Nonstandard Semantics for Modal Logic and the Concept of a Logically Possible World

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1 Logically Possible Worlds

The idea that a logically possible world is identical with or can be described as a maximally consistent proposition set is a fundamental assumption of the conventional model set theoretical semantics for modal logic. Although the concept is formally unproblematic, philosophically there are serious difficulties in the standard definition of a logically possible world. I want to raise philosophical objections to the model set theoretical concept of a logically possible world, and then sketch a proposal for modal semantics that strikes at the root of the problem in order to avoid these limitations. The conflict to which I call attention has recently been discussed as a dispute between *modal realism* versus *modal actualism*. It will quickly emerge that I am a modal actualist and that I oppose modal realism.

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2 What's Wrong With Standard Modal Semantics

The definition of a logically possible world as a maximally consistent proposition set is the heart and soul of conventional model set theoretical semantics for modal logic. It is easy in retrospect to understand why. The first modal syntax and axiom systems developed by C.I. Lewis in 1918 were formally uninterpreted until Saul A. Kripke and Jaakko Hintikka independently worked out model set theoretical semantics for modal logics and quantified modal logics in the mid-1960s. [1] Building on the firmly established mathematical foundation of Zermelo-Fraenkel set theory, Kripke and Hintikka provide an exact formal interpretation of these logical languages that have proven invaluable in the analysis and formal modeling of philosophical, scientific, and everyday discourse. [2]

The emergence of a powerful mathematical method for interpreting modal logic made a deep impression on the analytic philosophical imagination in the second half of the twentieth century. It quickly became a favorite tool for symbolizing many difficult logical concepts. The idea that a logically possible world is a maximally consistent proposition set has been so integral to standard model set theoretical semantics that it has been accepted as part of the same remarkable package, without much philosophical objection, and, indeed, without much philosophical question or scruple. The brilliance and usefulness of these semantic models and the unified interpretation of the variety of modal logics that they afford have made model set theoretical semantics a powerful paradigm of analytic philosophy, comparable in impact, and deservedly so, only to Russell's theory of definite descriptions.

The defects of standard model set theoretical semantics are less immediately appreciated; partly, no doubt, because proponents are thoroughly convinced of its usefulness and committed to its truth. The concept of a logically possible world as a maximally consistent proposition set can easily appear philosophically problematic, once we look beyond the pragmatics of formal analysis. There is no way to sugarcoat the fact that if sets exist, as mathematical realism classically implies,

1. C.I. Lewis, *A Survey of Symbolic Logic* (Berkeley and Los Angeles: University of California Press, 1918).

2. Saul A. Kripke, "Semantical Considerations on Modal Logics", *Acta Philosophica Fennica*, 16, 1963, pp. 83–94. Kripke, "Semantical Analysis of Modal Logic I: Normal Modal Propositional Calculus", *Zeitschrift für mathematische Logik und Grundlagen der Mathematik*, 9, 1963, pp. 67–96. Jaakko Hintikka, "The Modes of Modality", *Acta Philosophical Fennica*, 16, 1963, pp. 65–81.

then any logically possible world as a maximally consistent proposition set equally exists, even in the case of a nonactual world consisting entirely of nonexistent objects and nonexistent states of affairs. Moreover, in the prevailing climate of extensionalism in philosophical semantics, it is unavoidable to make logically possible worlds into something existent even when they are nonactual. How can we refer to and truly predicate properties of nonactual merely logically possible worlds, how can we say anything about them and how can they stand as true predication subjects, if they do not exist? The idea that nonactual merely logically possible worlds are maximally consistent proposition sets combined with a default platonistic or realist ontology of mathematical entities according to which proposition sets exist is nevertheless profoundly confused.

Among the indefinitely many logically possible worlds there is intuitively only one that actually exists, the one and only actual world in which we actually existent entities happen to reside. The actual world alone exists, and it includes all and only the genuine entities, all existent individuals, states of affairs, and whatever else actually exists as constituting the actual world, while merely logically possible worlds do not exist. If to insist that the actual world is uniquely existent is unobjectionable, then it is impossible to see how we can tolerate the definition of a logically possible world according to which all logically possible worlds exist, including all nonactual merely logically possible worlds. This, regrettably, is the inescapable conclusion if logically possible worlds are defined as maximally consistent proposition sets, and if sets as abstract mathematical entities exist. If sets exist, then the conventional concept of a logically possible world as a proposition set requires a counterintuitive distinction between existence and actuality, between what exists and what is actual, or perhaps between what exists and what actually exists.

