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THE AREA AND THE SIDE I ADDED: SOME OLD BABYLONIAN GEOMETRY

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Abstract. — There was a standard procedure in Mesopotamia for solving quadratic problems involving lengths and areas of squares. In this paper, we show, by means of an example from Susa, how area constants were used to reduce problems involving other geometrical figures to the standard form.

Résumé (La surface et le côté que j'ai ajouté : un problème de géométrie babylonienne)

Il y avait en Mésopotamie un procédé standard pour résoudre des problèmes quadratiques impliquant des longueurs et des surfaces de carrés. Nous montrons, sur un exemple de Suse, que des constantes géométriques ont été employées pour ramener des problèmes concernant d'autres figures au format standard.

INTRODUCTION

One of the central topics of Old Babylonian mathematics is the solution of 'quadratic' or area problems. Høyrup has developed a convincing geometric interpretation for the procedures by which many of these problems were solved. However, it has not previously been recognized how Old

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Babylonian scribes applied these techniques when the underlying figure was not a square. Here, we show by means of an exemplar from Susa that the solution to this problem lays in an elegant usage of another ubiquitous element of Old Babylonian mathematics, the technical constant, or coefficient.

Among the corpus of Old Babylonian geometric problems is a group of exercises that begin with variations on the phrase, "The area and side of my square I added..." The resulting total is given, and the problem for the student is to determine the length of the side of the square.[1] On first translation, these problems were seen as exercises in pure algebra, dressed in physical guise. If the unknown length of the side of the square is denoted ℓ, then the sum of the area and side is $\ell^2 + \ell$, and the goal of the problem is to solve for the side ℓ. On the other hand, sides and squares are geometrical objects, and if treated geometrically, then addition of lengths and areas makes no sense: the equations are not homogeneous. If Mesopotamian scribes of the second millennium are not to be considered to have had a concept of abstract algebra, a conceptual development that lay far in their future, then the geometrical viewpoint appears to present an insurmountable hurdle.

The resolution of this apparent dilemma has been developed by Høyrup over the last twenty years or so, culminating in the exposition in [Høyrup 2002]. By a close reading of the specific mathematical terminology involved, Høyrup showed that the stumbling block for a modern understanding of ancient geometry lay in our inheritance of the Greek categories of inhomogeneous lines and areas, and specifically in the Euclidean notion that a line has no breadth. Høyrup has convincingly demonstrated that in the pre-Euclidean world of the Old Babylonian scribes, lines are best understood as having unit breadth. That is, in algebraic contexts, we should view the side ℓ as in fact an area $1 \cdot \ell$, where

[1] The phrase itself occurs in the first problem of BM 13910 (discussed below) as 'a.šà^{lam} ù *mi-it-har-ti ak-mur-ma*', which Høyrup renders as "the surface and my confrontation I have accumulated" [Høyrup 2002, p. 50]. The rest of the problems on BM 13901 are related – there are assorted additions, subtractions and combinations. Similar problems are to be found in AO 6770, AO 8862, BM 85200 + VAT 6599, TMS 8, TMS 9, TMS 16, and YBC 6504. Additionally, BM 80209 contains statements of similar problems involving circles, but with no procedure given, and TMS 20, the main text discussed here, contains both statements and procedures for solving the problem in the case of the *apsamikkum* (see below).

the factor 1 is usually hidden from view. In geometric terms, the length added to the area becomes a rectangle of length 1 adjoined to a square.

More than the statement of the problem, it is a detailed analysis of the individual steps of the solution procedure that provides the most forceful evidence of the persuasiveness of Høyrup's case. His approach is based upon a very deep understanding of the precise usage of the technical vocabulary of Old Babylonian mathematics and a fine judgment for the careful way it is employed. It is clear from his work that Old Babylonian categories of thought relating to mathematical operations were not the same as ours and that in many cases, they made finer distinctions than our more abstract approach allows.[2] Høyrup has developed an English vocabulary to reflect these fine distinctions, but we will simplify some of the terminology here when the shades of meaning do not affect our argument. We also stress that Høyrup's analysis goes far beyond the problems considered here.

