# Revue d'Histoire des Mathématiques

# On the exchanges between W. Doeblin and B. Hostinský

Laurent Mazliak

Tome 13 Fascicule 2

## 2007

# SOCIÉTÉ MATHÉMATIQUE DE FRANCE

Publiée avec le concours du Ministère de la culture et de la communication (DGLFLF) et du Centre national de la recherche scientifique

### REVUE D'HISTOIRE DES MATHÉMATIQUES

#### RÉDACTION

Rédactrice en chef :

#### COMITÉ DE LECTURE

Jeanne Peiffer	P. Abgrall France
5	T. Archibald Canada
Rédacteur en chef adjoint :	J. Barrow-Greene Grande-Bretagne
Philippe Nabonnand	U. Bottazzini
Membres du Comité de	JP. Bourguignon France
rédaction :	A. Brigaglia
Michel Armatte	B. Bru France
Liliane Beaulieu	P. Cartier France
Bruno Belhoste	JL. Chabert France
Alain Bernard	F. Charette France
Jean Celevrette	K. Chemla France
Olivier Darrigol	P. Crépel France
Anne-Marie Décaillot	F. De Gandt France
Marie-José Durand-Richard	S. Demidov Russie
Étienne Ghys	M. Epple Allemagne
Christian Gilain	N. Ermolaëva Russie
Jens Hoyrup	H. Gispert France
Agathe Keller	C. Goldstein France
Karen Parshall	J. Gray Grande-Bretagne
Dominique Tournès	E. Knobloch Allemagne
*	T. Lévy France
Secrétariat :	J. Lützen Danemark
Nathalie Christiaën	A. Malet Catalogne
Société Mathématique de France	I. Pantin France
Institut Henri Poincaré	I. Passeron France
11, rue Pierre et Marie Curie	D. Rowe Allemagne
75231 Paris Cedex 05	C. Sasaki Japon
Tél.: (33) 01 44 27 67 99	K. Saito Japon
Fax: (33) 01 40 46 90 96	S.R. Sarma
$M\acute{e}l$ : revues@smf.ens.fr	N. Schappacher Allemagne
Url:http//smf.emath.fr/	E. Scholz Allemagne
Directeur de la publication :	S. Stigler États-Unis
Stéphane Jaffard	B. Vitrac France
Stephane Janaru	

Périodicité : La Revue publie deux fascicules par an, de 150 pages chacun environ.

- Tarifs 2007 :prix public Europe : 65 ; prix public hors Europe : 74 ;prix au numéro : 36 .Des conditions spéciales sont accordées aux membres de la SMF.
- Diffusion : SMF, Maison de la SMF, B.P. 67, 13274 Marseille Cedex 9 AMS, P.O. Box 6248, Providence, Rhode Island 02940 USA

© SMF Nº ISSN : 1262-022X

Maquette couverture : Armelle Stosskopf

Revue d'histoire des mathématiques 13 (2007), p. 155–180

#### ON THE EXCHANGES BETWEEN WOLFGANG DOEBLIN AND BOHUSLAV HOSTINSKÝ

#### LAURENT MAZLIAK

ABSTRACT. — We present the letters sent by Wolfgang Doeblin to Bohuslav Hostinský between 1936 and 1938. They concern some aspects of the general theory of Markov chains and the solutions of the Chapman-Kolmogorov equation that Doeblin was then establishing for his PhD thesis.

Résumé (La correspondance entre Wolfgang Doeblin et Bohuslav Hostinský)

Nous présentons les lettres envoyées par Wolfgang Doeblin à Bohuslav Hostinský entre 1936 et 1938. Elles présentent des aspects de la théorie générale des chaînes de Markov que Doeblin mettait en forme pour sa thèse ainsi que des considérations sur la résolution de l'équation de Chapman-Kolmogorov.

> Action must always be the daughter of rigor before being the sister of dream G. Canguilhem

("Avant d'être la sœur du rêve, l'action doit être fille de la rigueur"

[Canguilhem 1996, p. 32])

Texte reçu le 30 mars 2006, révisé le 19 février 2007.

L. MAZLIAK, Laboratoire de Probabilités et Modèles aléatoires, et Institut de mathématiques, projet *Histoire des sciences mathématiques*, Université Paris VI, 4 place Jussieu, 75252 Paris Cedex 05 (France).

Courrier électronique : mazliak@ccr.jussieu.fr

<sup>2000</sup> Mathematics Subject Classification: 01A60, 60J10, 60J27, 60J35.

Key words and phrases : Markov chains, Markov processes, Chapman-Kolmogorov equation, history of probability calculus.

Mots clefs. — Chaîne de Markov, processus de Markov, équation de Chapman-Kolmogorov, histoire des probabilités.

<sup>©</sup> SOCIÉTÉ MATHÉMATIQUE DE FRANCE, 2007

#### L. MAZLIAK

#### INTRODUCTION

On June 17th, 1936, Maurice Fréchet (1878–1973), one of the most prominent representatives of the French mathematical school as well as a creator of the modern school of probability calculus, wrote to his colleague and friend Bohuslav Hostinský (1884–1951), Professor of Theoretical Physics at Brno University in Czechoslovakia:

"I have a new pupil named Döblin who is studying probabilities in chains and will soon publish in the Comptes-Rendus<sup>1</sup> some results that I feel are interesting. He has much extended the results for the  $p_{i,k}^{(n)}$  obtained by M. Hadamard at the Bologna conference and those of Romanovsky in his last memoir in the *Acta*."<sup>2</sup>

Since the beginning of the 1990s, there has been a resurgence of interest in Wolfgang Doeblin (1915–1940), an amazing and engaging character, and above all in his magnificent mathematical achievements. With 13 papers and 13 notes to the *Comptes rendus*, he succeeded in giving new ideas to deal with the theory of Markov chains and stochastic processes, ideas which were to bear fruit for years and are still today fundamental in probability theory. The solemn opening of a fourteenth publication, sent as a sealed letter <sup>3</sup> to the Academy in 1940 while Doeblin was in the Army, allowed the addition of more wonderful pages to this fertile work.

The following paper is a general presentation of ten letters from Doeblin to Hostinský that we have found in Brno. As a collaboration between the *Revue d'histoire des mathématiques* and the *Electronic Journal for History of Probability and Statistics* (EJHPS), <sup>4</sup> the paper is completed with the integral

<sup>&</sup>lt;sup>1</sup> Comptes-Rendus stands here for *Comptes rendus hebdomadaires de l'Académie des sci*ences de Paris.

<sup>&</sup>lt;sup>2</sup> J'ai un nouvel élève Döblin s'occupant des probabilités en chaîne et qui publiera bientôt aux Comptes-Rendus des résultats que je crois intéresants. Il a beaucoup étendu les résultats pour les transitions obtenues par M. Hadamard à la conférence de Bologne et ceux de Romanovsky dans son dernier mémoire des *Acta*." The correspondence between M. Fréchet and B. Hostinský may be found in the archive department of Masaryk University, Brno for Fréchet's letters and the archive department of Paris Academy of Science for Hostinský's letters (fonds Fréchet, box F8).

<sup>&</sup>lt;sup>3</sup> In French *pli cacheté.* This is the name for a paper, sent by the author to the Academy for registration, when for any reason he does not want to or cannot propose it for publication. The letter is not opened until the author, or his heirs claim it to be, or after a period of a hundred years. During German occupation of France for example, this procedure was used by scientists condemned to silence for racial or political reasons.

<sup>4</sup> http://www.jehps.net

publication of the ten letters in the *Traces and Documents* section of the volume 3, n<sup>o</sup> 1, June 2007 of the EJHPS. The numbering of the letters mentioned hereafter is that of this publication in the EJHPS.

