

# Logic of existence and Logic of knowledge. Epistemic and non epistemic aspects of Logic.<sup>55</sup>

*Michel Bourdeau*  
CNRS Paris

**Résumé :** Contrairement à ce qui a parfois été dit, la logique classique et la logique intuitionniste ne s'opposent pas comme une logique de l'existence à une logique de la connaissance. Les considérations épistémologiques trouvent naturellement leur place dans le cadre de la logique classique, sans qu'il soit nécessaire de faire intervenir aucun principe intuitionniste ; il suffit pour cela de reconnaître que la logique ne peut se passer de la notion d'assertion, ou si l'on préfère de jugement. Les deux premières parties montrent donc les raisons qui conduisent à passer de l'énoncé à la proposition, puis de la proposition au jugement. Cette dernière distinction était parfaitement claire pour Frege, qui l'avait intégrée à sa conception de la logique ; dans la mesure où elle peut passer pour résumant la distinction entre l'épistémique et l'ontologique, cela montre que le point de vue classique ne saurait être caractérisé par l'adhésion à une logique de l'existence. La dernière partie examine alors le contrecoup de la distinction entre proposition et jugement sur les autres concepts logiques, notamment ceux de preuve et de vérité.

55. I am indebted to Per Martin-Löf for many conversations that have considerably helped me to shape the view reported herein, as well as to Göran Sundholm, who first introduced me to constructive logic.

About fifty years ago, Heyting presented intuitionistic logic as a logic of knowledge, in contradistinction to classical logic, understood as a logic of existence<sup>1</sup>. When a friend of mine quoted the passage, I was quite surprised. He intended it as a still accurate characterisation of intuitionism, whereas it did not fit with the way I (and, I believe, many people) understand these concepts. It was not an easy task to explain to my friend that his view was mistaken; an easily understandable mistake, but still a mistake. Heyting took from Hilbert his idea of the way logic has to be constructed, but there is no necessity to follow him on that point. What is more important, it is beyond any doubt that many classical logicians, even some of the most famous, would totally agree with what Heyting gave as the starting point of a logic of knowledge, namely the fact that “in order to apply a logical rule, we must know that its premises are true”. In that sense, it is simply not true that, as Heyting put it, “the logic of knowledge originated as the logic of the mathematical infinite”<sup>2</sup>; the logic of knowledge has quite different sources.

It is true that classical logic, as commonly understood today, is a logic of existence<sup>3</sup>. But this common understanding is an historical fact. Before Hilbert, classical logicians, namely Frege or Russell, were aware of the epistemic aspects of logic. The real disagreement is not whether we need or not a logic of knowledge; it is rather about the order of priority between the two components. So, it is not true that classical logic and intuitionistic logic are to each other as logic of knowledge and logic of existence. Rather, there is a logic of the knowledge in classical logic, as much as there is a logic of existence in intuitionistic logic. What I want to show is that there are epistemic and nonepistemic aspects of logic, and that both are present on each side.

Nevertheless, Heyting was right in stressing the difference between these two aspects of logic. The situation is well expressed in the first table<sup>4</sup>:

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1. “Intuitionism in Mathematics”, in R. Klibansky (ed.): *Philosophy in Mid-Century, A Survey*, La Nuova Italia Editrice, Firenze, 1958, vol. I, p. 107.

2. “La conception intuitionniste de la logique”, in *Etudes Philosophiques*, 1956, p. 228 and 230. For instance, Frege states explicitly that “the application of logical laws can guarantee the truth of a judgement only in so far as the judgements to which the justification resort are themselves true”. *Kernsätze zur Logik*, in *Nachgelassene Schriften*, Meiner, Hamburg 1978, p. 190. However, one should notice that the key word, *to know*, does not occur in the sentence.

3. Of course, there is no explicit concept of existence in classical propositional logic; but bivalence amounts to the acceptance of the realist concept of existence.

4. Cf. P. Martin-Löf: “Truth and knowability: on the principles C and K of Michael Dummett”, in *Truth in Mathematics*, H. G. Dales and G. Oliveri, eds, Clarendon Press, Oxford, 1998.

EPISTEMIC	NON EPISTEMIC
Judgement	proposition
demonstration	proof
evidence	truth
inference	consequence
premise	hypothesis

Table 1.