THE SIDE AND THE SQUARE

The classic example of this approach, explained many times by Høyrup himself and numerous other commentators is the first problem on the Old Babylonian tablet BM 13901.[3] The problem reads as follows:

The area and side of my square I added: 0 ; 45.[4]

[2] For example, Høyrup distinguishes four groups of terms for what we consider multiplication. Briefly, these may be described as: the a.rá or 'steps of' from the multiplication tables; terms derived from *našûm*, 'to raise or lift' for multiplication by constants, etc.; *espum*, a doubling or more general repetition, and terms based on *šutakūlum*, which generate physical areas from bounding lengths and widths. For more details see [Høyrup 2002, pp. 18–40] in general and [Muroi 2002] on terms for multiplication.

[3] The text was first published by Thureau-Dangin [1936] and re-edited by Neugebauer [1935/37]. It is of unknown provenance and currently housed in the British Museum. Høyrup includes in [Høyrup 2002] many of the problems (Problems 1, 2, 3, 8, 9, 10, 12, 14, 18, 23, 24) and has published a complete edition in [Høyrup 2001]; his transliteration mostly follows that of Neugebauer, with a few minor differences in restoration of broken passages. I have followed Høyrup's transliteration, but the translation below is mine.

[4] Abstract numbers in Old Babylonian mathematics were written in a sexagesimal, or base 60, place-value system. The value of a sign depended on the sexagesimal 'column' in which it occurred. However, these numbers contained no explicit reference to absolute size in terms

You, put down 1, the projection.
Break 1 in half.
Multiply $0;30$ and $0;30$.
Join $0;15$ to $0;45$: 1.
1 is the square root (of 1).
Subtract the $0;30$ which you multiplied in the 1: $0;30$.
The side (is) $0;30$.

Before illustrating Høyrup's geometric interpretation of this problem, it is worth giving a formal analysis of the steps of the algorithm for comparison with later problems. This approach is adapted from Ritter, who introduced a similar technique for comparing medical, divinatory and jurisprudential texts with mathematical ones in Egypt and Mesopotamia in [Ritter 1995a;b; 1998]. Ritter's main concern was to show that divination, medicine and mathematics formed a common intellectual domain for Old Babylonian scribes. Of course, the technical vocabulary of these three areas is quite discipline-specific, so that whereas Høyrup has focused on the terminology of mathematics, Ritter was drawn to analyzing the underlying grammatical and organizational structure of the texts. In particular, he showed the consistent ways in which grammatical signifiers marked off sections of the texts. As with Høyrup, Ritter's analysis is much wider and deeper than the specific cases considered here.

In order to analyze the underlying structures of Old Babylonian mathematical procedures, Ritter introduced a 'schematic form' of representation to show the linkages between individual arithmetic steps. The example he chose in [Ritter 1995a] to illustrate this technique was BM 13901, giving a full analysis of the first three problems and an abbreviated description of the remainder. As part of his analysis, Ritter showed that there are some core, basic algorithms in Mesopotamian (and Egyptian) mathematics, as well as many variations on that core. Ritter [1998] returned

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of the base unit. In order to align columns correctly for addition, size had to be inferred from context. There are several conventions for transliterating cuneiform numbers. Here, we follow the widely-used Neugebauer convention where each sexagesimal place is separated by a comma, and the 'sexagesimal point' separating multiples from fractions is denoted by a semi-colon. That is, 1,30 represents $1\times 60+30=90$, but $1;30$ represents $1+\frac{30}{60}=1\frac{1}{2}$. For convenience of the reader, we have silently introduced an absolute size for the problems we discuss, but we stress that this notation is not present in the cuneiform original.

	Computation	*Symbolic Instruction*
Step 0	Put down 1	w
Step 1	Break 1 in half: 0 30	$R_1 := \frac{1}{2}w$
Step 2	Multiply 0 30 and 0 30: 0 15	$R_2 := R_1^2$
Step 3	Join 0 15 to 0 45: 1	$R_3 := R_2 + D$
Step 4	Square root of 1: 1	$R_4 := \sqrt{R_3}$
Step 5	Subtract 0 30 from 1: 0 30	$R_5 := R_4 - R_1$

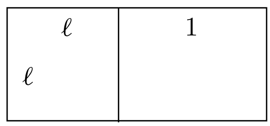
Table 1. BM 13901, Problem 1

to the same example when comparing the structure of the algorithm to that of the problem in Str. 368. The notation he used in the latter paper was somewhat different to the earlier version; it is this later technique we have adapted here. As some of the points we wish to emphasize are a bit different to Ritter's focus, we have changed the notation somewhat. Ritter's approach has also been extensively used by Imhausen [2002; 2003] in studying Egyptian mathematics.