Letters from Doeblin to Hostinský are interesting above all as they are a first-hand document on the very beginning of the international career of the young mathematician. They are kept at the Archive department of Masaryk University, Brno, in the boxes *correspondence with foreign scientists* of the Hostinský fund.

#### 1. WOLFGANG DOEBLIN

Before briefly tracing the life of the young mathematician, it is necessary to comment on the transcription of his patronymic. We indeed had the unusual choice between three possible names. When he became a French citizen in 1938, Wolfgang chose the official name of Vincent Doblin: it is under this name that he would receive his military distinctions, and it is the name which was carried afterwards by his younger brother Claude, who died in Nice in December 2005 and who made the decision to allow the publication of his brother's last memoir [Doeblin 2000]. Whereas his name at birth was Döblin, Wolfgang had chosen to spell it as *Doeblin* to simplify pronunciation in French. It is under this spelling that all his mathematical papers were signed and that he signed all his professional letters as well. This is the reason that we also adopt this spelling in the sequel.

On Wolfgang Doeblin's life, his engagement, his tragic end and his mathematical works, one may consult [Bru 1993] and [Doeblin 2000], as well as Paul Lévy's paper [Lévy 1955]. Doeblin has more or less been the only young mathematician for whom Lévy expressed admiration.

Wolfgang Doeblin was born on March 17th, 1915 in Berlin, in a family of the Jewish intellectual class. His father, Alfred Döblin (1878–1957), was a neurologist but had met spectacular success as a writer after the publication of his novel *Berlin Alexanderplatz* in 1929. Alfred's anti-nazi engagement obliged the entire family to seek exile as early as March 1933, immediately after Hitler assumed power in Germany. The family established in Paris, and the young Wolfgang was registered at the Sorbonne to follow lectures of the *Licence de mathématiques* from October 1933. There, he followed Darmois' teaching (who was, the same year, replacing Fréchet for the lessons on Markov chains). In June 1934, Wolfgang had already passed the majority of the exams composing the three-years course of the Licence, and moreover seemed to have chosen to specialize in probability theory. In the middle of the 1930s, the *Institut Henri Poincaré* (IHP) had become an international center for the field under Borel's initial impulse taken over by Darmois and Fréchet. The study of Markov chains was especially active. Using his extensive personal contacts, Fréchet invited to the IHP all the renowned specialists of the subject to hold conferences: Bernstein, Hostinský, von Mises, Onicescu. Let us observe that these invitations were not all fruitful, especially when they concerned Soviet scientists. The administrative archives of the IHP offer testimony to the desperate efforts made by Borel to use his important political influence (he was deputy and a former Minister of the Navy) to overcome the increasing difficulty for travelling since 1932.

The choice of probability theory by the young Wolfgang was reasonable from another point of view. French leading mathematicians of the time were used to ostracism against probability since this field was not considered as *real mathematics*. Because he did not belong to any of the oligarchies of the French university (mostly *École normale supérieure* and *École polytechnique*), Wolfgang knew that he could not have benefited from the advantages offered to the members of these institutions who chose Analysis, for example. Certainly, they would generally not have chosen Probability. In January 1936, Doeblin officially registered for the PhD with Fréchet as advisor. The four months gap between the academic year beginning with October 1935 and January 1936 may be due to Darmois' advice to await Fréchet's return before beginning work on Markov chains. (Fréchet left Paris for a great journey to the East-European countries lasting from October to December 1935.)

Within six months, Doeblin obtained the essential results for his thesis about finite Markov chains and began to prepare articles with this material without the approval from Fréchet, who was hostile to publication before the thesis defence (see [Mazliak 2007, Letter 4]). The first group of letters sent to Hostinský (Letters 1 to 7) traces the steps of these first writings. In June 1936, Wolfgang solved Lévy's conjecture on the dispersion of sums of random variables. Lévy seems to have been quite impressed to the point that he immediately presented a common note with Doeblin to the Comptes rendus, though he had never before had coauthors. The note [Doeblin & Lévy 1936], which appeared on June 22nd, 1936, is Doeblin's first publication. Subsequently, the publications would follow at a regular rate. After the study of the discrete case, Doeblin began to deal with the construction of stochastically interpretable solutions to the Chapman-Kolmogorov equation, and he quickly obtained significant results also in that domain (see the section 3 below). In October 1937, he actively participated in the Geneva International Colloquium on Probability Theory, where he met the major scientists involved in the field (except once again the Soviet mathematicians).

In March 1938, Doeblin obtained the title of Doctor in Mathematics (at the age of 23): the thesis jury was composed by Borel (President), Fréchet who wrote the report, and Garnier. Doeblin, his parents and his two youngest brothers, had obtained French nationality in October 1936, and Wolfgang, who had refused to be exempted of military service, must therefore go to the army after having finished his studies and defended the thesis.He was intially supposed to enter the armed forces during the Summer of 1938. But the Czechoslovakian crisis postponed the call – as well as certainly dampening Wolfgang's eagerness for engagement against Nazi Germany. The end of his exchanges with Hostinský brings a stirring testimony to that fact. Doeblin entered the Army in November 1938.

In September 1939, the general mobilization made him stay in the active Army. He was quartered at Givet in the Ardennes with the 291st Infantry Regiment, integrated into the defensive sector of the Ardennes whose atmosphere is magnificently described in Julien Gracq's [1958] novel A Balcony in the Forest. He was enrolled as telephone operator, and his task was to maintain contact with the Headquarters. During the long months of the phoney war, Doeblin took as much care as he could of the writing of his research about constructing a rather general diffusion process as solution of the Chapman-Kolmogorov equation: this constitutes the currently famous *Pli cacheté* opened in 2000 and published in [Doeblin 2000]. In April 1940, Doeblin's regiment was sent to the defensive sector of the Saar, on the Maginot line. He was there on May 10th, 1940 when the sudden German attack in the Ardennes began. Caught in the trap, Doeblin's regiment desperately withdrew to the Vosges and capitulated on June 22nd, 1940. On June 21st, Doeblin had committed suicide in Housseras (a small village in the Vosges, few kilometers from the main town of Épinal) at the precise moment when German troops were in sight of the place.

#### 2. BOHUSLAV HOSTINSKÝ

Bohuslav Hostinský belongs to the previous generation. His studies and professional career are quite similar to those of many other Czech mathematicians. Born on December 5th, 1884 in Prague, he was the son of a famous musicologist, a friend of Smetana and of the emblematic Thomas Masaryk, and a well-known member of that part of Czech intelligentsia claiming independence from Vienna. In 1902, Bohuslav entered the Philosophical faculty of the Czech university in Prague and

159

followed studies in Physics and in Mathematics, graduating in teaching for high schools and also writing a doctorate in Mathematics on the subject of Lie's spherical geometry. During the year 1908-1909, he received a grant from the Ministry of Education that allowed him to go to Paris and to study at the Sorbonne. He listened to Picard, Poincaré and Darboux teaching and read their works. His Parisian stay had a major influence on his scientific development. It allowed the preparation of a Habilitation degree Hostinský passed in November 1911 in Prague. His thesis, under the title On the geometrical methods in the theory of functions, was approved by a referee committee including Petr, Sobotka and Strouhal, all of them his former professors in Prague University. In 1912, Hostinský was declared soukromy docent, the equivalent of the Privatdozent of the German universities, that is to say benevolent teacher. He needed to keep a position as a high-school teacher in the Reálka (equivalent of the German Realschule) of Prague-Vršovice and at the same time began (in 1912) to give conferences at the university on high-level mathematical themes such as the theory of analytical functions, differential geometry of curves, differential equations, and geometrical application of differential equations. At the eve of his nomination in Brno, during the academic year 1919-1920, he taught Volterra's theory of integral equations.