It starts with a pair of concepts, that seem today somehow old-fashioned: judgement and proposition. But, as soon as you have acknowledged the distinction, you find that this bipolarity, to use Husserl's terminology, extends far beyond, to other notions. For instance, you have two notions of truth: truth of a judgement (or, in order to avoid ambiguity, evidence of a judgement), and truth of a proposition. Likewise, there are two notions of proof: proof, or demonstration, of a judgement, and proof, or verification, of a proposition. If you follow this path, you will see that you can recover distinctions that were usual before modern practice of logic had them almost condemned to oblivion; for instance, between inference and consequence. Inference is an activity, done by a cognitive agent; consequence, on the other hand, is an ontological relation.

A second table helps to see Heyting's mistake<sup>5</sup>. To stress the epistemic aspects of logic is by no means to speak in favour of intuitionistic logic. It is only to speak in favour of a more comprehensive view of logic.

	FREGE	GENTZEN
formal language	interpreted	non interpreted
truth applies	to propositions	to wwfs
are derived	judgements	wwfs
inference starts	from premises	from hypotheses
key notion	truth	consequence

Table 2.

The fact that, nowadays, many people have difficulty in accepting the idea that we need a logic of knowledge or that logic and the theory of knowledge are closely related, shows mainly how much we still remain

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5. Cf. G. Sundholm: "Implicit Epistemic Aspects of Constructive Logic", *Journal of Logic, Language, and Information* 6(1997), p. 191-212.

under the influence of Hilbert's paradigm. We may add that Frege and Russell wrote before the linguistic turn, so that the rediscovery of their point of view is in agreement with the move from philosophy of language back to philosophy of mind.

There is another way to look at the situation. Whoever discovers for the first time Per Martin-Löf's work may be very puzzled by the way he takes proposition and judgement as his two key concepts. Among analytical philosophers, for instance, it is a common place that, as much in logic as in philosophy, one of the main progress in twentieth century had been to have get rid of these concepts. You don't talk about judgement, but about propositional attitude; and not even about proposition, but about sentence. Why, then, reintroduce such obscure and obsolete notions?

In this respect, one possibility would have been to examine the objections that were made, and try to answer them; but we face another task, and I shall stick to the question: how to make sense of proposition and judgement? — So, in order to show that there is place for epistemic notions in logic, it seems appropriate to travel back from Frege to Gentzen: how can you go first from sentence to proposition, and then from proposition to judgement?

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*From sentences to propositions.* In order to motivate the first problem, let's consider a possible objection. "Look", one could say, "there is simply no concept of proposition to be found in contemporary logic. How then can you locate precisely at that place the disagreement between classical and intuitionistic logic?" But this would beg the question, and show mainly the influence of Hilbertian or Quinian paradigm. The answer, — and a very usual one, indeed —, is simply that there is another way to conceive logic, where proposition is a central notion; note that we still say "propositional logic", rather than "sentential logic".

The relationship between sentence and proposition is an extremely confused matter. It is almost impossible to say anything without being, from the start, exposed to fundamental objections. I shall stay at a very elementary level; I don't pretend to any historical accuracy either<sup>6</sup>. I am just trying to present, in a cursory way, the difference between sentence and proposition. There will be three steps: why were propositions

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6. Cf. *v. g.* G. Nuchelmans: "Proposition" in *Historisches Wörterbuch der Philosophie*, Wissenschaftliche Buchgesellschaft, 1989, t. 7, col. 1508-1524.

introduced? why people like Quine wanted to get rid of them? Why are the objections not cogent? Why are we still entitled to speak of propositions?

It seems that during a long time, no clear difference was drawn. In the middle ages, for instance, the philosophical and logical notion was *propositio*, and logicians did not use *sententia*. Furthermore, there was *prima facie* no problem looking at propositions as something indifferently linguistic or mental, so that our problem did not exist, or at least, was not formulated in the same way. French grammar, as it was taught at school, talked of *proposition*, which was the translation of *oratio*, and gave it the same definition as to *oratio perfecta*: a proposition was the expression of a complete thought. It seems that it was the same with the German *Aussage* or *Satz*.

Present use of *proposition*, and present debates about it, have a semantic motivation. One wishes to draw a clear cut, sharp distinction between what we say and what we mean, between the expression and its meaning. Instead of saying: the proposition is the expression of a complete thought, we say nowadays: a sentence is the expression of a proposition; conversely, a proposition can then be defined as the meaning of a sentence.

There are several answers to the question: why introduce one more level between the word and the object?<sup>7</sup>

The existence of meaning seems to be one of the most primitive, one of the most universal experiences of mankind. We mean (or the sentence means) something, without this something being necessarily the case, without this something obtaining in the real world. Words are like arrows and the speaker like an archer: he can miss the goal, he is nevertheless aiming at something. This confidence in the existence of meaning is strongly entrenched in linguistic practice. At early stages of generative grammar, for instance, Chomsky, who rejected the appeal to semantic data in linguistic analysis, used to say that transformations were meaning-preserving. When we translate from a language to another one, something is lost, but we all know that something is preserved, and that something is what we call meaning<sup>8</sup>.