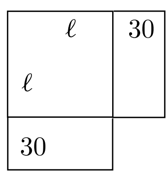
In this formalism, three types of information are differentiated. First, there are the data that are explicitly given in the statement of the problem; these data are denoted $D_1, \ldots, D_m$. In the case of BM 13901, Problem 1, the only datum given is the total area, which we denote by $D = 0;45$. Secondly, there is the implicit data, in this case the projection, which always has unit length, this we denote by $w = 1$. Finally, each arithmetical step of the algorithm produces a result, and we denote the result of step n by R_n. Using this approach, we may present the problem above in the form of Table 1.

One advantage of this particular approach to Old Babylonian mathematics is that it helps to foreground the fact that, as far as possible, each step of the algorithm uses the result of the immediately preceding step as one of its inputs, so that Step 4 uses R_3 (the result of Step 3), Step 3 uses the result of Step 2, and Step 2 uses the result of Step 1. The final Step 5 uses the result of the preceding step as well as the result of an earlier step, here Step 1. This important characteristic is lost in an algebraic description of the problem as an equation such as $x = \sqrt{(\frac{1}{2}w)^2 - \frac{1}{2}w}$.

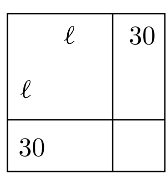
In contrast to this formal symbolic analysis of the steps of the algorithm, Høyrup offers a very concrete, physical interpretation of the procedure in terms of cut-and-paste, or 'naïve' geometry. In this view, the area (a.šà) is a physical square, and the side is a physical rectangle projecting a distance 1 from the square.



The instructions in the problem call for the projection to be broken in half. This may be viewed as a literal tearing in half of the unit rectangle, with the torn off piece being moved to construct a gnomon.



Since the original projection was torn in half and one half moved, the projecting halves allow for a literal completion of the square, where the computation determines the area of this square as $0;30$ times $0;30 : 0;15$.



The area of the new square is calculated (as $0;45 + 0;15 = 1$), and the length of the side of a square of area 1 is determined (a physical interpretation of the computation of a square root). Now the width of the adjoined, torn off rectangle is subtracted from the length of the side of the newly-formed square to find the length of the side of the original square, and thus the original unknown. Høyrup's interpretation thus gives a physical meaning to each algorithmic step.[5]

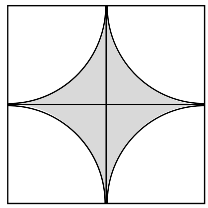
Although Høyrup has shown that there are a large number of problems involving squares and rectangles that are solved using his approach, it has not been clear that Old Babylonian scribes had generalized the procedure to apply to other geometric shapes. After all, a basic requirement of the procedure is that it begins with a square. However, Høyrup has shown that a number of 'non-normalized' problems, including several from BM 13901 itself, were solved by essentially the same procedure. Additionally, Høyrup has commented on the presence of similar problems involving circles on the tablet BM 80209, studied by [Friberg 1981]. This text presents a catalog of problems, mostly involving circles and including some problems of the variety, "to the area I added y times the circumference". However, the tablet gives no hint of the solution procedure. Fortunately, one other text, TMS 20, does show how this problem was handled in the case of the geometric figure called an *apsamikkum*, and the approach shows an elegant use of geometrical coefficients, a central tool in Old Babylonian mathematics.

THE *APSAMIKKUM*

There has been some debate about both the etymology and the meaning of the term *apsamikkum*. Here, we do not need to discuss the etymology, and Robson [1999] has conclusively demonstrated the primary meaning as the central, shaded, figure in the diagram below.

The *apsamikkum* arises naturally in a geometry that widely uses inscribed and circumscribed figures. This is not a central part of modern geometry and there is no good corresponding term for the figure in English. Robson uses 'concave square', while Muroi [2000] follows

[5] Together, Høyrup's and Ritter's approaches provide a powerful methodology for analyzing Old Babylonian mathematics. Although each approach has been well-developed separately, the two do not appear to have been combined before, perhaps because Høyrup's approach concentrates on vocabulary and geometry while Ritter's emphasizes grammar and algorithms. For other approaches to algorithmic aspects of Old Babylonian mathematics, see especially [Knuth 1972] and [Robson 1997].



the *Chicago Assyrian Dictionary* and translates the term as 'regular concave-sided tetragon'. For convenience, we shall use *apsamikkum* as a loan-word.

