beta}_t) - b_1(1 - \tilde{\beta}_t)D_t^*P_t^*$$

$$k(\tilde{u}_t) = \begin{cases} k \text{ if } \tilde{u}_t > 0 \\ 0 \text{ otherwise} \end{cases} \qquad (2.8)$$

$$a_1(\tilde{\beta}_t) = \begin{cases} a_1, \text{ if } \tilde{\beta}_t > 0 \\ 0 \text{ otherwise} \end{cases}$$

The value of the contract at time t is thus $\tilde{C}_t^1 - \tilde{C}_t^2$. Over T periods, the present value of the savings realized by the contracts are $\tilde{V}(T)$:

$$\tilde{V}(T) = \sum_{t=0}^{T} (\tilde{C}_t^1 - \tilde{C}_t^2)(1 + i)^{-t} \qquad (2.9)$$

where « i » is the riskless discount rate used by the firm in its budgeting analysis. Since $\tilde{C}_t^1$ and $\tilde{C}_t^2$ are random variables, so is the summation of their difference over time. Thus, $\tilde{V}(T)$ expressing the value of a contract is also a random variable whose probability distribution can be computed by simulation. This will be done next.

3. THE SIMULATION PROGRAM AND MODEL

The simulation program we use for the purpose of analysis is the IBM/CSMP (Continuous Systems Modeling Program) [5]. This is a simulator of continuous systems and combines the capabilities of ordinary analog simulators with these of the mathematically oriented digital computer languages such as FORTRAN IV. The latter is used as the source language for 95 % of the operations required in a CSMP program; the remaining 5 % are not easily performed in FORTRAN IV (Level E) and are therefore coded in Assembler Language.

The CSMP program is generally divided into three sections : (see the Appendix) (1) INITIAL (2) DYNAMIC and (3) TERMINAL. The INITIAL segment performs two basic functions : *a)* to specify the terms which are to be maintained constant throughout the program and *b)* to specify the initial conditions for the equations describing the evolution of dynamic systems. The DYNAMIC segment, following the INITIAL one includes the actual program. That is, the equations describing the evolution of the variables in time are specified. Within this segment we can refer to a large family of subroutines called MACROS. These subroutines may be drawn from a CSMP library or may be given as FORTRAN subroutines. Finally, the TERMINAL section specifies the variables to be printed and plotted, the integration routine, the step size, accuracy, and the length of the planning horizon. These values need not be constant and may be changed for each run.