Secondly, there is a more specific reason for a logician: somehow,

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7. Notice that the question takes for granted that meanings do not belong to the word, to the linguistic level, which is far from being evident and depends upon the definition you give of the concept of word.

8. To believe that you have language as soon as you have sound is an illusion. Cacophony like *Blituri* doesn't belong to language. In order to be able to be *true*, sounds have therefore to be first meaningful.

logic has to do with truth and the truth predicate seems to apply to propositions rather than directly to sentences. So, it is better not to define logic as a *scientia sermocinalis*, but to take instead meaning as the object of logic.

This position has been strongly criticised; amongst its most vehement opponents are Russell (after 1913 of course !) and Quine. — Meanings, we are told, are mysterious entities. We know what is a word, what is an object; but all we know about meanings is that they are neither words nor objects, and this negative characterisation is not sufficient. Furthermore, to accommodate abstract entities is condemned by Occam's razor: *entia non sunt multiplicanda praeter necessitatem*. According to Russell, it is a mistake to believe that in “ $x$  thinks that  $p$ ” thinking is a relation between  $x$  and  $p$ . Logical analysis shows that, in order to account for propositional attitudes, we don't need propositions: it is enough that  $x$  be acquainted with the constituents of the proposition  $p$ , and it is therefore better to view propositions as another case of incomplete symbols, to apply the principle of abstraction (better named the principle that allows you to dispense with abstraction), and to stipulate: as far as possible, substitute logical constructions for inferred entities<sup>9</sup>.

As for Quine, he devoted the first paragraph of his *Philosophy of Logic* to this topic: “My objection to recognising propositions does not arise primarily from philosophical parsimony — from a desire to dream of no more things in heaven and earth than need be. Nor does it arise, more specifically, from particularism — from a disapproval of intangible or abstract entities. My objection is more urgent. If there were propositions, they would induce a certain relation of synonymy or equivalence between sentences themselves: those sentences could be equivalent that expressed the same proposition. Now, my objection is going to be that the appropriate equivalence relation makes no objective sense at the level of sentences”<sup>10</sup>. The argument leans heavily upon holism: if we can't define an equivalence relation between sentences, it is because there is no possibility to attribute any clear import, either empirical or cognitive, to a sentence taken in isolation.

It is not necessary to enter any further into the details of the argument to understand that the rejection of propositions, like the rejection of modal logic (one more point upon which Russell and Quine agree!) is an outcome of the extensionality thesis, and that there is no more necessity in the former than in the latter. We are facing an old controversy, and it

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9. Cf. v. g. B. Russell: *Mysticism and Logic*, Longmans, 1918, p. 155 and 219.

10. W. Quine: *Philosophy of Logic*, Prentice Hall, 1970, p. 3.

would be very presumptuous, and very naive, to believe that it has been definitively settled. To be sure, we know how to define truth directly for sentences, and not derivatively from the truth of propositions. But is it sufficient to get rid of propositions? Such solutions are often artificial, and anyway, that does not mean that the adverse position has been refuted. On the contrary, it seems quite fair to say that, today, the extensionality thesis has been abandoned, and that nobody will follow Quine in his strict adherence to it.

If you accept intensional logic, there is no reason to reject propositions. Thanks to possible world semantics, they have become again respectable entities, and they play an important role in, for instance, Montague Grammar. In the controversy between one-layer and two-layer semantics, we have taken side with Frege as against Russell. Concerning the former, it may be useful to draw attention to one point. Usually, he is remembered for having held that propositions are the meaning of sentences: in a two level semantics, sentences must have both sense and reference, and that will be respectively proposition and truth value. So propositions are conceived primarily as one of the two semantic values of a sentence. This account is somehow misleading. For Frege, the semantic approach to proposition is only derivative. If we need propositions, it is in order to account for truth, much more than for building a theory of meaning<sup>11</sup>. If you prefer, truth theory belongs to epistemology and not to semantics, as is usually believed today. Propositions have to be conceived primarily as truth bearers. The fact that truth is essentially a property of propositions, not of sentences, agrees with another neglected aspect of Frege's thinking. The father of modern logic is not what we call a linguistic philosopher; one of his *leitmotivs* was to struggle against grammar, to free logic, and the mind, from the yoke of language. It is therefore not surprising that he did not take a linguistic point of view as his point of departure.