In Old Babylonian mathematics, a crucial role is played by the so-called 'coefficients'. As Mesopotamian geometry is profoundly computational, so there is a need for constants or coefficients linking the various parameters of a geometrical object. These coefficients have been studied in depth by Robson [1999]. Each geometrical object has a defining parameter from which the other parameters are derived. In the case of the *apsamikkum*, the defining parameter is one quarter-arc, which is thus taken to have length 1. The quarter-arc of the *apsamikkum* is also a quarter-arc of a circle, and *apsamikkum* coefficients are related to those for the circle. The defining parameter for the circle was the circumference and the crucial relations for our purposes are that the diameter of a circle was taken to be $\frac{1}{3} = 0;20$ of the circumference and the area to be $\frac{1}{12} = 0;5$ times the square of the circumference.[6] Thus a circle with a circumference of 4 would be taken to have a diameter of $\frac{1}{3} \times 4 = 0;20 \times 4 = 1;20$ and an area of $\frac{1}{12} \times 4^2 = 0;5 \times 16 = 1;20$. Since the *apsamikkum* with a quarter-arc of length 1 would have a perimeter of 4, it can be considered as inscribed in a square of side $1;20$, this is the coefficient of the 'diagonal' of the figure. The area of the *apsamikkum* may be easily obtained as the difference between the area of the square of side $1;20$ and an inscribed circle of diameter $1;20$; it is $0;26,40$. One other *apsamikkum* coefficient is the transversal, the difference between the diagonal of the square and diameter of the inscribed circle; it is $0;33,20$. More complete descriptions of the derivations of these coefficients can be found in [Muroi 2000] and [Robson 1999].

[6] These formulas are equivalent to setting $\pi = 3$.

There are only a few surviving texts with *apsamikkum* problems; one of these is TMS 20.[7] This text was originally published by Bruins & Rutten [1961] as text number 20 in *Textes mathématiques de Suse*. The tablet is badly broken and only part of it survives. However, the extant part is sufficient to show that originally the tablet had one problem written on each side. The problem on the obverse can be restored as beginning, "The area, the length, and the diagonal I added: 1;16,40". The procedure to be followed is given and the solution is determined: the length of the side was 0;30. The problem on the reverse can be restored as beginning, "The area and the side of the *apsamikkum* I added: 0;36,40". Again, the procedure is given, and the side is found to be 0;30. As the second problem is simpler we consider it first. Although the tablet is quite broken, the mathematical content can be restored with confidence from the remaining numbers and terminology and parallels between the two problems, even if there is room to doubt precise phrasing of certain terms. The translation below is adapted from Bruins and Rutten and no indication is given here of breaks and restoration from the original.

The area and the side of the *apsamikkum* I added: 0 ; 36,40.
You, put down 0;36,40.
Multiply 0;26,40, the coefficient, by 0;36,40: 0;16,17,46,40 you see.
Turn back. 1, for the length, put down.
Break 1 in half: 0;30 you see.
Square 0;30: 0;15 you see.
Add 0;15 to 0;16,17,46,40: 0;31,17,46,40 you see.
What is the square root? 0;43,20 is the square root.
Subtract 0;30 from 0;43,20: 0;13,20 you see.
Find the reciprocal of 0;26,40, the coefficient: 2;15 you see.
Multiply 2;15 by 0;13,20: 0;30, you see.
0;30 is the length.

As with the problem from BM 13901, we first present the algorithm used in tabular form. In this case, the given or known data at the start of the problem are the total area, $D = 0;36,40$, the coefficient for the

[7] The others are BM 15285 and TMS 21. See [Robson 1999] and [Muroi 2000] for descriptions of the problems in these texts.