Nor does it help to retreat to a redefinition whereby logically possible worlds are only described or represented rather than constituted by maximally consistent proposition sets. The problem in that case is just the opposite of the one that plagues the interpretation of logically possible worlds as maximally consistent proposition sets. If all logically possible worlds are described alike by maximally consistent proposition sets, then we face the equally difficult question of what it is, as far as the conventional semantics of modal logic is concerned, that is supposed to single out the actual world as having special ontic status from all other nonactual merely logically possible worlds.

To this pertinent ontological query, there is no satisfactory answer in a conventional modal model set theoretical framework. While modal

logicians have formalized appropriate model set theoretical relations among logically possible worlds, they have not looked into or tried in any meaningful way to characterize the metaphysics of being, or to establish the principles of pure philosophical ontology for actually existent entities. It is clear, in reading any standard work of formal modal semantics, that the definition of a logically possible world in set theoretical terms offers no answer at all to the question of what it means for something to exist, or in particular of what it means for a logically possible world to be correctly identified as the uniquely existent actual world. The usual practice is for a model set theoretical semantics to define an enormous number of combinatorially generated logically possible worlds, typically by the equivalent of a Lindenbaum maximal consistency recursion, each as a distinct maximally consistent proposition set, and then simply to declare that one of these sets is to be 'distinguished' as the actual world. Notations differ, but it is common practice to adopt the mnemonic symbol *alpha*, 'α', or the *at*-sign, '@', to designate the actual world in modal semantics as '$w_@$'.[3] The philosophical question that urgently remains to be pressed in light of these devices is by virtue of what features a logically possible world, if universally defined as a maximally consistent proposition set, is correctly identified as the actual world?

It is possible for many things in the actual world to be other than they are. Life need not have evolved, the Third Reich might have won World War II, or I might have gone to Mexico to snorkel instead of Austria to ski. The actual world is only one possibility among limitlessly many others. Of all the logically possible worlds, only one is actual. We might not yet know exactly what this means, or why it should be so, if we have not yet grasped what makes the actual world actual, and what distinguishes it from all nonactual merely logically possible worlds. To that extent, also, we do not yet fully and accurately understand what is

3. Among other sources, see David K. Lewis, *Counterfactuals* (Oxford: Blackwell Publishing, 1973). Some developments of model set theoretical semantics for modal logic do not refer to the actual world as such, but make only passing reference to the fact that the actual world is to be included as one among all logically possible worlds, and considering only generalized accessibility relations relative to any arbitrary world α. A good example of this approach to set theoretical modal semantics is Brian F. Chellas, in his *Modal Logic: An Introduction* (Cambridge: Cambridge University Press, 1980), who does not define the concept of a logically possible world but treats it as primitive or intuitive. A generalized semantics of the sort Chellas considers must be supplemented with a method of designating the actual world for applications such as the interpretation of counterfactual propositions, in which the facts of the actual world are contrasted with propositions that are supposed to be true in alternative logically possible worlds via some specification of world-to-world modal accessibility relations.

meant by the concept of a logically possible world. This is precisely what is missing from the definition of a logically possible world in conventional model set theoretical modal semantics. We transgress the limits of pure logic if we try to say that the actual world is the one that we logicians psychologically experience, or that has the objects or states of affairs that we encounter in sensation. It nevertheless appears that efforts to single out the actual world in conventional modal semantics never amounts to anything more than this kind of implicit appeal to extralogical empirical facts.

3 General Existence Conditions for Entities

Where, then, can we start, if in the semantics for modal logic we must start again? We can say that the actual world is the world consisting of all and only existent states of affairs involving all and only existent objects, and conversely that an existent object or state of affairs is an object or state of affairs that belongs to the actual world, as opposed to a merely logically possible world.

We do not necessarily know from this alone what either of these statements is supposed to mean in the absence of a unified account of the existence of entities, including actually existent logically possible objects, states of affairs, and the actual world. It is only if we come into possession of a correct metaphysics of being that we can hope to explain what it means for a logically possible world to be actual, and why there exists exactly one actual world. We need, in other words, to answer satisfactorily the question why or with what justification a particular logically possible world is rightly designated as the actual world, and distinguished thereby from all nonactual merely logically possible worlds.

The next important step in analyzing the concept of being required by a philosophically astute modal semantics is to clarify the concept of an actual world. Kripke, in his lectures on *Naming and Necessity*, reminds us that logically possible worlds are not viewed through high-powered telescopes to discover what objects they contain.[4] We similarly need to

4. "A possible world isn't a distant country that we are coming across, or viewing through a telescope. Generally speaking, another possible world is too far away. Even if we travel faster than light, we won't get to it. A possible world is *given by the descriptive conditions we associate with it...* 'Possible worlds' are *stipulated*, not *discovered* by powerful telescopes. There is no reason why we cannot *stipulate* that, in talking about what would have happened to Nixon in a certain counterfactual situation, we are talking about what would have happened to *him*." [Kripke 1980, 44]

understand that the actual world is not selected from a beauty pageant lineup of all logically possible world hopefuls, only one of which is to be 'designated' as actual. We need to disabuse ourselves of misleading ways of imagining the semantics of modal logic, and the existence requirements for logically possible objects, states of affairs, and the actual world. The actual world does nothing to deserve its actuality in competition with nonactual merely logically possible worlds. It is the actual world as a matter of fact because it satisfies the requirements of being, whatever these turn out to be.