The foundation of a Czech university in Brno in 1919 was the conclusion of a long process, illustrating the fluctuation of Austrian politics towards its Czech dependences between autonomy and supervision. In Moravia, there had been a university in Olomouc since 1573, which was transformed into a high-school in 1782. It was renamed University in 1827 until 1851 when it was closed. Until 1919, the only university in the Czech lands was therefore Prague University, which divided into two in 1882: the prestigious German university and the newly created Czech university. A Technical Superior School was opened in Brno in 1873. Brno, today the second town of the Czech republic, is the main town of Moravia, a border region with Austria north of Vienna. An area known for its strong Germanspeaking minority, the town (Brünn in German) had benefited during the 19th Century from an explosive industrialization and had also become an important intellectual center. It is the birthplace of Gödel and the place where Richard von Mises and Georg Hamel began their careers in the beginning of the 20th Century [Sišma 2002]. The town is now universally known through the names of Georg Mendel, who established the first principles of genetics and of Leoš Janaček, a major composer of the early 20th Century. At the end of the 19th Century, new discussions had begun to propose the creation of a new university in Moravia, either in Brno or in

Olomouc. A crucial point was to decide whether there would be two universities (a German and a Czech one) like in Prague. To examine the question, a commission, including Otakar Hostinský, Bohuslav's father, was established in Prague. It declared in favor of the creation of a new Czech university in Moravia and provided a list of possible professors. Nevertheless, the creation of a Czech Technical Superior School in Brno delayed the realization of the project and World War I brought a completely new situation. With the independence of Czechoslovakia, the creation of a University in Brno was decided and it was given the name of Masaryk. Bohuslav Hostinský obtained the position of Professor of Theoretical Physics. He became very much involved in scientific and academic politics, reaching a status somewhat similar to that of Fréchet's position in Strasbourg, France. As was the case with his French colleague, Hostinský belonged to a generation for which WWI represented a deep rupture (even if Bohuslay, contrary to Fréchet, had not served in the Army due to his bad health). Fréchet in Strasbourg and Hostinský in Brno wanted to contribute to the edification of a new world. Several times rector and dean of Masaryk University, Hostinský was deeply interested in developing Czech mathematical publications and making them known abroad. In 1919, Fréchet contacted Hostinský in order to ask him to list the scientific publications that existed in Prague, and a captivating scientific correspondence between the two men began that was destined to last for 35 years. A study of this correspondence was begun in [Havlova et al. 2005] detailing the steps leading to Hostinský's interest in probability theory.

The social papers in Hostinský's archives show that he played the role of a kind of academic ambassador and was subsequently invited to visit ministries and politicians in the countries where he traveled. A passionate lover of France, he was also an active supporter of French culture in Czechoslovakia and had been for several years president of an active French cultural circle in Brno. In his correspondence with Fréchet, the latter questioned him several times regarding the situation in Czechoslovakia.

It is interesting to observe that when Hostinský was appointed in Brno, he had published only five papers with a physical theme. Until 1915, he had been exclusively involved in differential geometry. In 1915 his first book *Differential geometry of curves and surfaces* appeared. It would certainly have been more natural for him to take a mathematical chair, but the only position of Professor of Mathematics in the new Masaryk University had logically gone to Matyáš Lerch (1860–1922), a world-famous specialist in number theory who had been Professor at the Brno Technical Superior School. Hostinský only gradually became interested in Physics. Jiří Beránek, who had been Hostinský's last assistant in Brno, mentions in [Beránek 1951] the importance of reading Borel's book, Introduction géométrique à quelques théories physiques which appeared in 1914 and was henceforth regularly quoted in Hostinský's works. During WWI, Hostinský became more and more interested in the study of the mathematical disciplines that gained increasing importance in physics, such as integral equations and of course probability theory. As already mentioned, this story was traced in [Havlova et al. 2005] and we shall not repeat it here. Let us only mention two other sources of this focus on probability theory. Karel Vorovka, a professor of philosophy in Prague University who had commented on Poincaré's conceptions on randomness, played an important part in directing his friend Hostinský's attention to Poincaré's works on probability. Moreover, Beránek writes that the article on Statistical Mechanics written by Paul and Tanya Ehrenfest in 1911 for the Mathematical Sciences Encyclopedia, translated and completed by Borel for the French version, also made a deep impression on Hostinský. The fact that Hostinský immersed himself in researches on probability is testified in his own diary, also kept in Brno university archives. Entries made prior to 1917 contain only consideration of differential geometry. In January 1917, Hostinský makes some observations about the study of card shuffle by Poincaré (after his 1912 book) and lottery problems. In the next section, we present the way in which the Czech mathematician got in touch with the major subject of his professional life, Markov chains.

#### 3. HOSTINSKÝ, DOEBLIN AND MARKOV CHAINS

During the Fall of 1935, Doeblin took up the study of Markov chains. More or less neglected after the founding works of Markov, the subject had encountered a spectacular renewed interest at the end of the 1920s.

Let us begin by giving a survey of the mathematical model in its simplest form. The law of large numbers, expounded by Jacques Bernoulli and polished afterwards by numerous mathematicians (de Moivre, Laplace, Tchebitcheff, to mention only a few), asserts that the arithmetic mean of independent trials of a random variable converges towards the mathematical expectation of this random variable. It is the very type of an ergodic property: in the limit, the temporal means merge into the spatial mean. Looking for an extension of this result to dependent variables ([Seneta 2003], [Mazliak 2006]), Markov had defined in [Markov 1906] his notion of chain. A *Markov chain* is a sequence of random variables  $(X_n)_{n>0}$  such that the dependence between the state at time n + 1,  $X_{n+1}$ , and the past trajectory  $X_0, X_1, \ldots, X_n$ , exists only through the knowledge of the present state  $X_n$ . In other words, if the  $X_n$  take their values in the (discrete) set I, one has

$$P(X_{n+1} = j \mid X_n = i, X_{n-1} = i_{n-1}, \dots, X_0 = i_0)$$
  
=  $P(X_{n+1} = j \mid X_n = i) = p_{i,j}.$ 

Therefore a transition matrix  $P = (p_{i,j})$  is associated with a Markov chain and one classically denotes by  $p_{i,j}^{(n)}$  the elements of the matrix  $P^n$ :  $p_{i,j}^{(n)}$ therefore represents the probability for the chain to jump from the state *i* to the state *j* in *n* steps. Markov had studied some properties of this model. In particular, he proved the convergence of  $p_{i,j}^{(n)}$  to a value  $p_j$  independent from *i* when the transitions  $p_{i,j}$  are all positive [Markov 1908]. This result would be discovered again some twenty years later by Lévy and Hostinský, who would hear about Markov's works only after 1928. Markov had also proved the extension of the Central Limit Theorem (convergence to Gauss distribution) for this case.

Independently from Markov, Poincaré had considered events in a chain in order to model the very specific situation of a card shuffle. Each time a pack of cards is shuffled, a new order of the cards appears. Poincaré modeled this process as successive passages through the different possible permutations by providing the probability  $p_{i,j}$  of transition from an order i to an order j. He proved that distribution of the cards tends to uniformization: the more the pack is shuffled, the more the distribution is close to uniformity. The technique used by Poincaré to achieve the proof of this convergence was based on linear algebra arguments which would much later become the natural mathematical basis for the study of Markov chains studies. Around 1930, Kaucky in [Kaucký 1930] and Von Mises in [1931] used the theory of Perron-Frobenius concerning the eigenvalues of stochastic matrices to analyze Markov chain's asymptotics (see also [Seneta 1981, pp. 131–132]). Poincaré's aim in considering the elementary situation of a card shuffle in his book [Poincaré 1912] was to illustrate that with the passage of time, a system of dynamical particles can forget its initial distribution in order to achieve a stationary distribution (uniform, in the case of card shuffle). Poincaré had in the same way introduced the method of arbitrary functions in order to justify the uniform distribution of small planets over the zodiac.