This remark helps to grasp an important point. There is not one but two approaches to the concept of proposition, and the former one is perhaps not the most appropriate. In order to understand why we need this concept, it is advisable to take our point of departure not in the philosophy of language, but in the philosophy of mind: propositions are much less meanings of sentences than contents of cognitive acts.

From this point of view, Prior spoke about 'objects of thought' and he explicitly mentions the relation with what Ryle had called the *ac-*

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11. Cf. M. Dummett: *Frege's Philosophy of Language*, Duckworth, Londres, 1981, p. 153; G. Frege: *Nachgelassene Schriften*, Meiner, Hamburg 1978, p. 222-3.

*cusative* of mental acts<sup>12</sup>. In particular, it was the approach of Frege and Russell. In spite of his antipsychologism, the former was perfectly aware of the close relationship between proposition and judgement<sup>13</sup>. Likewise, for Russell, the two concepts were closely related: propositions were useful foremost as objects of propositional attitudes. We know that he borrowed part of his theory of proposition from Moore, whose seminal paper was entitled: *On the Nature of Judgement*; he turned to propositional attitudes when he was convinced by Wittgenstein to abandon his former theory of judgement<sup>14</sup>.

However, this dependency may be used as the basis for an objection. "Theory of judgement", the expression sounds very old fashioned. Modal logicians are prone to accept propositions, but it does not appear that they want to restore theories of judgement. If such is the natural framework for propositions, they are liable to collapse with it. Furthermore, by appealing to judgement in order to explain propositions, it may seem that we are begging the question. Our goal was to show the epistemic aspects of logic. But with this second approach, we are taking the point for granted: we take judgement to be our point of departure, we go neither from sentence to proposition, as announced, nor from proposition to judgement, but instead from judgement to proposition. We have gone too fast. So, let's go back and go more carefully through the second step.

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*From proposition to judgement.* Our aim is to show that a logic of knowledge is in no way characteristic of intuitionistic logic. In other words, we want to justify epistemic considerations without stepping outside the classical understanding of logic, without appealing to any intuitionistic principles. In order to do that, we agreed to focus on judgement, as we focused before on proposition, in so far as judgement is the

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12. A. N. Prior: *Objects of Thought*, ed. by P. Geach & A. J. Kenny, Oxford, Clarendon Press, 1971, p. 3. Ryle introduced this new terminology in his article *Are There Propositions* (Proceedings of the Aristotelian Society 30(1929-1930), p. 91-106) in order to avoid the ambiguity associated with the concept of object and, so to speak, to 'desobjectify' the intentional correlate of psychic acts.

13. For instance he wrote: "I use the word *proposition* <Gedanke> more or less like the logicians use the word *judgement*. To think is to grasp a proposition. After one has grasped a proposition, one can recognise it as true, which is judging; and then, express this recognition, by asserting the proposition". *Nachgelassene Schriften*, p. 201; cf. the letter to Hugo Dingler, dated 6/2/1917.

14. B. Russell: *My Philosophical Development*, Unwinn Books, Londres, 1975, p. 87-8.



paradigm of epistemic notions. But the discrepancy between the two tasks is obvious. The present one is much more difficult and controversial. Therefore, it may be appropriate to begin with a few words about the two notions of epistemology and ontology. We don't use them in any unusual sense and it could have been left implicit, but, by explaining their relationship, we provide a first reason why epistemic considerations are inescapable.

Epistemology is nothing else than the theory of knowledge: it studies our theoretical relations with the world, the way we come to know what we know. We may be mistaken. So, the main task of a theory of knowledge is to separate true knowledge from false knowledge. Truth is the cornerstone of a theory of knowledge. Anticipating what follows, it is already possible to see why we need judgement. In the last run, all errors are errors of judgement. There would be no error of reasoning if our judgements were always correct, because judgement is the place of truth, and therefore of error too. One may wonder if the place that is given to argumentation by some analytical philosophers doesn't lead one to forget the role of judgement, the subordination of reasoning to judging. As to ontology, it is the study of being *qua* being, or, in a more modern conception, of the thing as it is by itself, independently of the way we know it. Some would say, it is the theory of the object in general, *Gegenstandstheorie*.