Without further ado, I now want to propose what I consider to be the correct analysis of the concept of being. The definition is *intensional*, involving an object's properties, in what I shall refer to as a *property combination*. A property combination is the set of properties nominalistically *associated* with an object, corresponding to the Fregean *sense* of an object's logically proper name or definite description. Accordingly, I shall say that to *be* is to have a *maximally consistent property combination*. An *entity* is an existent object, state of affairs, or the actual world as a whole, which by the present proposal is one that has a maximally consistent property combination. A *nonexistent* intended object, including fictional objects that belong to the semantic domain of a logic of fiction, is an object whose property combination is either inconsistent, containing both a property and its complement, or incomplete, failing to contain either a property or its complement. A *logically possible world* is a logically consistent states of affairs combination; a *nonfactual, merely logically possible world* is a submaximally possible states of affairs combination, and *the actual world* is a maximally consistent possible states of affairs combination. I shall refer to a theory of this kind as a *combinatorial analysis of the concept of being*.

If an object has both or is lacking both a property and property-complement pair from its property combination, then the object fails to exist. An entity cannot be both red and non-red or fail to be either red or non-red, round and/or non-round, and so on. In the first case, its property combination would be inconsistent, and in the second case incomplete or submaximal. Maximal consistency is evidently necessary for existence; whether it is also sufficient is a question that forces us to reconsider the definition of a logically possible world as a maximally consistent proposition set.

A defender of conventional modal semantics is committed to denying that maximal consistency is sufficient for an object to be actual or actually existent. Modal logicians, recognizing the difficulty, and desperate to find a way to resolve conflicting intuitions, typically relativize

existence to particular logically possible worlds. They say, for example that the Statue of Liberty exists in the actual world, but that in other logically possible worlds there exist objects like the Fountain of Youth or city of Eldorado that do not exist in 'our' world. [5] Conventional modal semantics thus entails that there are objects and states of affairs that truly exist in worlds that truly do not exist. Such a position appears logically incoherent, in opposition to the pre-analytic intuition that all and only the entities and states of affairs belonging to the actual world exist.

Why, then, does conventional modal semantics insist that all logically possible worlds are maximally consistent proposition sets? Perhaps it is because these logicians perceive that the actual world is maximally consistent, and that it is, after all, a logically possible world. The more important question that is not usually asked is rather, what if it is maximal consistency that makes the actual world actual, a property shared by no merely logically possible nonactual world? It may be less objectionable in trying to work out the formal semantic structures for interpreting the propositions of modal logic not to become embroiled in the metaphysics of being for the actual world or the states of affairs by which it is constituted or the entities it contains. In metaphysics, on the other hand, in ontology's inquiry into the concept of being, we cannot afford to ignore such problems. It appears that conventional model set theoretical modal semantics, neglecting the difficulty of what it means for the actual world to be so designated and distinguished from among all merely logically possible worlds, fails to recognize drastically counterintuitive results even within its limited project of formalizing modal semantics. Conventional modal logic, with a mathematician's typical insouciance about the philosophical problems of ontology, does not seem to care at all about the question of being — but it should.

By defining existence as the maximal consistency of an object's property combination, we explain what it means for the actual world to exist, and we recover consistency in maintaining that only objects and states of affairs in the actual world exist, and that only the actual world exists. We can then say that for the actual world to exist, to be designated or distinguished as actual from among all other logically possible worlds, means for it to be a maximally consistent states of affairs combination.

5. An example of this widespread practice is found in [Forbes 1985, 28]: "The discussion of the previous section should have imparted a general picture of what model theory for quantified S5 is going to look like. As in the sentential case, there will be a set of possible worlds, but in addition, each world will be assigned a set of objects, the things which exist at that world".