As already mentioned, it is through his interests for statistical mechanics that Hostinský had read Poincaré's book around 1915, and had the idea of applying the method of arbitrary functions to the classical problem of Buffon's needle. Hostinský's aim was to counter an observation already formulated by Carvallo in 1912: the classical solution of Buffon's problem requires implementation of the experience on an infinite plane on which the needle could be thrown anywhere, and this is clearly unrealistic. Taking advantage of the end of World War I and of the thirst of the new Czechoslovakia to establish relations with universities of the winner's side (especially France), Hostinský proposed a French translation of his Czech paper on the subject (see [Hostinský 1920]), which attracted Fréchet's curiosity. At the same time, Fréchet was indeed learning the probability calculus that he should teach his new students in Strasbourg. This story has been traced in detail in [Havlova et al. 2005].

Hostinský's interest in probability theory henceforth developed and he was obviously especially interested in studies concerning the ergodic principle. He should therefore have been quite surprised when Hadamard in 1927 revisited Poincaré's card shuffle problem. In a note to the Comptes rendus in July 1927, [Hadamard 1927], Hadamard gave two new proofs of Poincaré's convergence result and partly restates the original proof included by Lévy in his 1925 course [Lévy 1925], a proof that nobody had noticed. In the Letters from November 3rd and 9th, 1928 in [Barbut et al. 2004], Lévy wrote to Fréchet that "it was from Hostinský that [he] had learnt that he had provided a new proof".<sup>5</sup> In November 1927, Fréchet traveled to Czechoslovakia and especially visited Brno, where he was welcomed by Hostinský, who mentioned Hadamard's note to him. In a letter sent from Strasbourg on January 4th, 1928, Fréchet wrote that he had included Hadamard's considerations in his lectures on analytic theory of probability. He also told Hostinský that an actuary from Brno, F.M. Urban, had already considered the same kind of problem in his book [Urban 1923]. This letter is therefore a partial answer to the question left unanswered by Bru [2003, p. 203]: how did Hostinský obtain information about Urban's book? It remains unclear how Fréchet himself had known of this book.

Hostinský published a note [Hostinský 1928a] in which he generalized the ergodic theorem to Markov chains with positive transitions. But, above all, he extended it to the case of continuous values defined through a functional transition kernel f(x, y). He was the first to deal with this situation: in particular he preceded Lévy and Kolmogorov who were to be two major leaders of the theory of stochastic processes in the 1930s. It

<sup>&</sup>lt;sup>5</sup> "C'est par Hostinsky que j'ai appris que j'avais fait du nouveau" ... [Barbut et al. 2004, p. 141].

is necessary to mention the amazing exception of Bachelier's works who had, as early as 1900, proposed a system of continuous transitions for modeling stock exchange fluctuations. In Hostinský's publications [Hostinský 1931] and [Hostinský 1932] devoted to the cycle of conferences he had given in Paris about Markov chains, Bachelier is only mentioned after Kolmogorov's paper [Kolmogoroff 1931]. Certainly Hostinský had not read Bachelier's works may be because of his poor reputation among French mathematicians. In [Barbut et al. 2004, pp. 26–28], one may find a description of Bachelier's unhappy story and, above all, references for studies on Bachelier. Victim of ostracism from his colleagues, Bachelier had discovered Hostinský's works only in 1936 at a time when he was a professor at the University of Besançon where mathematical publications were not available. Having found there some results he had himself obtained twenty years ago, he wrote a furious letter to Hostinský on May 14th, 1936 with a comment about the results for which he claimed antecedence. The correspondence seems to have immediately stopped.

Let us come back to 1928. Hadamard, who presented Hostinský's note to the Academy seems to have been excited about the possible developments of Hostinský's model: it might have been an important step for proving a sufficiently general version of the ergodic principle. Hadamard added a remark in this direction at the end of Hostinský's note. In fact, this proof of the general ergodic principle, given by Birkhoff [1931], had arrived only three years later and from completely different considerations. Birkhoff proved the result as a consequence of a recurrence theorem for repeated spatial transformations, under the hypothesis that the invariant sets through the transformation are of measure 0 or 1. Hadamard went back to work and published himself a new note on the subject [Hadamard 1928], immediately followed by another note by Hostinský [1928b]. When the 1928 Bologna International Congress of Mathematicians opened, the theory of events in a chain was an important subject for discussion among those who dealt with probability. It was in particular the subject of Hadamard's talk, a quite remarkable event if we keep in mind the lack of consideration in the French mathematical school for probability. The link with Markov's works seems to have been unnoticed up to this moment, at least by Hostinský, Fréchet and Hadamard. Bru [2003] (as has already been mentioned by Von Plato [1994]) proposes the reasonable hypothesis that it was in Bologna, the last mathematical international congress (outside USSR and before the 1960s) where the Soviet delegation was important, that the link was established through Polya and Bernstein, who were more or less the only ones aware of Markov's works.

Even Romanovsky, who was to become a great specialist in Markov chains, may have discovered the interest of the Markov model through Hostinský in Bologna. It is usually considered that Romanovsky and Hostinský (who used the expression in a note to the *Comptes rendus* as early as July 8th, 1929) created the name *Markov chains*.

At the beginning of the 1930s, Hostinský was therefore one of the main specialists in the topic of Markov chains. He exchanged letters with a great number of scientists throughout Europe. In particular, his correspondence with the young Soviet probabilistic school began just after the Bologna conference. From 1929 on, the exchanges between Fréchet and Hostinský had almost exclusively dealt with two subjects: the general theory of Markov chains, and the solutions of the Chapman-Kolmogorov equations, that is to say precisely the two themes in which Doeblin was going to be interested in his scientific career.

Fréchet rapidly began to teach Markov chains. He wrote:

"We [i.e. Hadamard and Fréchet] are so penetrated with the importance of this question that one of us [Fréchet] had made from this subject the substance of several semestrial lectures at the Faculties of Science in Strasbourg and Paris."  $^6$ 

In a series of publications after 1932, Fréchet had built a first really general theory of Markov chains in discrete or continuous cases. Let us in particular mention the important paper [Fréchet 1932] where he obtained the limit form of the dispersion, independently from Potoček, a Hostinský student who had also found the same result in [Potoček 1932]. At the end of 1935, Fréchet, who had spent several days in the Soviet Union, had learnt from Kolmogorov the last advances of the Soviet school on the topic of Markov chains. As soon as he had returned to Paris, he advised Hostinský to look at them. At this very moment, Doeblin was presenting the first results he obtained to Fréchet.

Doeblin's works were synthetized by Lévy [1955] for the first time. Doeblin successfully achieved a complete classification of Markov chains with discrete time and space, simultaneously with and independently from Kolmogorov's own classification published in [Kolmogoroff 1936]. Starting from the results that Fréchet had obtained in the regular case where the transitions are positive, Doeblin proves that the state space can be split in two subset categories: the *final groups* that the chain cannot leave anymore

<sup>&</sup>lt;sup>6</sup> "Nous sommes si pénétrés de l'importance de cette question que l'un de nous [Fréchet] en a fait le sujet de plusieurs cours semestriels aux Facultés des sciences de Strasbourg et de Paris." [Fréchet & Hadamard 1932, p. 5].

once it entered them, and the *passage groups* (see [Mazliak 2007, Letter 6]). Moreover, Doeblin also shows that one must analyze the periodicity of the states from each group and distinguish those which are *cyclic* from those which are *uncyclic*. Doeblin was also interested in the case of non-discrete valued Markov chains for which he extended the results of convergence and periodicity obtained by Fréchet [1932] as an extension of Hostinský's [1932] model. Here, the notions of passage or final groups are naturally much harder to define than in the discrete case, and Doeblin used measured properties for that purpose. It was the object of the second chapter of his thesis ([Doeblin 1938]).