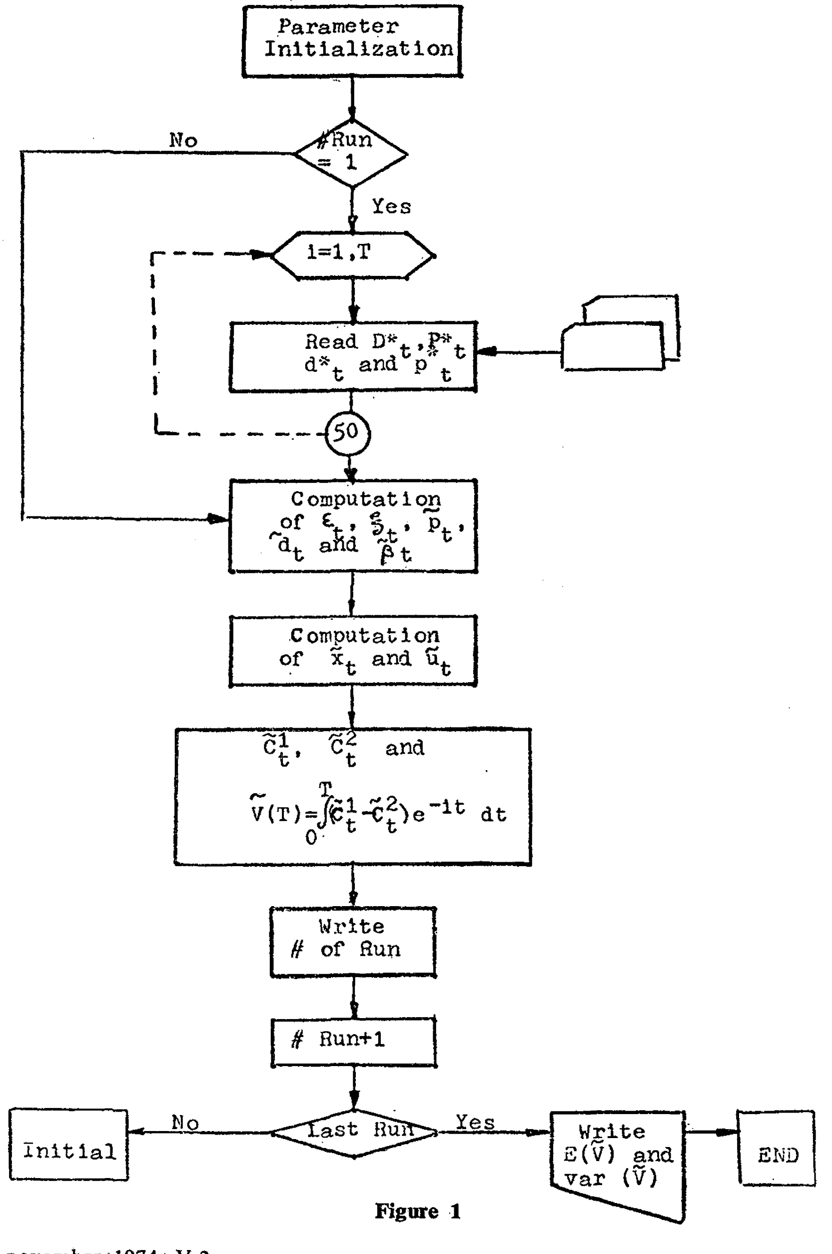

Figure 1

In our program (see the appendix) whose flow chart is given in Figure 1, we first read in a data set containing the values D_t^*, P_t^*, d_t^* and p_t^*. The initial segment includes the number of simulation runs (T), as well as all parameters and initial conditions used in describing the evolution of prices, demand, and their uncertainty. The dynamic segment compute the time variables $\tilde{d}_t$, $\tilde{p}_t$, $\tilde{\beta}_t$, $\tilde{C}_t^1$, $\tilde{C}_t^2$ iteratively for all $t \in [O, T]$ and $\tilde{V}(t)$ the contract value at t.

In the terminal section, the program monitors the number of runs, prints out $\tilde{V}(T)$ for each run as well as the mean and variance of the probability distribution of $\tilde{V}(T)$. Finally, a subroutine is used to plot the frequency distributions of $\tilde{V}(T)$.

Results of our simulation study are considered next.

4. RESULTS

Five contract terms were simulated and a simulated probability distribution for the value of each of the contracts was found.

Figure 2 outlines both the terms of the contract as well as the forecasted trends in prices and demands.

Time Period	Contract 1		Contract 2		Contract 3		Contract 4		Contract 5			
	D_t^*	P_t^*	D_t^*	P_t^*	D_t^*	P_t^*	D_t^*	P_t^*	D_t^*	P_t^*	d_t^*	p_t^*
1	102.00	1.00	106.00	1.00	102.00	1.02	102.00	1.06	100.00	1.06	100.00	1.00
2	110.16	1.05	114.48	1.05	110.16	1.07	110.16	1.09	108.00	1.11	108.00	1.05
3	117.30	1.08	121.90	1.08	117.30	1.10	117.30	1.12	115.00	1.14	115.00	1.08
4	122.40	1.11	127.20	1.11	122.40	1.13	122.40	1.15	120.00	1.18	120.00	1.11
5	132.60	1.15	137.80	1.15	132.60	1.17	132.60	1.20	130.00	1.22	130.00	1.15
6	140.76	1.20	146.28	1.20	140.76	1.22	140.76	1.25	138.00	1.27	138.00	1.20
7	151.98	1.28	157.94	1.28	151.98	1.31	151.98	1.33	149.00	1.36	149.00	1.28
8	164.22	1.35	170.66	1.35	164.22	1.38	164.22	1.40	161.00	1.43	161.00	1.35
9	173.40	1.44	180.20	1.44	173.40	1.47	173.40	1.50	170.00	1.53	170.00	1.44
10	183.60	1.55	190.80	1.57	183.60	1.60	183.60	1.63	180.00	1.66	180.00	1.57
11	188.70	1.68	196.10	1.68	188.70	1.71	188.70	1.75	185.00	1.78	185.00	1.68
12	192.78	1.75	200.34	1.75	192.78	1.78	192.78	1.82	189.00	1.85	189.00	1.75
13	198.90	1.84	206.70	1.84	198.90	1.88	198.90	1.91	195.00	1.95	195.00	1.84
14	208.08	1.90	216.24	1.90	208.08	1.94	208.08	1.93	204.00	2.01	204.00	1.90
15	220.32	1.96	228.96	1.96	220.32	2.00	220.32	2.04	216.00	2.08	216.00	1.96