This in turn is equivalent to the actual world's consisting only of objects and states of affairs whose property combinations are maximally consistent, and to the states of affairs combination of the actual world in its entirety as an existent entity being maximally consistent. These are all purely logical concepts, by which the actual world is distinguished as a maximally consistent states of affairs combination to be represented linguistically as a uniquely maximally consistent proposition set, unlike, by the proposed definition of existence, all nonactual merely logically possible worlds. [6]

A metaphysics of being as maximal consistency for all entities, including objects, states of affairs, and the actual world, is unified, economical, and comprehensive. Is it also true? The sticking point, for most contemporary modal logicians, is undoubtedly the implication that nonactual merely logically possible worlds are submaximally consistent states of affairs combinations: possible, because they are at least internally consistent in their constitutive states of affairs by which properties are attributed to objects; submaximal, in that they are incomplete in the sense of containing property and property complement gaps, projected into their constitutive gappy incomplete states of affairs combinations. Standard modal semantics regards all logically possible worlds as descriptively maximally consistent proposition sets. [7]

6. I explore these topics in greater depth and historical context in Dale Jacquette, *Ontology* (Acumen Publishing, 2003). See especially Part I on Pure Philosophical Ontology, including Chapters 2–5, on 'Combinatorial Ontology', 'Why There is Something Rather Than Nothing', 'Why There is Only One Logically Contingent Actual World', and 'Concepts of Existence in Philosophical Logic and the Analysis of Being qua Being'. The key to understanding why there can be at most one maximally consistent states of affairs combination is that if we think of the actual world as maximally consistent, and imagine the occurrence of something else, a putatively distinct but equally actual world, even if it were to consist only of a single maximally consistent object or state of affairs, it would fall under the combinatorial analysis of the concept of an actual world and hence would necessarily *de facto* be only a distinct part of the one and only actual world or literal universe. The two hypothetical actualities would logically definitionally be absorbed into one; metaphorically speaking, like two drops of mercury touching.

7. The commitment to the idea of a logically possible world as maximal is clearly stated in Forbes, *The Metaphysics of Modality*, p. 8: "A possible world is a complete way things might have been — a total alternative history ... In terms of our model theory, the requirement that worlds be complete is reflected in the constraint that every sentence letter occurring in the argument in question be assigned one or other truth value at each world." This intuitive statement of the semantic concept of a logically possible world is equivalent to the conventional definition of a world as a maximally consistent proposition set. Forbes significantly adds, ibid.: "We shall see in §4 of this chapter that we can get by without this sort of completeness, but that we pay a price in terms of simplicity."

4 Maximal Consistency of the Actual World

We now set the stage for three arguments in support of the conclusion, contrary to conventional model set theoretical modal semantics, that nonactual merely logically possible worlds should not be interpreted semantically as maximally consistent, but as at most only submaximally consistent states of affairs combinations. Against the assumptions of conventional modal semantics, it seems correct on the contrary to hold that only the actual world is maximally consistent.

The upshot of a combinatorial analysis of the concept of being is that all nonactual merely logically possible worlds are submaximal. Furthermore, and most importantly, the combinatorial theory proposed here does not equate logically possible worlds with maximally or submaximally consistent proposition sets, but with maximally or submaximally consistent states of affairs combinations. The present proposal is thereby distinguished from efforts to relate logically possible worlds to propositional set theoretical quasi-filters. If we wish, in response to related semantic needs, as several writers have proposed, we can further speak of impossible worlds as involving inconsistent submaximal or even maximal states of affairs combinations. [8] The reward for defining existence as maximal consistency is that we thereby provide a clearcut logical foundation for distinguishing the actual world as uniquely existent, without making every logically possible world an existent proposition set.

The difficulty to be answered in defending the theory is that it seems we can also freely describe maximally consistent proposition sets in the same way that we can describe other mathematical entities. Nothing prevents a conventional modal logician from devising Lindenbaum-style recursions whereby all the distinct complete and consistent sets of propositions are projected. The method is to consider each proposition in turn and add it to a given set if and only if it is logically consistent with the propositions already collected in the set until there are none left, and otherwise adding its negation, following the process in the case of every logically distinct combination of propositions until every proposition or

8. Graham Priest, guest-edited issue of *Notre Dame Journal of Formal Logic*, 38, 1997, on 'Impossible Worlds', including contributions by Edwin D. Mares, "Who's Afraid of Impossible Worlds?", pp. 516–526; Daniel Nolan, "Impossible Worlds: A Modest Approach", pp. 535–572; and David A. Van der Laan, "The Ontology of Impossible Worlds", pp. 597–620. See Jaakko Hintikka, "Impossible Possible Worlds", *Journal of Philosophical Logic*, 4, 1975, pp. 475–484. Nicholas Rescher and Robert Brandom, *The Logic of Inconsistency: A Study in Non-Standard Possible-World Semantics and Ontology* (Totowa: Rowman and Littlefield, 1979), distinguish between 'inconsistent' and 'impossible' worlds.

its negation is incorporated. A similar recursion is followed in consistency and completeness proofs in standard logical metatheory, which no doubt played an important role historically, along with the default realist or platonistic ontology for mathematical entities, in the received concept of a logically possible world as a maximally consistent proposition set. If we are platonic realists in the applied scientific ontology of mathematics, then we may be strongly inclined if not irrevocably committed to regarding such sets as themselves existent mathematical entities to which we can appeal *ad libitum* in theory construction, especially in designing a model set theoretical semantics for modal logic.