	Computation	*Symbolic Instruction*
Step 0	Put down 0 ; 36,40	D
Step 1	Multiply the coefficient 0 ; 26,40 by 0 ; 36,40: 0 ; 16,17,46,40	$R_1 = cD$
Step 2	Put down 1	ℓ
Step 3	Break 1 in half: 0 ; 30	$R_3 = \frac{1}{2}\ell$
Step 4	Square 0 ; 30: 0 ; 15	$R_4 = R_3^2$
Step 5	Add 0 ; 15 and 0 ; 16,17,46,40: 0 ; 31,17,46,40	$R_5 = R_4 + R_1$
Step 6	Find the square root of 0 ; 31,17,46,40: 0 ; 43,20	$R_6 = \sqrt{R_5}$
Step 7	Subtract 0 ; 30 from 0 ; 43,20: 0 ; 13,20	$R_7 = R_6 - R_3$
Step 8	Find the reciprocal of 0 ; 26,40: 2 ; 15	$R_8 = \bar{c}$
Step 9	Multiply 2 ; 15 by 0 ; 13,20: 0 ; 30	$R_9 = R_8 R_7$

TABLE 2. TMS 20, Reverse

length, $\ell = 1$, and, the area coefficient of the *apsamikkum*, $c = 0;26{,}40$. In Step 8 of Table 2, $\bar{c}$ denotes the reciprocal of c.

A comparison of Tables 1 and 2 reveals the similarities between the approaches to the two problems and also shows how the Mesopotamian scribes applied the technique to geometrical figures that were not squares, by following essentially the approach developed for non-normalized square problems (see [Høyrup 2002] on BM 13901, Problems 3, 7, 14, and 24, for instance). Note that Steps 3–7 of Table 2 are identical procedures to those of Steps 1–5 of Table 1, with the single crucial exception that in Step 3 of Table 1, the area of the figure is given as initial data and based upon a square, whereas the equivalent Step 4 in Table 2, uses a value that needed to be calculated earlier in Step 1. This shows that the result of Step 1 of the second procedure is conceived of as giving the area of a figure based on a square. But Step 1 multiplies the given area of the figure by its coefficient. Hence, the scribe is using the area coefficient in two different ways.

Recall that the *apsamikkum*, in common with other geometrical figures in Old Babylonian mathematics has a defining component, in this case one arc, and a number of associated constants or coefficients, of which the primary one is that determining the area (see [Robson 1999, p. 56]). That is, in algebraic terms, whereas the area, A, of a square of side ℓ is obtained as $A = \ell^2$, the area of any other geometric figure is obtained as $A = \mathit{coefficient} \cdot \ell^2$. Let c be an abbreviation for *coefficient.* Then $A = c\ell^2$ is precisely the area of a rectangle with sides ℓ and $c\ell$. Geometrically, such a rectangle may be obtained from the original length ℓ by first scaling the length to produce $c\ell$ and then constructing the rectangle from the two lengths, or by forming a square of side ℓ and then scaling it. Algebraically, the question is whether $c\ell^2$ should be viewed as $c\ell \cdot \ell$, or as $c \cdot \ell^2$. For the *apsamikkum*, there is no direct evidence, but for other figures, such as the circle, Høyrup's delimiting of the different uses of types of 'multiplication' shows that the area is viewed as a scaled square.[8] Also, from the algorithmic perspective, forming a square and then scaling it by the coefficient is a cleaner and simpler process than first multiplying the given length by the coefficient and then retaining the original length for the second multiplication.

Step 1 of the algorithm above takes the given data, in this case the total area, and multiplies it by the coefficient. As is easily seen in the algebraic view, what is obtained is

$$\mathit{coefficient} \cdot A = (\mathit{coefficient})^2 \ell^2 = (\mathit{coefficient} \cdot \ell)^2.$$

Thus, the result of multiplying the area of a geometric shape by the coefficient is a square of scaled length. Bruins and Rutten, in the original publication of the text [Bruins & Rutten 1961], commented that the problem is solved by means of a "*surface 'fausse'*" but without emphasizing the underlying geometry. The area coefficient can hence be used to 'square' any figure. Once a square has been obtained, the procedure followed is the standard one for problems based on a square, and the scaled length is derived. Recall that in the canonical problem, the given area is that of the square adjoined by a unit rectangle. In this problem, the given area is that of the *apsamikkum* plus a unit rectangle. When this total is scaled by

[8] This fact was pointed out to me by a referee of an earlier version of the paper.

the area coefficient, the result is both a scaled square from the *apsamikkum* and a scaled unit rectangle, which can then be naturally adjoined.