One used to say that ontology is to epistemology as ancient philosophy to modern philosophy. But this remark turns the necessity for epistemic considerations into a mere historical fact. Some more elaborate argument is in order. — The question is simply this: what can we say confidently of the thing in itself, independently of the way we know it? Can we say more than what we know? This does not amount to a denial that the thing may be independent of us. We may easily concede that it has its objectivity, its subsistent being, that it is not our creation. Still, it is hopeless to try to put into parentheses our access to it and to take, so to speak, the point of view from nowhere. — In this sense, a logic of existence can't be independent of a logic of knowledge. It is only a fragment, a by-product of the latter: it is the theory of knowledge minus the access to the world; it is what is left when we forget the way we have come to know, and philosophy reminds us that it is dangerous to go too far away from the source of knowledge.

In spite of being one of the main spokesmen of classical logic, Frege had a theory of judgement: there is place, in his philosophy, for a logic

of knowledge, besides the logic of existence<sup>15</sup>. However, it is not unfair to say that his influence has been rather in the opposite direction. Until recently, everything happened as if he had no theory of judgement at all. The attack against psychologism can easily be understood as an attack against psychology “überhaupt”. Theory of knowledge includes, as a component, a theory of cognitive acts, that may in turn be viewed as a sort of psychology. Wittgenstein, for instance, has very well expressed this idea when he said that epistemology is nothing more than the philosophy of psychology (*Tractatus*, 4.1121). — In this respect, the comparison with Husserl is illuminating. The father of phenomenology fought against psychologism too; but for him this did not mean fighting against psychology, and he wanted to maintain the link between logic and the theory of knowledge. In the case of Frege, the situation is very different and his antipsychologism led him to reject epistemology altogether. Logic is one thing, psychology another. It is not surprising if, in comparison with Husserl, Frege has not much to say about the theory of knowledge; the epistemic dimension of logic is recognised, but it stays in the background, and, perhaps unwillingly, Frege contributed to the decay of the theory of judgement.

So, why do we need to go forward from proposition to judgement? The simplest answer comes from the incomplete character of propositions. Propositions are not the concern of logicians alone. In business, in diplomacy, in negotiations in general, propositions are made, are accepted or rejected. It is the same with logic. It is in the nature of proposition to be asserted or denied, and this is nothing else but judging. — More explicitly: in order to define logic, we need truth; even if we think that the core of logic is the study of the relation of consequence. Consequence cannot be defined without resorting to truth: the consequence is valid if it transmits to the conclusion the truth of the premises.

One may be tempted to say that, truth — namely truth of a proposition — being an ontological notion, truth theory does not commit us to step outside ontology to go into epistemology. To this remark one will reply that, although truth is indeed a property of propositions, *to recognise* the truth of a proposition (recognition that can in turn be true or false, correct or incorrect) is precisely what we mean by judgement. Truth, in this sense, is an epistemic notion too; an unknown truth is something strange, and in particular something of which we cannot give any example. In other words, we need to distinguish carefully, as Frege

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15. Bell: *Frege's Theory of Judgement*, Oxford, Clarendon Press, 1989.

did, between the proposition and its assertion; or, in the reverse order, between the act and its object. According to Dummett, the study of assertoric force incorporated in Frege's "judgement stroke" belongs to speech act theory. But, in order to see the link with epistemology, it is enough to recall that thinking has often been viewed as a mental language. Speech act theory appears then, in some respects, as inherited from the theory of mental act, as it was developed in classical epistemology. However, it still remains to explain why logic cannot be a mere theory of objects.

As a first approximation, we can answer that the distinction between proposition and judgement is clearly given in language. Consider: *if p then q* on one hand, and: *p. Therefore q* on the other. In both cases, we have two propositions, *p* and *q*. But in the first case, we have only one judgement; in the other, we have two judgements. When I say: *if it rains, I stay home*, I am not asserting that it rains, even less that I stay home. I am just asserting that the first proposition entails the second one. And the consequence may be valid, even if the sun shines, even if I am in the street. But when I say: *It rains. So I stay home*, I am giving both propositions as true. The fact was well known by traditional logicians, who spoke respectively of hypothetical and of categorical judgement. If *modus ponens* is sometimes called the detachment rule, it is because, in order to conclude *p* from *q*, you need another premise, namely an hypothetical one, which asserts that a relation of consequence holds between *p* and *q*, and allows you therefore, given the antecedent *p*, to detach the consequent *q*.