The combinatorial analysis of the concept of being as maximal consistency does not cast doubt on the possibility of such a procedure. On the contrary, we assume that maximally consistent proposition sets, regardless of how they are classified in applied ontology, can be thought of as being compiled combinatorially in just this way. The issue is rather whether the resulting maximally consistent sets of propositions deserve to be called or considered as describing nonactual logically possible *worlds*. As indicated, the identification or description of a logically possible world as a maximally consistent proposition set, or logically derivative from a maximally consistent states of affairs combination, is warranted only in the unique case of the actual world. Nonactual merely logically possible worlds, by comparison, are fictional creatures of conventional modal semantics, and as such inherently predicationally incomplete, involving inherently incomplete states of affairs combinations.

The following arguments provide good reasons for adopting the unconventional concept of a logically possible world that has now been introduced in a reformed semantics of modal logic. To dramatize the limitations of conventional modal semantics, we argue that submaximal consistency affects even the proposition sets standardly associated with logically possible worlds, thus opening the door to logically possible worlds as maximally or submaximally consistent states of affairs combinations.

Objection 1: *Kripkean Transworld Identity Stipulations are Inherently Submaximal*

The first objection to considering maximally consistent proposition sets as nonactual merely logically possible worlds depends on Kripke's answer to the transworld identity problem. Objects and states of affairs in the actual world might have been so different than they actually are that it appears impossible even in theory and certainly in practice to identify the same objects from world to world by positive correspondence with their descriptions in any given world.

Kripke sidesteps the problem by arguing that transworld identity is not a matter of discovery, but of decision. We stipulate, in Kripke's terminology, that there is a nonactual logically possible world in which Richard Nixon's chromosomal endowment is so radically altered prior even to his development in the womb that at no time within that world is he recognizable as the Richard Nixon we know from experience of his appearance in the actual world, but exactly resembles Marily Monroe.[9] Kripke's response to transworld identity problems has gained wide acceptance among modal logicians. Taken literally, although Kripke does not acknowledge the consequence, Kripkean transworld identity stipulation implies a constitutional incompleteness in the proposition sets associated with any nonactual logically possible world, by which they can only be submaximal even if logically consistent. Stipulation involves real-time human decision-making that is incompatible with the possibility of including all the items in a consistent proposition set needed in order to qualify as maximally consistent. We, finite creatures that we are, can only submaximally stipulate so much in describing distinct logically possible worlds in the limited time we can devote to such theorizing, in which this or that is different from the actual world, and leave the rest unspecified.

Objection 2: *Submaximal Consistency is Adequate for the Modal Semantics of Nonactual Worlds*

Secondly, it is significant that submaximally consistent proposition sets are adequate for the formal semantics of modal logic in the case of all nonactual logically possible worlds. It is good enough for the purposes of formalizing a general semantics of modal logic to recognize maximal consistency only in the case of the actual world. There is nothing we can practically do with maximally consistent proposition sets in understanding the truth conditions for sentences in modal logic that we cannot do with submaximally consistent sets or submaximally consistent states of affairs combinations.

As a further theoretical advantage, submaximally consistent proposition sets do not incur the difficulties of conventional modal semantics. They encourage an answer to the question of being with respect to worlds, explaining the actual world as maximally consistent in its fully consistent complement of actually existent states of affairs as determined

9. See Hugues Leblanc, "On Dispensing With Things and Worlds", in *Logic and Ontology: Studies in Contemporary Philosophy*, edited by Milton K. Munitz (New York: New York University Press, 1973); reprinted in Leblanc, *Existence, Truth, and Provability* (Albany: State University of New York Press, 1982), pp. 103–119. Forbes, *The Metaphysics of Modality*, especially pp. 70–89.

by the actually instantiated properties of actually existent entities. They further avoid the need to relativize existence to specific logically possible worlds, dispensing with the confusing assertion that the Fountain of Youth exists *in* a particular world, whatever this is supposed to mean, when we assume on the contrary without qualification that the Fountain of Youth does not exist or does not actually exist. The very idea of 'existence *in* a (nonactual) world' ought to be avoided if at all possible, because it saddles modal logic with a problematic way of distinguishing the actual world from alternative logically possible worlds. The actual world in that case cannot be identified as the world containing all and only existent states of affairs involving all and only existent entities. All worlds, paradoxically, then, each have their own world-indexed existent entities. We are further obliged in that case to index actually existent entities to the predesignated actual world, whose facts and objects are existent-$w_{@}$, rather than existent$-w_1$, $-w_2$, $-w_3$, and so on, where $@ \neq 1, 2, 3, \ldots$

What, then, could possibly justify conventional modal logic's demand that nonactual merely logically possible worlds are maximally consistent rather than submaximally consistent proposition sets or states of affairs combinations? Why go maximal except in the unique case of the actual world? The only imaginable reason has to do with an intuition about the meaning of the word 'world', according to which submaximal consistency does not deserve to be called or associated with a *world*, or that a world in the true sense of the word, even if it is nonactual and merely logically possible, must be maximal. The reasoning, however, seems unfounded and there are overriding counterobjections in favor of the interpretation of nonactual merely logically possible worlds as submaximally consistent states of affairs combinations.