Figure 2

Figures 3 through 7 describe the probability distribution of the value of contracts based on two hundred simulated runs. Each distribution has, as

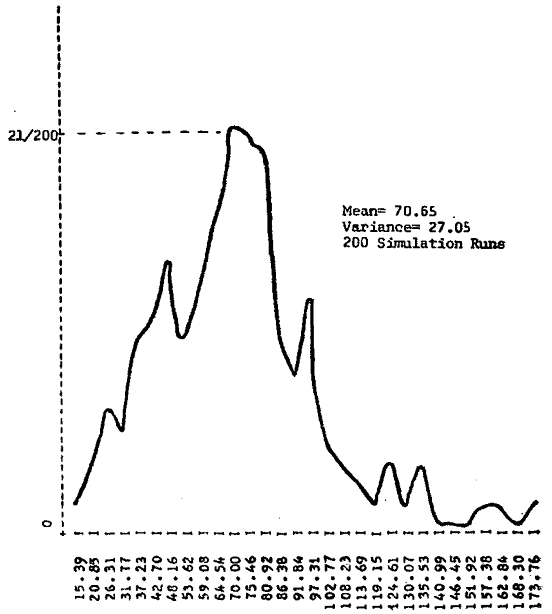

Figure 3

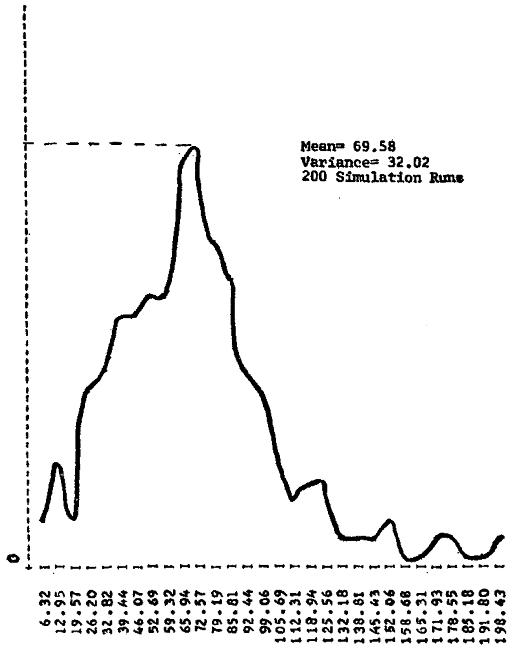

Figure 4

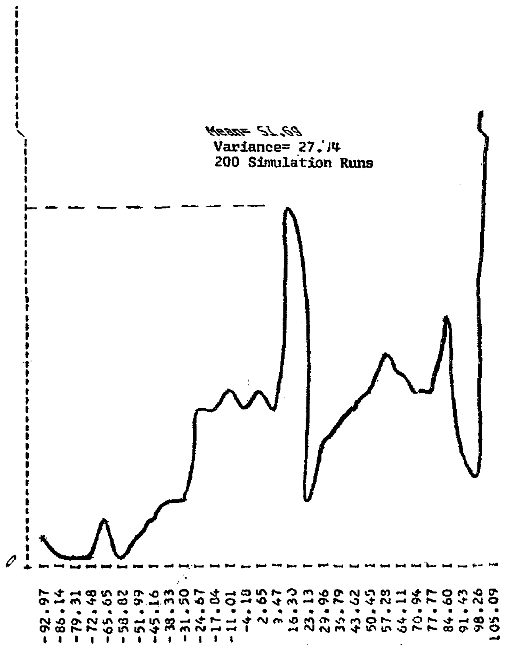

Figure 5

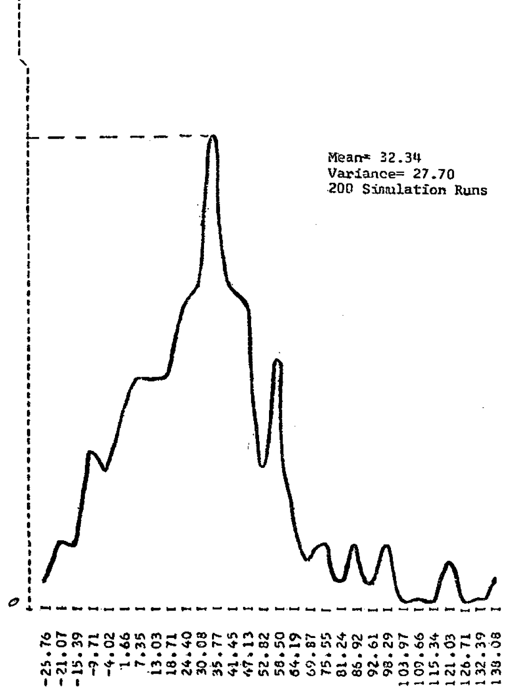

Figure 6

indicated in Figures 3-7 an expected value and a variance which can be used in determining the most beneficial contract terms.

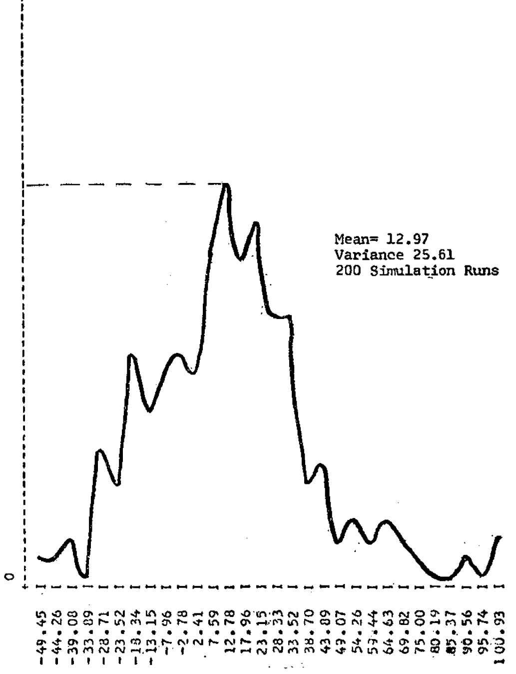

Figure 7

A comparison of the contract terms on the basis of their expected value indicates that contract 1 is the best one. That is, if $E_i\{\tilde{V}(T)\}$ is the expected value of contract i, given by :

$$E_i\{\tilde{V}(T)\} = \int_{-\infty}^{\infty} \tilde{V}(T) f_i\{\tilde{V}(T)\} \mathrm{d}\,\tilde{V}(T) \tag{4.1}$$

where $f_i\{\tilde{V}(T)\}$ is the simulated probability distribution of the contract value, then

$$E_1\{\tilde{V}(T)\} > E_j\{\tilde{V}(T)\} \qquad j = 2, \dots 5 \tag{4.2}$$

This implies in our model that equating contracted prices with forecasted prices and contracting a supply rate slightly larger (2 %) than the demand rate is most beneficial. In fact, supply rates 4 % larger than demand rates turn out to be equally good. Here the expected value is slightly smaller for contract 2 than contract 1 while the variance, given by equation (4.2), is slightly larger for contract 2

$$\mathrm{var}_i\{V(T)\} = \int_{-\infty}^{\infty} [\tilde{V}(T) - E_i\{\tilde{V}\}]^2 f_i\{\tilde{V}(T)\} \mathrm{d}\tilde{V}(T) \tag{4.3}$$

A comparison of the graphical plots in Figures 3 and 4 shows that the difference between the first two contracts is reflected in the skewness of their probability distributions. The first contract is slightly skewed to the right while the second one slightly skewed to the left.