So far, it has just been shown that the distinction is familiar to logicians; but this does not entitle to say that judgement belongs to the subject matter of logic; and even less that the business of logic has to do primarily with judgement. Nevertheless, we must admit that the fundamental unit of logic is judgement, not proposition. The point deserves some comments. Propositions are to judgements as logical constants are to logical rules. Propositional connectives, as their name says, connect propositions. But logical rules, or rules of inference, do not. As Frege saw, and said, clearly, premises and conclusions are asserted propositions, that is, judgements. To posit judgement, and not proposition, as the cornerstone of logic amounts to deny that its subject matter is what medieval logicians used to call *syncategorematics*. The business of logic has to do primarily with logical rules and only subsidiarily with logical constants. We have just reached the core of the argument. To repeat: since logicians have to deal as much with judgements as with propositions, in logic there is place, not just for ontological considerations, but

for epistemic ones. And since the result has been obtained without stepping outside the classical framework, it is simply untrue that classical logic is to intuitionistic logic as the logic of knowledge is to the logic of existence.

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Now, let me try to unfold some of the consequences of this main result. The distinction between the epistemic and the ontological pervades the entire field of logic, and we meet that distinction again when considering notions like consequence, truth or proof.

First, as already seen, we have to distinguish between inference and consequence. The point has been very clearly made by G. Sundholm<sup>16</sup>. It is characteristic of current logical theory that the two concepts are taken as almost totally synonymous. However, their grammar is obviously different. “To infer” is constructed with a personal subject, while “to follow” commands an impersonal one: I infer something from something else, but something follows from something. One speaks of rule of inference, not of rule of consequence. In spite of that, according to modern logical orthodoxy, to say:

the consequence  $C$  follows from  $A_1, \dots, A_k$  holds

amounts to the same as to say:

the inference  $A_1, \dots, A_k$  . Therefore  $C$  is valid.

Both are defined in the same way:  $C$  is true whenever all the  $A_j$  are true. It is a good example of the impoverishment induced by the disappearance of the epistemic point of view: a distinction that was clearly made by traditional logic has been almost totally blurred today<sup>17</sup>.

Secondly, we have to distinguish between two kinds of proof. The idea was adumbrated by Hilbert. On one hand, there is the formal proof, which is purely mechanical, which seeks no help from intuition and proceeds instead blindly, relying exclusively upon the strict application of

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16. G. Sundholm: “Inference versus Consequence”, in *Logica Yearbook*, Czech Acad. Sc., Prague, 1998.

17. Actually, the situation is not so simple. One oscillates in reality between two positions. Sometimes, the two notions are merely identified; sometimes, as in Tarski 1936 paper *On the Concept of Logical Consequence*, inference is reduced to consequence: the inference is valid if the consequence is valid. The distinction is too well entrenched to be completely lost. But, without the distinction between proposition and judgement, there is no more ground left to differentiate inference and consequence.

a set of pre-established rules; on the other hand, the metamathematical proof, which is informal, much closer to mathematical practice, which appeals to intuition (*inhaltlich*), and where understanding recovers its rights. But the distinction we are dealing with is drawn along another line: according as it applies to a judgement or to a proposition, a proof will be conceived as an act or as an object.

The proof as act corresponds approximately to the usual notion. Proofs exist in textbooks, or in mathematical reviews, or in class rooms; but before being written or spoken, they have to be conceived, to exist in the mind of the mathematician. The material proof serves only as an help, designed to produce the understanding of the reader, or of the student. A proof is essentially a mental act, by which a judgement becomes evident. The word *demonstration* aptly reminds us that, in this sense, to prove is to show, that a proof is designed to produce insight.

One of the original contributions of contemporary type theory is to introduce a new idea of proof: the proof as object, in a sense different of the metamathematical one, that is without the splitting of levels characteristic of Hilbertian proof theory. If, in a typed lambda calculus, we take the type of the term as a proposition, then the term itself turns out to be (or to encode) a proof of the corresponding proposition. But a lambda term is a mathematical object, and so is the proof. — It is not even necessary to resort to lambda calculus, and de Bruijn has given a very natural argument in defence of this new idea<sup>18</sup>. Mathematical demonstrations require us to apply theorems in the same way we apply functions. In order for a machine to do mathematics, we have to consider proofs as mathematical objects. In this approach, proposition-as-type loses its status of principle, to become a mere corollary of a more fundamental idea, namely the idea of proof as object. Both cases ask for an extension of our notion of mathematical objects. Primitively, functions were merely tools for manipulating numbers. It took a long time to view them as a new kind of mathematical object. In the same way, today's mathematical practice encourages us to enlarge once again our concept of object, in order to incorporate proofs, that is, lambda terms.

But to develop further those ideas would take us too far away. The point was only to notice that the principle of proposition-as-type supports a splitting of the proof notion which, unlike the Hilbertian one, agrees with two philosophical distinctions. Indeed, we have not only the

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18. N. G. de Bruijn: "On the roles of types in mathematics", in *The Curry-Howard Isomorphism* (Ph. de Groote, ed.) Cahiers du centre de logique 8, département de philosophie de l'université catholique de Louvain, Louvain la neuve, 1995.

traditional act-object scheme; it must be clear that the aforementioned principle demands that the proof object be the proof of a proposition, as much as the proof act is the proof of a judgement.