We have already mentioned the interest of Hostinský for the Chapman-Kolmogorov equation. In 1929 at the Warsaw conference of the mathematicians from Slavic countries, Hostinský revealed the first results he had obtained for continuous time diffusions : these results were developed afterwards and presented in his Paris conferences published in [Hostinský 1931]. Through Polish scientists, he got information on the works of Smoluchowski on Brownian motion. Let us in particular mention his exchanges with Wladislaw Natanson (1864-1937), a physicist from Cracow who had studied in St Petersburg (where he followed Markov's lectures) and who had worked with Smoluchovski in Lwów and Cracow. At the end of 1931, Hostinský had read Kolmogorov's [1931] paper on Markov processes where Kolmogorov (briefly) mentions Hostinský's note on the ergodic theorem for continuous-valued Markov chains as an origin for his own theorem on ergodicity of Markov processes. To take position on possible anteriority problems, Hostinský had added at the end of [Hostinský 1932] a Note III called "On ergodic principle", where he mentions the exact simultaneity of Kolmogorov's results with his own. During the period 1934-1936, Fréchet had also published several papers on continuous time Markov processes defined through solutions of the (Bachelier-Smoluchowki)-Chapman-Kolmogorov equation. It was therefore natural for Doeblin to have become involved in such a subject. The mentioned functional equation is the one satisfied by the transition probabilities from a state to another of a particle following a Markovian motion trajectory. If one denotes by F(x, y; s, t) the probability for the particle to jump from state x at time s to state y at time t, one has

(1) 
$$F(x, y; s, t) = \int_{-\infty}^{+\infty} F(z, y; u, t) \, \mathrm{d}F_z(x, z; s, u).$$

If this equation had already been formed by Smoluchowski and Chapman in their study of Brownian particle (not to mention also Bachelier), it was Kolmogorov who firstly made it a tool for defining a Markov process in [Kolmogoroff 1931]. Since then, several mathematicians have worked in order to find solutions to the functional equation (1), among them Bernstein, Hostinský, Fréchet, Khinchin, Petrowski, Feller, ... Each of them chose an analytical approach based in particular on parabolic PDEs extending the heat equation for Brownian motion. As often in considerations about stochastic processes, specifically probabilistic aspects appear less obviously in the continuous case which may therefore be dealt with using methods of Analysis. It is when jumps are introduced in the processes that original methods must be found, using the random process structure. Pospišil, Hostinský's student, had proved a first existence and uniqueness result for the Chapman-Kolmogorov equation corresponding to a process taking values in a discrete subset of  $\mathbb{R}$ . He used for that a local condition of a stochastic type: where Hostinský had considered the existence of a derivative for the conditional density linking state x and state y, Pospišil considered an integrated form of this hypothesis : the existence of the derivative of the transition probability from the state xto a measurable subset R of the state space. This hypothesis is weaker, but above all it is of a more probabilistic nature than Hostinský's which is purely analytical. This allowed Pospišil to avoid the singularity problem in x which prevented Hostinský from obtaining a unique solution. Hostinský had rapidly sent Pospišil's paper to Fréchet, maybe with the intention to see it published again in a French journal. However, it seems that Fréchet had problems obtaining a clear idea of its contents and therefore gave it to several students for interpretation. On June 17th, 1936, at the end of the letter (already mentioned at the beginning of the present paper) where he first mentions Doeblin, he wrote to Hostinský:

"I have also given Pospiszil's [*sic*] paper to Döblin, and he does not understand it either. I would really like to know what it does contain. Could you send an exemplary to M. Ostenc, teacher at the *lycée St Charles* in Marseilles, and to M. Döblin, Student, Institut H. Poincaré, 11, rue P. Curie."<sup>7</sup>

Émile Ostenc was another of Fréchet's students who worked on ergodic theory for Markov chains: he is quoted by Doeblin [1937]; he did not finish his thesis and left for secondary teaching. Let us mention also that after the aforementioned paper, Pospišil's interests had changed and he began

<sup>&</sup>lt;sup>7</sup> "J'ai donné à lire à Döblin aussi le mémoire de Pospiszil, il n'arrive pas non plus à le comprendre. Je voudrais tout de même bien savoir ce qu'il contient. Pourriez-vous en faire envoyer un exemplaire à M. Ostenc, Prof. au lycée St Charles à Marseille, et à M. Döblin, Étudiant, Institut H. Poincaré, 11, R. P. Curie." (Archive department of Masaryk University, Brno)

to work on general topology, another topic represented in Brno by a first rate mathematician, Cech. Pospišil tragically died in 1944 following several months of internment in a German concentration camp [Cech 1947]. Pospišil's paper was published in *Časopis* by Hostinský, and Doeblin seems to have been its most attentive reader: Bru [1993, p. 8] mentions the existence of an off-print annotated by Doeblin. Contrary to Bru's suggestion in [Doeblin 2000, Note 7, p. 1136], Doeblin already knew Pospišil's paper when Hostinský came to Paris at the end of 1936 to give lectures at the IHP where he quoted this piece of work. Probably he had been struck by the originality of the 'integrated' hypothesis proposed by Pospišil. For Doeblin, it had become clear that what was important was to find hypotheses and solutions to the Chapman-Kolmogorov equation which are interpretable from a stochastic point of view, contrary to conditions of an analytical regularity type which may be meaningless with respect to the process structure. In Letter 8, [Mazliak 2007], Doeblin mentions that it was Hostinský's observation in Geneva that enlighted this point for him. In the draft of his talk at Hadamard's seminary about Chapman's equation, Doeblin himself decribes the genesis of these ideas [Doeblin 2000, pp. 1129-1134]. Pospišil's work had therefore motivated Doeblin's first work on the Chapman-Kolmogorov equation. Doeblin submits his paper for publication to Hostinský in July 1938 (Letter 8) and mentions that the results contained in it were obtained some six months before. The gripping story of this paper marked the abrupt interruption of the exchanges between Doeblin and Hostinský (see Letters 9 and 10, in [Mazliak 2007], as well as 4.2 below).

#### 4. THE TEN LETTERS

Doeblin seems to have got rid of all the letters sent to him by Hostinský after the sudden stop of their correspondence. Though we do not know the level of the young man's esteem for the Czech mathematician, it seems clear that for Doeblin, Hostinský was a scientist of the old times, rather unable to follow the most recent methods of probability calculus introduced, for instance, by the Soviet school (Kolmogorov), and who was not in a position to extract all the wealth of the models he was studying. Besides, there is an indication about Doeblin's state of mind in a letter sent by Doeblin to Fréchet in September 1936 where he wrote: "with M. Paul Lévy I think that one must draw all the conclusions from a study if one does not want

#### L. MAZLIAK

to be called stupid"<sup>8</sup>. Nevertheless, Doeblin was looking for opportunities for publication, and he had written for more than two years to Hostinský, seeking his interest and approbation, probably advised to do so by Fréchet, who had regularly asked his friend to publish his own papers. Fréchet in particular submitted Doeblin's thesis for publication in Brno to Hostinský in 1938, who explained that he was not in a position to accept so long a text. Doeblin's thesis was eventually published in Romania (!).