It is evident that the two first contracts are better than the remaining ones. The third contract, differing from the first by a 2 % increase in contracted prices, has for example a peculiar distribution whose expected value although positive can lead to significant losses. An approximation of this distribution by a normal one will obviously be a poor one as our distribution has at least two modes. The fourth and fifth contracts have distributions similar to the first and second ones. Nonetheless, they can (with significant probability) lead to a negative value for the contract as well as lower expectations than contracts 1 and 2. It is noteworthy, however, that *all* contracts have a positive expected value. That is, on the average, it pays off to enter a contractual agreement, even at contracted prices and contracted delivery rates above forecasted prices and demands.

Evaluation of contract terms on the basis of mean and variance criteria may not be sufficient (see Markowitz [6]). It may, therefore, be necessary to compare contract terms on the basis of stochastic dominance (see Hanoch and Levy [4]). If we let $F_i\{\tilde{V}(T)\}$ be the cumulative probability distribution of $\tilde{V}(T)$, the value of contract term i,

$$F_i\{\tilde{V}(T)\} = \int_{-\infty}^{\tilde{V}(T} f_i\{\tilde{V}(T)\}\,d\tilde{V}(T) \qquad (4.4)$$

A contract term i is stochastically dominant to contract j if :

$$F_i\{\tilde{V}(T)\} > F_j\{\tilde{V}(T)\} \qquad j \neq i \qquad (2.13)$$

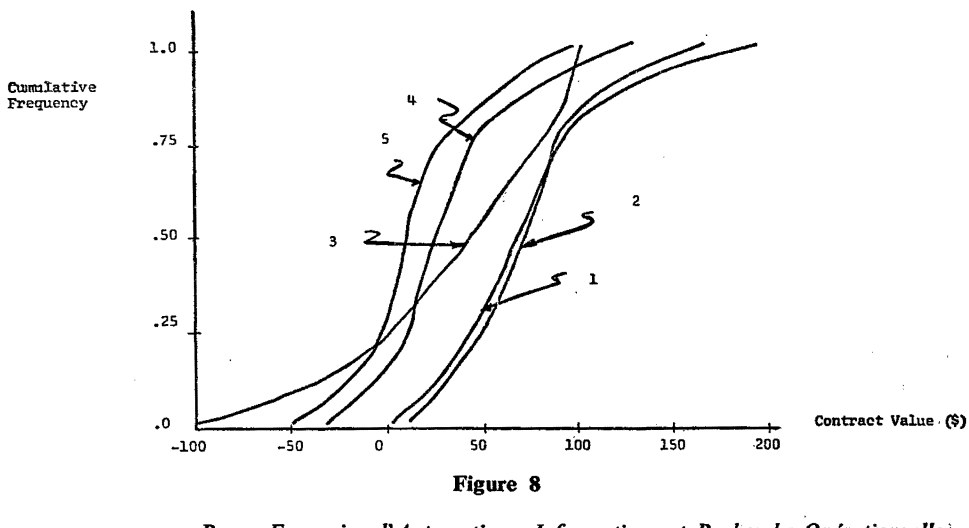

Figure 8

Using Figure 8, which represents the cumulative probability distribution of returns, we note that the first and second contract curves dominate all other contract curves. Following this simulation study, we may thus state that the first two contracts are best in terms of the expected value, stochastic dominance criteria, and a mean variance criterion.