Objection 3: *Nonactual Worlds by Definition are Submaximally Consistent*

A third justification for distinguishing the actual world as the only maximally consistent proposition set or states of affairs combination, by contrast with the submaximal consistency of nonactual merely logically possible worlds, is based on a trilemma. The argument reveals a deeper reason why nonactual merely logically possible worlds are submaximally consistent. A maximally consistent set of propositions, even on the weakest accessibility relations between logically possible worlds in model set theoretical semantics, must include true or false propositions about the actual states of affairs obtaining in the actual world. Nonactual worlds must look over their shoulders at what is happening in the actual world, so to speak, and make some statement about the situation

there, in order to be maximal. Otherwise, there will be propositions that are entirely left out of their proposition sets, which by definition are thereby submaximal.

The proposition set of a nonactual merely logically possible world as a result has a conflicting set of responsibilities if it is to be maximally consistent. It must pretend, in certain cases, that the Fountain of Youth 'actually' exists, or exists in its associated world, and must accordingly include a proposition to this effect, while at the same time declaring that the Fountain of Youth does not exist, or does not actually exist or exist in the actual world. We have already seen that indexing the truth of propositions to particular logically possible worlds within proposition sets associated with worlds is a philosophically questionable practice in modal semantics. Now we are prepared to see worse problems arise whether or not world-indexing of propositions is tolerated. Consider a proposition set S for a nonactual merely logically possible world w_i, that is striving for maximal consistency in the spirit of conventional model set theoretical modal semantics. Set S either includes or does not include the proposition that the Fountain Youth does not exist, and either does so indexically by making reference to the truth of the proposition in the actual world, or nonindexically.

Thus, there are three possibilities:

(1) If S does not include any proposition expressing the fact that the Fountain of Youth does not exist (in actual world $w_@$), then S is submaximal, even if it is logically consistent.

(2) If S includes a proposition expressing the fact that the Fountain of Youth does not exist indexically by referring to the proposition's truth in or with reference to the actual world $w_@$, 'The Fountain of Youth does not exist (in $w_@$)', then it must also assert the existence of the Fountain of Youth in or with reference to w_i, 'The Fountain of Youth exists in w_i' ($i \neq @$). Then, problems of indexicality for an extensionalist model set theoretical logic aside (and they are considerable, including the danger of outright logical paradox), S contains propositions that acknowledge by their explicit indexing that the Fountain of Youth does not actually exist; in effect declaring its own falsehood, a false description of the world, thereby rendering S unfit as a description of w_i.

(3) If S includes a proposition nonindexically expressing the fact that the Fountain of Youth does not exist (in $w_@$) and nonindexically expressing the fact that the Fountain of Youth does exist (in w_i, $i \neq$

@, as before), then, without benefit of the indices indicated in parentheses, S is inconsistent, even if maximal.

We want to know what it means for the actual world to be distinguished as actual, by comparison with all nonactual merely logically possible worlds. We also want to be able to say that only the actual world exists, that all and only the objects and states of affairs in the actual world exist, that existence is not to be relativized to worlds, but that 'existence' means real existence or actuality. Thus, it seems we have no choice but to rethink the conventional wisdom of standard model set theoretical modal semantics. Set S is logically inconsistent if it recognizes the facts of the actual world but shuns indexicality, submaximal if it ignores the facts of the actual world, and inadequate as a description of w_i if it embraces indexicality in order consistently to include the facts of the actual world, such as the fact that the Fountain of Youth does not actually exist.

It may finally appear as somewhat of a relief to consider that, even if nonactual merely logically possible worlds are defined combinatorially rather than set theoretically, all the standard set theoretical machinery of conventional model set theoretical modal semantics on which we have come to rely can remain in place, leaving the formal semantics of modal logic untouched. We can preserve set theoretical relations among sets of worlds just as before, even if worlds are not themselves sets, and in particular even if they are not maximally consistent proposition sets, but maximally or submaximally consistent states of affairs combinations. A logically necessary proposition, if there are any, is still one that is true in every logically possible world, involving the stipulation-exempt properties of abstract entities exclusively; a logically possible proposition is still one that is true in at least one logically possible world; and we can still invoke differential accessibility relations between logically possible worlds to interpret iterated and especially quantified iterated alethic modalities.