The first letter was sent on June 29th, 1936 and the last one in September 1938, but most of them were sent before March 1937, while Doeblin was busy with discrete time Markov chains. On August 17th, 1937, as Doeblin was helping Fréchet to correct the proofs of his treatise on arbitrary functions's methods, Doeblin finished the writing of his thesis and wrote that he wished "to work on the general case of Smoluchowski's equation soon after his thesis defense". Afterwards, he was mostly interested in Chapman-Kolmogorov's equation and wrote again to Hostisnký only in the Summer 1938 in order to propose the publication of his paper extending the study of Pospišil [1936] (see Letter 8, [Mazliak 2007]).

Hostinský came to Paris at the end of 1936 and he must have met Doeblin at this time. They also met in October 1937 at the Geneva conference already mentioned several times. De Finetti's notes of this conference, published in [de Finetti 1939] mention many observations by Doeblin and Hostinský.

Let us finally give a short overview of the letters' content.

#### 4.1. First series (Letters 1 to 7)<sup>9</sup>

The first letter is simply dated from June 29th: the year is not specified but is clearly 1936, due to the following letter and the announcement of the publication of the article [Doeblin 1936a] presented the same day by Borel at the Academy. Doeblin mentions that he had read Hostinský's note "Sur les mouvements qui dépendent du hasard", also presented by Borel on June 22nd, 1936, the same day that Doeblin's first note (the note [Doeblin & Lévy 1936]) was published. In this note, Hostinský studies a coupled motion of two mobiles with elementary methods, and Doeblin writes that the results he had himself obtained for simple Markov chains can be extended to Hostinský's case.

<sup>8 &</sup>quot;Avec Monsieur Paul Lévy j'estime qu'il faut tirer toutes les conclusions de ses travaux si on ne veut pas être taxé d'incapable." [Bru 1993, Letter III]

<sup>&</sup>lt;sup>9</sup> The text of the Letters is to be found in [Mazliak 2007].

Letter 2, a postcard sent from Paris on July 3rd, 1936, is an erratum of the previous one.

In Letter 3, without date but to which Hostinský answered on October 20th, 1936, Doeblin, who is writing his thesis (less than a year after having begun it ...) asks Hostinský to provide some bibliographical references on Markov's works.

Letter 4 to follow is also without date. Doeblin mentions there that he had only published the two notes of Summer 1936 and this allows the letter to be dated between July and November 1936, as on December 7th, 1936, Hadamard presents Doeblin's note [1936b] to the Academy. In the letter, Doeblin describes the main results of an attached note. This is the first version of the paper [Doeblin 1937] published in the Scientific Publications of Masaryk University by Hostinský. Following [Litzman 1997], Hostinský's role was essential for establishing a system of regular and standard publications in Brno University. At the beginning of his correspondence with Hostinský, Fréchet, who had contacted him in order to list the possible Czech publications in French, gives advice for making these publications accessible to foreign readers by systematically adding French summaries of the published papers [Havlova et al. 2005]. About the article [Doeblin 1937], it is interesting to observe that Doeblin had considerably improved the writing of the final version, as here it is somewhat cursory and confused. Moreover, the results are completed by those expounded in Letter 6.

Letter 5 is a small postcard stamped on November 21st, 1936 at 18.30. In red, Hostinský has written: "answered 23.XI.36" and this quick reaction may be a sign of his amazement about Doeblin's inventiveness. Doeblin has indeed observed that there is a very simple way to deal with the coupled motions studied by Hostinský: by considering the process taking values in a product space, one defines a simple Markov chain for which limit theorems are obtained in the usual way. This may be the first consideration of coupling which is a great contribution of Doeblin for the treatment of Markov chains [Liggett 1993]. At the mentioned date, Hostinský was, besides, preparing for his trip to Paris where he would give lectures on Markov chains at the IHP. In the paper published after these lectures, he proves the convergence of the transitions for his coupled motion using Doeblin coupling [Hostinský 1937, n° 13, p. 98].

In Letter 6 from January 21st, 1937, Doeblin completes his series of results on the general analysis of discrete Markov chains. They will be partly incorporated into the paper [Doeblin 1937]. In fact, as Doeblin mentions himself, he is involved in the studies of a general model the conclusion of which will be the object of his big paper [Doeblin 1940]. The following Letter 7 is dated from March 11th of the same year. Doeblin sends back the proofs of [Doeblin 1937] that Hostinský should have sent in February. He seizes the opportunity for evoking his project to study the behavior of chains with long-time dependence (with the meaning that the probability to be in a given state at time *n* depends on what has happened in the *m* previous states). This scheme will be the subject of a study made in common with Fortet which will lead to the publication of a paper in the *Bulletin de la SMF* [Doeblin & Fortet 1937]. This kind of theme had already been considered by [Onicescu & Mihoc 1935]. As Doeblin does not mention this paper in the present letter, it may be Hostinský who had revealed its existence to him. The difficulty for dealing with the problem is presented here by Doeblin in a heuristic way: to find a good formulation to quantify the decreasing (with time) dependence of the values taken by the chain. In [Doeblin & Fortet 1937], the authors will formulate two possible controls for this decrease that they call *type A* and *type B*.

#### 4.2. Second Series (Letters 8 to 10)

There is an interval of sixteen months between Letters 7 and 8. Meanwhile, Doeblin and Hostinký met in October 1937 at the Geneva conference. This important scientific event, already mentioned several times above, gathered the most eminent specialists in probability theory of the moment except the Soviet ones, whose travel had been forbidden by their government. This was precisely the peak of the *Ejovchtchina* massive campaign of arrests of the years 1936-1938: the order 00447 of the NKVD (the People's Commissariat for Internal Affairs) on the elimination of anti-soviet individuals, signed by Ejov in July and applied since August 5th, 1937 is usually considered as the beginning of the massive purge. It was a time of total blackout in the USSR and the borders remained hermetically closed. On this subject, the interested reader may for instance consult [Conquest 1990]. We have already mentioned the revelations by Doeblin and Hostinský at the conference, reported by de Finetti [1939]. It is probable that Doeblin and Hostinský had also spoken about the possible extension of Pospišil's results to solve the Chapman-Kolmogorov's equation.

Letter 8 is a postcard, dated from July 17th, 1938 and received in Brno on July 20th. Doeblin wrote his personal address on it, 5 square Delormel, Paris 14<sup>e</sup>, where he lived with his family. He declares to Hostinský that he has found a solution of the Chapman-Kolmogorov equation under more general conditions than Pospišil's, as well as the interpretation of the corresponding pure jump process. He suggests the publication in a Czechoslovakian journal to Hostinský. The latter begins by agreeing to publish the paper in *Časopis pro Pestovani Matematiky*, of which he was the editor in chief. Doeblin (in the next Letter 9 dated from August 26th, 1938) thanks him for that. He adds a series of comments about the fact that the analytical solutions of the Chapman-Kolmogorov equation are, more or less, easy to interpret as stochastic processes. On this topic the reader may refer to the detailed study of the resolution of this equation in [Doeblin 2000, pp. 1104–1108].

Doeblin mentions also the talk he has given at Hadamard's seminar on the Chapman-Kolmogorov equation. It is certainly the talk the notes of which are transcribed and edited in [Doeblin 2000, pp. 1129–1138]. These notes establish the link between Doeblin's work and the work of his predecessors Hostinský (the memoir [Hostinský 1932] above all) and Pospišil on one hand, and Kolmogorov on the other hand. It is interesting to observe the comments on Hostinský's conferences at the IHP at the end 1936-beginning 1937, where he discussed Pospišil's paper. In Fréchet's fund in the Archives of the Science Academy in Paris a copy is kept of [Hostinský 1932] annotated by Doeblin. It is probable that the latter constructed his talk following Hostinský's plan, not without expressing major doubts about the way Hostinský used purely analytical methods to obtain solutions. In the case where the state is concentrated in the set [a, b], Doeblin insisted on the fact that Hostinský has not sufficiently taken into account the singularities appearing in a and b, criticisms he mentions again in the present letter.