At the end of the problem, the correct length is recovered from the scaled length by multiplying the latter by the reciprocal of the coefficient, as is done in Step 9 above.

For the *apsamikkum*, we have that the area is equal to that of a rectangle of sides ℓ and $0;26{,}40\,\ell$. Multiplying the other side of the rectangle by the same coefficient gives a square of sides $0;26{,}40\,\ell$. From this point, the problem can proceed exactly as in the case of a square, allowing only for a rescaling at the end of the problem. Thus the general problem of working with arbitrary geometrical figures for which an area coefficient is known has been reduced to the standard case, and the algorithm needed is merely a variant of the standard 'core' algorithm.

The problem on the obverse of the tablet is similar, with the exception that the diagonal is also included in the initial total.

The area, the side, and the diagonal I added: 1 ; 16,40.

You, 0 ; 26,40, the coefficient of the *apsamikkum*, by 1 ; 16,40 multiply: 0 ; 34,4,26,40 you see.

Turn back. 1, for the length, and 1 ; 20 for the diagonal which you do not know add: 2 ; 20 you see.

Break 2 ; 20 in half: 1 ; 10 you see. Square 1 ; 10: 1 ; 21,40 you see.

1 ; 21 ; 40 to 0 ; 34,4,26,40 add: 1 ; 55,44,26,40 you see.

What is the square root? 1 ; 23,20 is the square root.[9]

1 ; 10 from 1 ; 23,20 subtract: 0 ; 13,20 you see.

The reciprocal of 0 ; 26,40 the coefficient, detach: 2 ; 15 you see.

2 ; 15 to 0 ; 13,20 multiply: 0 ; 30 you see.

0 ; 30 is the length.

Here, the given or known data are the total area $D = 1;16{,}40$, and the coefficients for the length, ℓ, the diagonal, $d = 1;20$, and the area of the *apsamikkum*, $c = 0;26{,}40$. Table 3 shows the procedure.

[9] The text is silent on how the square root of 1 ; 55,44,26,40 is determined, as also for the equivalent problem on the reverse. Frustratingly little is known about Old Babylonian square root algorithms. Some recent attempts to unravel the situation, responding to the explanation in [Neugebauer & Sachs 1945, p. 43], have been made by Friberg [1997, p. 318] discussing TMS 20 in the context of other texts, [Fowler & Robson 1998] and [Muroi 1999].

	Computation	*Symbolic Instruction*
Step 1	Multiply the coefficient 0;26,40 by 1;16,40: 0;34,4,26,40	$R_1 = cD$
Step 2	Add 1 and 1;20: 2;20	$R_2 = \ell + d$
Step 3	Break 2;20 in half: 1;10	$R_3 = \frac{1}{2}R_2$
Step 4	Square 1;10: 1;21,40	$R_4 = R_3^2$
Step 5	Add 1;21,40 and 0;34,4,26,40: 1;55,44,26,40	$R_5 = R_4 + R_1$
Step 6	Find the square root of 1;55,44,26,40: 1;23,20	$R_6 = \sqrt{R_5}$
Step 7	Subtract 1 ; 10 from 1;23,20: 0;13,20	$R_7 = R_6 - R_3$
Step 8	Find the reciprocal of 0;26,40: 2;15	$R_8 = \bar{c}$
Step 9	Multiply 2;15 by 0;13,20: 0;30	$R_9 = R_8 R_7$

TABLE 3. TMS 20, Obverse

A comparison of Tables 2 and 3 shows that the procedure is essentially identical, with the exception of the addition needed for the sum of the length and diagonal coefficients.

Although the examples given here are for the *apsamikkum*, it is clear that the procedure would allow a scribe to solve similar problems involving any geometrical figure for which the coefficients were known. In particular, the circle problems from BM 80209 mentioned above would be amenable to an identical treatment. The standard Old Babylonian techniques not only could be applied to a wider set of problems than has previously been noted, they were so applied.

Høyrup has observed the use of scaling for non-normalized problems such as BM 13901, Problem 3 [Høyrup 2002, p. 55]; here we see the role that the basic geometrical coefficients played in reducing problems involving non-square figures to the standard format, and that this technique involved a very conscious 'squaring' of the original figure.

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