5 Alethic Modality and the Logic of Fiction

We are now in a position to appreciate the significance of the logic of fiction in a combinatorial semantics of alethic modal logic. The conventional approach is to develop a model set theoretical semantics for modal logic and then to apply modal logic in trying to understand the logic of fiction. If the objections raised here are sound, then this otherwise reasonable strategy has things reversed. Logically possible worlds

other than the actual world in that case are mere semantic fictions, so that we stand in need first of an adequate logic of fiction in order to formalize an exact interpretation of alethic modality.

All nonactual merely logically possible worlds are not rightly identified with sets, let alone maximally consistent proposition sets, but with appropriately submaximally consistent states of affairs combinations that are not associated with any existent entity. The worlds to which they correspond are fictional, not only in the loose sense of characterizing nonexistent incomplete objects, but in the more accurately delimited sense of being described in a work of discourse. The fictions in which nonactual merely logically possible worlds are presented might be stories, novels, poetry, and other forms of entertainment literature, in scientific writings, including theories of natural phenomena that happen to be false, and in Kripke-style stipulative but intensionally combinatorially interpreted modal semantic constructions. This is as it should be if we assume that the actual world is uniquely existent and that nonactual merely logical possible worlds do not exist even and especially as abstract mathematical or propositional structures. When we produce a formal semantics for alethic modal logic, on the present account, we refer to the uniquely existent maximally consistent actual world, and we also engage in fiction, creating an imaginary order of nonactual merely logically possible submaximally consistent worlds that are different in their nonexistent constituent facts and usually also in their nonexistent constituent objects from the actual world. [10]

10. Recent sources on the modal realism-actualism controversy include Charles Chihara, *The Worlds of Possibility: Modal Realism and the Semantics of Modal Logic* (Oxford: The Clarendon Press, 1998). Robert Stalnaker, 'On Considering a Possible World as Actual I', *Proceedings of the Aristotelian Society, Supplement*, 75, 2001. Merrie Bergmann, 'A New Argument from Actualism to Serious Actualism', *Noûs*, 30, 1996. Phillip Bricker, 'Isolation and Unification: The Realist Analysis of Possible Worlds, *Philosophical Studies*, 84, 1996. Selmer Bringsjord, 'Are There Set Theoretic Possible Worlds?', *Analysis*, 45, 1985. John Divers, 'Modal Fictionalism Cannot Deliver Possible Worlds Semantics', *Analysis*, 55, 1995. Patrick Grim, 'On Sets and Worlds: A Reply to Menzel', *Analysis*, 46, 1986. Bob Hale, 'Modal Fictionalism - A Simple Dilemma', *Analysis*, 55, 1995. Allen Hazen, 'Expressive Incompleteness in Modal Logic', *Journal of Philosophical Logic*, 5, 1976; 'a logically consistent states of affairs', *Philosophical Studies*, 84, 1996. T. Jager, 'An Actualist Semantics for Quantified Modal Logic', *Notre Dame Journal of Formal Logic*, 23, 1982. Michael Jubien, 'Actualism and Iterated Modalities', *Philosophical Studies*, 84, 1996. Jaegwon Kim, 'Possible Worlds and Armstrong's Combinatorialism', *Canadian Journal of Philosophy*, 16, 1986. Alan McMichael, 'A New Actualist Modal Semantics', *Journal of Philosophical Logic*, 12, 1983; 'A Problem for Actualism About Possible Worlds', *The Philosophical Review*, 92, 1983. Christopher Menzel, 'On Set Theoretic Possible Worlds', *Analysis*, 46, 1986; 'Actualism, Ontological Commitment, and Possible Worlds Semantics', *Synthese*, 85, 1990. Alvin Plantinga, 'Actualism and Possible

It might be wondered why we could not similarly dissociate maximally consistent proposition sets from logically possible worlds in conventional model set theoretical semantics. What final advantage is gained by invoking property or states of affairs combinations in place of sets in combinatorial modal semantics? The trouble is that if we try to separate worlds from maximally consistent proposition sets, then, conventionally, what is a logically possible world if it is not an existent set? If worlds are only remotely associated or correlated with proposition sets, then they cannot be referred to or have constitutive properties truly predicated of them in a conventional extensionalist semantic framework. When we hear a philosopher say, 'It is logically possible that pigs fly means that there is at least one logically possible world inhabited by airborne swine', we are to understand this as equivalent to a logician's tale, beginning with the preamble, 'Once upon a time . . . '