Finally, we have two notions of truth, too. Truth of a proposition is possession of a proof, that is to say, of a proof object. In constructive type theory, *A is true* is short for *a proof of A exist*. In this way, constructivism is able to accommodate itself to the correspondence theory of truth. A proposition is true if there exists in the world something to which it corresponds, namely its proof object. Instead of calling the lambda term a proof object, one could as well call it a truth maker. Actually, it is what we do when we talk about verification: a truth-maker being nothing else than a veri-fier, with the stems translated from Latin to English.

So, the constructivist does not reject the correspondence theory of truth<sup>19</sup>; he only claims that it does not say all the truth about truth, and that there still remains much to understand. Besides propositions, there are judgements; truth can't be only correspondence, it is evidence too; and just as, on the epistemic side, proof is called demonstration, truth is called evidence. A demonstration is what makes a judgement known, that is to say, evident; conversely, *a judgement is demonstrated* means: it is known as true. In no other place can one see so clearly the link between judgement and the theory of knowledge. Just as a proposition is defined by its verification conditions, a judgement is defined by its assertion conditions. If you prefer, to say what is a proposition is to say what you have to know in order to assert: *A is a proposition*; and to say what is a judgement is to lay down what you have to know in order to judge. (Notice the shift from the ontological to the epistemic, from the question: *what is...?*, to the question: *what do we have to know in order to...?*) — More specifically, truth is knowledge dependent. Brouwer's famous dictum: *there are no non experienced truth* must be understood to be about judgements. It agrees with Husserl's definition of evidence as experience of truth (*Erlebnis*, not *Erfahrung*).

However, this is not the last word about truth, not even truth of a judgement: besides evidence, we have correctness too. A judgement is not always known, that is to say, made evident. We have to distinguish between the judgement as act and the judgement as object, between the act of judging and what is judged. This ultimate splitting, which, you will notice, takes place this time inside epistemology, offers the opportunity to rectify a very current misunderstanding.

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19. P. Martin-Löf: "Truth and knowability: on the principles C and K of Michael Dummett"; see footnote 4.

It is quite natural to set ontology over against epistemology as objectivity to subjectivity. Then, the epistemic notion of truth would at the same time be a subjective one. If this applies to evidence, because the subjective dimension of experience is not to be contested, it does not apply anymore to judgement in general. — Before being known, a judgement was already true. In spite of the view commonly attributed to him, the intuitionist is not committed to hold that truth is a human creation. It would not have been possible for the judgement to become true if, before hand, it had not been, in some respect, already true. This last aspect of truth, we shall call correctness (rectitude, *Richtigkeit*).

Correctness is epistemic truth, without being experienced, subjective truth. One may then wonder: if it is so, how can we still maintain that correctness is knowledge dependent? The answer lies in the distinction between actual and potential. Just as evidence means actual truth, correctness means potential truth: a judgement is correct if it *can* be known. — We are prone to think that, because it is not experienced, potential truth is not knowledge dependent. But we are then forgetting that potentiality presupposes actuality. When you try to define correctness, you cannot get rid of knowledge, you are ineluctably brought back to it. To repeat: a judgement is correct if it can be *known*, that is, if it can become evident, experienced. One is used to thinking that power precedes act. That may be the case chronologically, but not logically. As Aristotle and after him Aquinas already saw, in the conceptual order, *actus est prior potentia*, because potentiality is nothing but potentiality to act. In order to be more precise, one would have to introduce another distinction between what is really possible and what is logically possible: potentiality is real, not logical possibility. But what we have just said is sufficient, I hope, to show that even correctness is knowledge dependent<sup>20</sup>.

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We took as our starting point Heyting's dictum according to which the idea of logic does not include epistemology. It would be rather a matter of choice: if you are intuitionist, you will favour a logic of knowledge; but you are always free not to follow him and to side with classical logic, rejecting then epistemic considerations to content yourself with a mere logic of existence.

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<sup>20</sup>. Cf. P. Martin-Löf: "A path from logic to metaphysics" in *Atti del Congresso Nuovi problemi della logica e della filosofia della scienza*, CLUEB, Bologna, 1991, p. 141-149.