5. CONCLUSION

The probability distribution of the value of contracts was determined by simulation methods. This entailed the modeling of future market exchange conditions, a firm's future needs and operational conditions as well as the uncertainty implied in these future conditions. The probability distribution thus defined allowed us to apply decisionmaking criteria under uncertainty to evaluate the desirability of particular contract terms. In practice, contractual terms involve two negotiating parties while this paper has focused on one party only. A more extensive model would emphasize simulation of the valuation criterion and uncertainties of both parties as well as simulation of their negotiating strategies. To do so, we would require a gametheoretic framework establishing the conditions under which the two parties can successfully resolve their differences as well as their behavioral strategies in moving towards a conflict resolution. This paper has focused instead on the problems faced by one party only and allows testing of the economic and risk implications of contracts. By varying the amounts of information available to the firm in conducting its bargaining strategy we can also determine the value of information in contract negotiations. Of course, we should stress that for implementation purposes, rapidity and flexibility in the access to computing facilities are important constraints to be added to the simulation program. On-line and interactive programming such as in time-sharing systems would then be necessary. The reader should, nonetheless, be aware that the results we obtained depend on knowledge of the theoretical probability structure of forecasts and their parameters. This is usually difficult to obtain but reasonable data analyses, econometric model building and subjective judgement can be used in improving our guess of the future environmental conditions to a firm.

APPENDIX

J	= number of runs	XT	$= \tilde{x}_t$
T	= contract period T	XDT	$= \tilde{x}_{t-1}$
GAMQ	$= \gamma_1$	UT	$= \tilde{u}_t$
GAMQ	$= \gamma_2$	AO	$= a_0$
ALP	$= \theta_0$	INUT	$= f(\tilde{x}_t)$
ALQ	$= \sigma_0$	OC	$= oc$
MUT	$= \mu_t$	PENAT	$= g(\tilde{\beta}_t)$
EPST	$= \varepsilon_t$	CT	$= \tilde{C}_t^1$
QSTAT	$= p_t^*$	D_1	$= \delta^2(t)$
PSTAT	$= P_t^*$	CCT	$= \tilde{C}_t^2$
QT	$= \tilde{d}_t$	X	$= - it$
PT	$= \tilde{p}_t$	DIFCT	$= (\tilde{C}_t^1 - \tilde{C}_t^2)\, e^{-it}$
BETAT	$= \tilde{\beta}_t$	OBJ	$= \tilde{V}$
QQT	$= D_t^*$	NBRUN	= Run number
PPT	$= P_t^*$	MEAN	$= E(\tilde{V})$
K	$= i$	STD	$= \sqrt{\mathrm{Var}(V)}$

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            ****CONTINUOUS SYSTEM MODELING PROGRAM****

      ***PROBLEM INPUT STATEMENTS***

TITLE SIMULATION MODEL OF A CONTRACT VALUE
FIXED I,J,T,IT,N,NBRUN
STORAGE QSTAT(25),PSTAT(25),QQT(25),PPT(25)
INITIAL
INCON INO=0.
PARAMETER T=15,J=200
PARAMETER NBRUN=0,N=1,ME=0.,SQ=0.
PARAMETER A0=.08,A1=5.,B1=.04,OC=I0.,K=.1
PARAMETER ALQ=10.,ALP=.12,GAMQ=.02,GAMP=.03
NOSORT
      PT=0.
      QT=0.
      QQ=0.
      XT=0.
      IF(NBRUN.GE.1) GO TO 60
      DO 50 I=1,T
      READ(5,100) QQT(I),PPT(I),QSTAT(I),PSTAT(I)
   50 CONTINUE
  100 FORMAT(4(F10.2,2X))
   60 CONTINUE
DYNAMIC
NOSORT
IT=TIME
QQ=QQT(IT)
VQ=GAMQ*TIME
VP=GAMP*TIME
SIGQ=ALQ*EXP(VQ)
SIGP=ALP*EXP(VP)
      IF(TIME.NE.1.) GO TO 80
      DO 70 I=1,N
EPST=GAUSS(21,0.,SIGQ)
MUT=GAUSS(21,0.,SIGP)
Y=RNDGEN(21)
   70 CONTINUE
      GO TO 90
   80 EPST=GAUSS(21,0.,SIGQ)
      MUT=GAUSS(21,0.,SIGP)
Y=RNDGEN(21)
   90 CONTINUE
QT=QSTAT(IT)+EPST
PT=PSTAT(IT)+MUT
      D2=1.
      IF(Y.LE..09) GO TO 1
      IF(Y.LE..19) GO TO 2
      IF(Y.LE..29) GO TO 3
      IF(Y.LE..49) GO TO 4
      BETAT=1.
      D2=0.
      GO TO 5
```