At the foot of the letter, Hostinský has written: "Answered on IX 18, 38 and I have a manuscript with annotations". The detail of this answer is not known but it is clear that Hostinský had rejected the paper. In a document kept in Doeblin's archive fund in Marbach, Doeblin mentions that Hostinský returned the manuscript with "a letter insolent to him containing no serious argument" [Bru 1993, note 10]. The last Letter 10 of the exchanges presented in our paper is very short and cutting: it has no date but it was clearly written just after the signature of the Munich agreement (September 30th, 1938). The explosive situation of the moment had most certainly urged Hostinský to redouble his carefulness before publishing a French paper, moreover written by the son of a well-known anti-nazi German refugee. Hostinský may not have dared to write down such an explanation. But it makes Hostinský's volte-face understandable, accepting and then rejecting Doeblin's paper. Moreover, in 1949, he confessed that he had felt admiration for the work. Doeblin offers that same explanation in the letter and his last sentence shows how lucid he was about the international situation. Doeblin's paper was eventually published in Sweden by Cramer and Feller in the *Skandinavisk Aktuarietidskrift* [Doeblin 1939].

Monsieur le Professeur, J'ai reçu votre lettre et mon manuscrit que je publicrais ailleno. Je est à peine besoin d'indiquer que je ru'ai rieu à ajouter ou à reprancher à ce pravail : la déconyrorition que je donne est démontrée sons mes tryrothères et nou admise a priori. Toute fois je comprends très tien que une reme telieco le vague ne voit par la nécessité de publier en ce moment un mémoire d'un étranger. Je n'ai jamais en anssi houte que maintenant et j'aurais préférré la guerre à ce qui se passe maintenaul. Agrées, monoieur le Professeur, l'esepression de mes seuliments respectueurs. D'. Dochim

Letter 10 (October 1938) from Wolfgang Doeblin to Bohuslav Hostinský © Archive Department, Masaryk University, Brno

#### **EPILOGUE**

The exchanges between Doeblin and Hostinský stopped on this abrupt interruption. France and Great-Britain had signed a blank cheque to Hitler to dismember Czechoslovakia, with the illusion that it was the price for peace. The French breaking of the alliance treaty linking France and Czechoslovakia was particulary bitter to live with on the Czech side. Fréchet wrote an embarrassed letter to Hostinský on September 30th, 1938 where he tried to justify the French position and at the same time, mentioned his understanding of Czech bitterness. "Though I deplore the conditions of the obtained agreement", he wrote, "I am very glad that

the independence of Czechoslovakia had been spared".<sup>10</sup> Contrary to Doeblin, Fréchet was not twenty years old, and, more important, had not been himself confronted with the brutality of the Nazi regime. On March 15th, 1939, German troops entered Prague and in November 1939, the German government ordered the closure of the Czech universities. Five and a half years of terror had begun.

One finds an amazing document in the Brno archives. When the great Italian mathematician Vito Volterra died in October 1940, a death which was partly a consequence of the racist laws imposed in Italy since 1938, his widow sent an announcement to Hostinský in Brno. The Italian postal service carefully added the mention Protettorato Boemo-Moravo. The envelope was subsequently unambiguously stamped with a swastika. At this point Doeblin had been dead for a long time. The purity of his last act echoes back to the end of his correspondence with the Czech mathematician.

If the ten letters described in the present paper do not add much to our understanding of Doeblin's significance in the history of probability theory and stochastic processes, they give an interesting insight on the young mathematician's coming on the international stage. They provide details on the chronology of Doeblin's work and on his mathematical agenda, and therefore form a valuable complement to the set of letters exchanged with Fréchet studied by [Bru 1993].

#### Acknowledgement

It is my pleasure to thank Pavel Šišma and Jiřína Kalendovska, as well as the Scientific Service of the French Embassy in Prague for offering all facilities for consultation. I would like to thank the officials of the Barrande Research Program between France and the Czech Republic for providing funds for extension of the present research to a general study of Franco-Czechoslovakian mathematical exchanges between the two World Wars. I want also to express my deep gratitude to the referees who allowed the present version to be so different from the first one (for sure, for its own good!), and to Prof. Guy Thompson and Susan and George Cardosi-Clark for their unvaluable help in improving the English translation of the French text.

<sup>10</sup> "En déplorant les conditions dans lesquelles l'accord a été obtenu, je me félicite que l'indépendance de la Tchécoslovaquie ait été épargnée." (The letter is kept in the archive department of Masaryk University, Brno).

#### REFERENCES

#### BARBUT (Marc), LOCKER (Bernard) & MAZLIAK (Laurent)

[2004] Paul Lévy-Maurice Fréchet: 50 ans de correspondance en 107 lettres, Paris: Hermann, 2004.

#### Beránek (Jiří)

[1951] Bohuslav Hostinský, Československý časopis pro fyziku, 1 (1951), pp. 90– 95.

#### BILOVA (Stepanka), MAZLIAK (Laurent) & ŠIŠMA (Pavel)

[2006] The axiomatic melting pot, *Electronic Journ@l for History of Probability* and Statistics, 2, n<sup>o</sup> 2 (2006); http://www.jehps.net/decembre2006. html.

#### BIRKHOFF (George D.)

[1931] Proof of Ergodic Theorem, Proc. N.A.S., 18 (1931), pp. 656–660.

#### BRU (Bernard)

- [1993] Doeblin's life and work from his correspondence, in Cohn (Harry), ed., *Doeblin and Modern Probability*, Providence: Amer. Math. Soc., 1993, pp. 1–64.
- [2003] Souvenirs de Bologna, J. Soc. Fr. Stat, 144 (2003), pp. 135–226.

#### **CANGUILHEM** (Georges)

[1996] Vie et Mort de Jean Cavaillès, Allia, 1996.

#### Čесн (Edvard)

 [1947] Vědecké pracé Bedřicha Pospíšila, *Časopis Pěst. Mat.*, 72 (1947), pp. 1– 9.

#### CONQUEST (Robert)

[1990] The Great Terror : a Reassessment, Oxford: Oxford University Press, 1990.

#### DOEBLIN (Wolfgang)

- [1936a] Sur les chaînes discrètes de Markoff, CRAS Paris, 203 (1936), pp. 24-26.
- [1936b] Éléments d'une théorie générale des chaînes constantes simples de Markov, CRAS Paris, 203 (1936), pp. 1210–1211.
- [1937] Le cas discontinu des probabilités en chaîne, Brno: Pub. Fac. Sci. Univ. Masaryk, vol. 236, 1937.
- [1938] Sur les propriétés asymptotiques de mouvements régis par certains types de chaînes simples, Thèse de doctorat ès Sciences Mathématiques, Paris, 1938.

- [1939] Sur certains mouvements aléatoires discontinus, Skand. Aktuarietidskr.,
  22 (1939), pp. 211–222.
- [1940] Éléments d'une théorie générale des chaînes simples constantes de Markoff, Ann. Sci. E. N. S., 57 (1940), pp. 61–111.
- [2000] Sur l'équation de Kolmogoroff, Pli cacheté à l'Académie des sciences, édité par
  B. Bru et M. Yor, numéro spécial des CRAS Paris, série 1, t. 331, 2000.