It must now be clear that such statements were incorrect. They expressed the influence of Hilbertian, formalist conception of logic, and it was enough to come back to Frege to recognise that by itself, classical logic does not preclude epistemic considerations. But this does not close the debate. Once conceded that there is room for knowledge-theoretical notions in logic, all the problems are not *ipso facto* settled, and it remains for instance to specify with more precision the respective parts of epistemic and non epistemic considerations. Furthermore, opposition to the theory of knowledge does not come only from the formalists: the early Wittgenstein cannot be suspected of sympathy for Hilbert, and Port Royal has been reproached for being “the source of a bad fashion of confusing logic with epistemology”<sup>21</sup>.

In this respect, the case of Frege is specially instructive. To be sure, the author of the *Begriffsschrift* is not Hilbert, but he is not Husserl either. We have appealed to him against the former, but an orthodox Fregean could object that we have given too much to the latter. For instance, the idea of truth as evidence is Husserlian; we would in vain look for something similar in Frege’s writings and it would not be difficult to present Fregean criticism to my position.

Frege and Husserl did not pursue the same goal. The *Nachlass* makes very clear that, although the ideography needs a judgement stroke, judgement, viewed as a psychic activity, has no place in logic: “Precisely because thinking is a psychic process, we don’t need to consider it in logic. It is enough that we grasp propositions and that we can recognise them as true. How this happens is another question.(...) For the success of a scientific inquiry, it is not bad that questions would not be mixed, in order not unnecessarily to complexify the situation (...). Therefore we will not enquire as to how thinking takes place.(...) To explain the process of thinking or judging is a possible task, but not a logical one”<sup>22</sup>.

This exclusion is related to Frege’s idea of logic, where judgement

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21. W. and M. Kneale: *The Development of Logic*, Clarendon Press, Oxford, 1970, p. 316.

22. “Das Erfassen dieses Gesetzes ist doch ein seelischer Vorgang! Ja! aber ein Vorgang, der schon an der Grenze des Seelischen liegt und der deshalb vom rein psychologischen Standpunkte aus nicht vollkommen wird verstanden werden können, weil etwas wesentlich dabei in Betracht kommt, was nicht mehr im eigentlichen Sinne seelisch ist: der Gedanke; und vielleicht ist dieser Vorgang der geheimnisvollste von allen. Aber eben weil er seelischer Art ist, brauchen wir uns in der Logik nicht darum zu kümmern. Uns genügt es ja, dass wir Gedanken fassen und als wahr erkennen können; wie das zugeht, ist eine Frage für sich.” *Logik* (1897), dans *Nachgelassene Schriften*, p. 157. Even worse, by stressing the role of acts, theory of knowledge is suspected of collusion with idealism (p. 155).



plays only a subsidiary role. In the *Kernsätze zur Logik*, he wrote: “14) The doctrines of concept and judgement serve only as preparation to the doctrine of consequence (*Folgern*). 15) The task of logic is the establishment of the laws according to which a judgement is justified by another one, without regarding whether those judgements are true or not”<sup>23</sup>. Without this narrow conception of logic, we would not have got the *Begriffsschrift*, but there was a price to pay for this achievement. Husserl represents another possibility: most people would hesitate to classify him among the logicians, and histories of logic do not mention his name; but he asked the questions Frege neglected to ask.

We have reached the end; so let’s sum up. The main point was to break with current logical orthodoxy and to advocate instead a return to a more classical point of view. Behind Heyting’s statement that I quoted at the beginning, we find the influence of the formalist paradigm, which had cut logic from its philosophical roots. In order to characterize intuitionism as a logic of knowledge, classical logic had to be first separated from the links it traditionally entertained with epistemology. Hilbert’s attack against intuitionism was so vehement that we often come to forget that he fought on two fronts: against Brouwer, but also against Frege. We focused upon the latter aspect as, from a purely philosophical point of view, it touches on much more fundamental questions<sup>24</sup>. As soon as this point is accepted, it appears that the main logical notions present two sides: an epistemic and a non-epistemic one. It is not the least merit of the epistemic conception of logic to remind us of the centrality of judgement.

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23. *Nachgelassene Schriften*, p. 190.

24. Furthermore, what I have said should be quite uncontroversial. It is not the least original either. On the long run, the heterodox logician is Hilbert. For many years, Per Martin-Löf has stressed the necessity to revert to this point of view, which turns out to be the point of view of Frege or Russell. One may notice that this move agrees with a more general one, through which philosophy of language has been largely superseded by philosophy of mind. If the coming back of psychology in the cognitive sciences has to be met with reservation, there are many good reasons to welcome the end of the systematic exclusion of a theory of cognitive acts.