```
    1 BETAT=.0
      GO TO 5
    2 BETAT=.25
      GO TO 5
    3 BETAT=.50
      GO TO 5
    4 BETAT=.75
    5 CONTINUE
SORT
XDT=DELAY(20,DELT,XT)
AT=QT-XDT-BETAT*QQ
UT=AMAX1(O.,AT)
AAT=-AT
XT=AMAX1(O.,AAT)
NOSORT
      INVT=AO*(XT*PT)
      IF(BETAT.EQ.1.) GO TO 9
      PENAT=A1+B1*(1.-BETAT)*QQT(IT)*PPT(IT)
    9 CT=QT*PT+OC
      D1=NOT(XT)
      CCT=INVT+BETAT*(QQT(IT)*PPT(IT))+UT*PT+D1*OC-D2*PENAT
      X=-K*TIME
      DIFCT=(CT-CCT)*EXP(X)
OBJ=INTGRL(INO,DIFCT)
TERMINAL
TIMER DELT=1.,FINTIM=15,OUTDEL=1.
METHOD RKSFX
      NBRUN=NBRUN+1
      IF(NBRUN.EQ.1) WRITE(6,100) T,J
  100 FORMAT(1H1,///,40X,18HCONTRACT PERIOD = ,I2,10X,17HNUMBER OF RUNS
     $= ,I3,////,30X,10HRUN NUMBER,5X,14HCONTRACT VALUE,//)
      WRITE(6,200) NBRUN,OBJ
  200 FORMAT(1H ,34X,I3,11X,F8.2)
      SQ=SQ+OBJ**2
      ME=ME+OBJ
      IF(NBRUN.GE.J) GO TO 12
      N=NBRUN*T+1
      TEST=1.
      CALL RERUN
      GO TO 13
   12 CONTINUE
      JREAL=J
      MEAN=ME/JREAL
      SQQ=SQ/JREAL
      X=SQQ-MEAN**2
      STD=SQRT(X)
      WRITE(6,300) MEAN,STD
  300 FORMAT(///////,1HO,54HPARAMETERS OF THE CONTRACT VALUE FUNCTION DI
     $STRIBUTION,//,20X,7HMEAN = ,F8.2,5X,21HSTANDARD DEVIATION = ,F8.2)
   13 CONTINUE
END
```

REFERENCES

[1] Aoki M., *Optimization of Stochastic Systems*, New York, Academic Press, 1967.

[2] Bley K. B., « Randomized and Stochastic Linear, Discrete Differential Games with Quadratic Payoff Functions », Ph. D. Thesis, School of Engineering and Applied Science, University of California, Los Angeles, August 1970.

[3] Borch K., « Indifference Curves and Uncertainty », *Swedish Journal of Economics*, March 1968, pp. 19-24.

[4] Hanoch G. and Levy H., « The Efficiency Analysis of Choices Involving Risk », *Review of Economic Studies*, July 1969, pp. 335-346.

[5] IBM/CSMP, « Continuous System Modeling Program », *Users' Manual*, GH20-0367-4, Program Number 360A-CX-16X, White Plains, New York, 1972.

[6] Markowitz H., *Portfolio Selection-Efficient Diversification of Investment*, New York, John Wiley & Sons, 1959.

[7] *New York Times*, December 19, 1972.

[8] Sage A. P. and Melsa J. L., *Estimation Theory with Applications to Communications and Control*, New York, McGraw Hill Book Co., 1971.

[9] Sobel M. J., « Non-Cooperative Stochastic Games », Report No. 21, Department of Administration Sciences, Yale University, Oct. 1969.

[10] Telser L. G., *Competition, Collusion and Games Theory*, Chicago, Aldine, Athevton, 1972.

[11] William W. W., « Conflict and Decentralization in Stochastic Differential Games », 1970, Joint Automatic Control Conference of the American Automatic Control Council, Georgia Institute of Technology, June 22-26, 1970.