#### DOEBLIN (Wolfgang) & FORTET (Robert)

[1937] Sur deux notes de MM. Kryloff et Bogoliouboff, *CRAS Paris*, 204 (1937), pp. 1699–1701.

DOEBLIN (Wolfgang) & LÉVY (Paul)

- [1936] Sur les sommes de variables aléatoires indépendantes à dispersions bornées inférieurement, *CRAS Paris*, 202 (1936), pp. 2027–2029.
- DE FINETTI (Bruno)
  - [1939] Compte-rendu critique du colloque de Genève sur la théorie des probabilités, Actualités Scientifiques et Industrielles, vol. 766, Paris: Hermann, 1939.
- **FRÉCHET** (Maurice)
  - [1932] Compléments à la théorie des probabilités discontinues "en chaîne", Ann. Sci. Sci. Norm. Sup. Pisa, Sér. II, II (1932), pp. 131–164.
  - [1938] Méthodes des fonctions arbitraires. Théorie des événements en chaîne dans le cas d'un nombre fini d'états possibles, *Traité de calcul des probabilités et de ses applications*, t. I, fasc. III, Paris: Gauthier-Villars, 1938.

#### FRÉCHET (Maurice) & HADAMARD (Jacques)

 [1932] Sur les probabilités discontinues des événements "en chaîne", Ann. Sci. Sc. Norm. Sup. Pisa, Sér. II, II (1932), pp. 131–164.

#### GRACQ (Julien)

[1958] Un balcon en forêt, Paris: J. Corti, 1958; English Translation: A Balcony in the Forest, Harvill paperbacks, 1992.

#### HADAMARD (Jacques)

- [1927] Sur le battage des cartes, CRAS Paris, 185 (1927), pp. 5–9.
- [1928] Observations sur la note précédente, CRAS Paris, 186 (1928), p. 86.

#### HAVLOVA (Veronika), MAZLIAK (Laurent) & ŠIŠMA (Pavel)

[2005] Le début des relations mathématiques franco-tchécoslovaques vu à travers la correspondance Hostinský-Fréchet, *Electronic Journ@l for His*tory of Probability and Statistics, 1, nº 1 (2005); http://www.jehps. net/mars2005.html.

#### HEYDE (Chris) & SENETA (Eugène), eds.

[1991] Statisticians of the Centuries, New-York: Springer, 1991.

#### HOSTINSKÝ (Bohuslav)

- [1920] Sur une nouvelle solution du problème de l'aiguille, Bull. Sci. Math., 44 (1920), pp. 126–136; article original : Bohuslav Hostinský, Nové řešení Buffonovy úlohy o jehle (New solution of Buffon problem on needle), Rozpravy Ceské Akademie, XXVI, II (1917), 13.
- [1928a] Sur les probabilités relatives aux transformations répétées, *CRAS Paris*, 186 (1928), pp. 59–61.
- [1928b] Complément à la note sur les probabilités relatives aux transformations répétées, *CRAS Paris*, 186 (1928), pp. 187–189.
- [1931] *Méthodes générales du calcul des probabilités*, Mémorial des Sciences Mathématiques, fasc. 52, Paris: Gauthier-Villars, 1931.
- [1932] Application du calcul des probabilités à la théorie du mouvement brownien, *Ann. Inst. H. Poincaré*, 3 (1932), pp. 1–74.
- [1937] Sur les probabilités relatives aux variables aléatoires liées entre elles. Applications diverses, Ann. Inst. H. Poincaré, 7 (1937), pp. 69–119.
- [1949] O výpočtu pravděpodobností, které se vztahují k časovému vývoji soustav, Aktuárské vědy, 8 (1949), pp. 61–67.

#### Kaucký (Jan)

[1930] Remarques à la note de M.V. Romanovsky sur les chaînes discrètes de Markoff, *CRAS Paris*, 191 (1930), pp. 919–921.

#### KHINCHIN (Alexandr)

[1933] Asymptotische Gesetze der Wahrscheinlichkeitsrechnung, Ergebnisse der Mathematik und ihrer Grenzgebiete, vol. 2, Berlin: Springer, 1933.

#### KOLMOGOROFF (Andrei)

- [1931] Über die analytischen Methoden in der Wahrscheinlichkeitsrechnung, *Math.Ann.*, 104 (1931), pp. 149–160.
- [1936] Zur Theorie der Markoffschen Ketten, Math. Ann., 112 (1936), pp. 155–160.

#### Lévy (Paul)

- [1925] Calcul des Probabilités, Paris: Gauthier-Villars, 1925.
- [1955] Wolfgang Doeblin (V. Doblin) (1915–1940), Rev. Hist. Sciences, 1955, pp. 107–115.

#### LIGGETT (Thomas M.)

[1993] The Coupling Technique in Interacting Particle Systems, in Cohn (Harry), ed., *Doeblin and Modern Probability*, Providence: Amer. Math. Soc., 1993, pp. 73–83.

#### LITZMAN (Otokar)

[1997] *Bohuslav Hostinský*, Osobnosti Přirodovědecke Fakulty Masarykovy Univerzity, Brno: Folia Historica, 1997.

#### Markov (A.A.)

- [1906] Extension of the law of large numbers to quantities dependent on each other, *Bull. Soc. Math. Kazan, Série* 2, XV (1906); available on http://www.jehps.net; original article: Андреи А. Марков: Распространение закона больших чисел на величины зависящия друг от друга.
- [1908] Extension of limit theorems of probability to sums of quantities connected in chain, *Bull. Acad. Imp. Sci. St Petersbourg*, 22 (1908), р. 9; original article: Андреи А. Марков: Распространение предельных теорем исчисления вероятностей на сумму величин, связанных в цепь.

#### MAZLIAK (Laurent)

- [2006] Markov et ses chaînes, Pour la science, 340 (2006), pp. 12-15.
- [2007] ed. Wolfgang Doeblin-Bohuslav Hostinský: Correspondence (1936– 1938), partie Traces et Mémoires, Electronic Journ@l for History of Probability and Statistics, 3, nº 1 (2007); http://www.jehps.net/juin2007. html.

#### ONICESCU (Octav) & MIHOC (Gheorghe)

[1935] Sur les chaînes de variables statistiques, *Bull. Sci. Math.*, 59 (1935), pp. 174–192.

#### POINCARÉ (Henri)

[1912] Calcul des Probabilités, 2<sup>e</sup> éd., Paris: Gauthier-Villars, 1912.

#### POSPIŠIL (Bedrich)

[1936] Sur un problème de MM. S. Bernstein et A. Kolmogoroff, *Casopis Peš. Mat.*, 65 (1936), pp. 64–76.

#### Ротоčек (Jan)

[1932] Sur la dispersion dans la théorie des chaînes de Markov, *Pub. Fac. Sci. Univ. Masaryk*, 154 (1932).

#### SENETA (Eugène)

- [1981] Non-negative Matrices and Markov Chains, 2nd ed., New-York: Springer, 1981.
- [2003] Statistical Regularity and Free Will: Quetelet and Nekrasov, *Int. Stat. Review*, 71 (2003), pp. 319–334.

#### Šıšма (Pavel)

[2002] Georg Hamel and Richard von Mises in Brno, *Hist. Math.*, 29 (2002), pp. 176–192.

#### URBAN (Friedrich M.)

[1923] Grundlagen der Warscheinlichkeitsrechnung und der Theorie des Beobachtungsfehler, Leipzig: Teubner, 1923.

#### VON MISES (Richard)

[1931] Warscheinlichkeitsrechnung und ihre Anwendung in der Statistik und theoretischen Physik, Leipzig-Wien: Deuticke, 1931.

#### von Plato (Jan)

[1994] Creating Modern Probability, Cambridge Studies, 1994.