Nor is completeness a feature of logically possible worlds that we can freely stipulate. If completeness itself could be stipulated, if we could simply declare and it would then be true that there is a maximally consistent state of affairs combination in which pigs fly, then when A stipulates a complete world in which pigs fly and B stipulates a complete world in which pigs fly, then, other things being equal, A and B presumably stipulatively identify the same world. If that in turn were true, however, then there should be no further questions, as there obviously are, about whether in the supposedly complete world A stipulates as in the supposedly complete world B stipulates donkeys as well as pigs fly, since either possibility could hold true in a complete world in which pigs fly. So, which is it? When A and B stipulate a complete, maximally consistent world in which pigs fly, are they stipulating a world in which donkeys fly or one in which donkeys do not fly? The stipulated worlds are clearly indeterminate in this respect; they are and should be understood, in the absence of further specification, as the same constitutionally incomplete stipulated nonactual merely logically possible worlds. Otherwise, we are committed to the possibility of distinct but logically indistinguishable worlds. This is precisely why an intensionalist property-oriented combinatorial semantics is offered as an alternative to traditional model set theoretical semantics for modal logic. It is also why we are forced to adopt a logic of fiction in order to do justice to the nonactual merely logically possible worlds by which logical possibilities relative to the ac-

Worlds', in *The Possible and the Actual*, edited by Michael J. Loux, Ithaca: Cornell University Press, 1979. Greg Ray, 'Ontology-Free Modal Semantics', *Journal of Philosophical Logic*, 25, 1996. Peter Van Inwagen, 'Two Concepts of Possible Worlds', *Midwest Studies in Philosophy*, 11, 1986. Stephen Yablo, 'How in the World?', *Philosophical Topics*, 24, 1996.

tual facts or actually existent states of affairs of the uniquely existent actual world are interpreted. [11]

We need an intensional logic of fiction rather than an extensional mathematical theory of sets for the semantics of modal logic. The reason is that nonactual merely logically possible worlds are the imaginative creatures of formal theoretical semantic fictions. Logically possible worlds other than the actual world are not real things, but modal the-

11. Chihara in *The Worlds of Possibility* argues that a Cantorian cardinality paradox afflicts Plantinga's set theoretical principles of modal semantics in *The Nature of Necessity* (Oxford: Oxford University Press, 1974). Chihara identifies the following assumptions in Plantinga's modal ontology, p. 125:

'1. States of affairs exist.

2. Every state of affairs S has a complement S'.

3. If S is a state of affairs, then necessarily either S or S' obtains.

4. Given any possible state of affairs S, there exists a set whose members are all those possible states of affairs whose members are all those possible states of affairs that include S.

5. Given any set b of states of affairs, the conjunction of that set, *all the members of b having obtained*, exists.'

Chihara infers the existence of at least one logically possible world by building up a set b by means of a Lindenbaum-style maximal consistency recursion, adding either S or the complement of S, S', for every S, and defining D as the state of affairs of all the states of affairs in b having obtained. Set D thereby satisfies the conventional model set theoretical definition of a logically possible world, which Plantinga also accepts, the existence of which, by Plantinga's principles, is set-theoretically guaranteed. Chihara then deduces a Cantorian paradox, p. 126: 'As shown above, we can conclude from Plantinga's prinicples that there is a set a whose elements are *all the possible states of affairs that obtain.* Then, according to Cantor's Theorem, the power set of a, 2^a, has cardinality greater than the cardinality of a. Hence, there cannot be a one-one correspondence between 2^a and any subset of a. But, as will be shown below, there is.' The proposed combinatorial analysis of alethic modality avoids Chihara's Cantorian paradox by detaching the concept of a logically possible world from that of a maximally consistent proposition or states of affairs set like D. Chihara's paradox refutes conventional model set theoretical concepts of a logically possible world, like Plantinga's. Importantly, the combinatorial model does not accept Plantinga's principle [2], that all states of affairs exist, since not all states of affairs combinations are maximally consistent. Nor does Chihara's proof of the existence of at least one logically possible world on Plantinga's realist assumptions go through in a combinatorial actualist modal ontology and semantics. At most, Chihara establishes a result that is limited in immediate relevance to set theory in the abstract, including conventional set theoretical models of logical possibility. In proving the existence of set D, Chihara establishes the existence of at least one maximally consistent states of affairs set. From a combinatorial modal actualist perspective, however, he does not thereby prove the existence of a logically possible world. The only logically possible world recognized as existent in an actualist combinatorial modal semantics is the actual world, and it is not an abstract set but a maximally consistent states of affairs combination.

oretical fictions. What Kripke and others do not seem to have fully appreciated, in the grip of the model set theoretical apparatus for interpreting modal logic, is that to stipulate a logically possible world is to fictionalize, to tell a partial, inevitably incomplete story about places and times that do not in any sense exist.

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