

PHILOSOPHIA SCIENTIÆ

MOGENS WEGENER

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Philosophia Scientiæ, tome 1, n° S1 (1996), p. 81-103

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'A–Priorism' in Poincaré, Eddington & Milne

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Philosophia Scientiae, Volume 1 (Cahier spécial 1), 1996, 81-103

« Reason does not extract its laws from nature, it prescribes them to nature ... by subsuming all phenomena under its own laws, reason is the source and origin of the general order of nature... »¹

« Nothing of all that which has been set forth about the universe could ever have been said if we had never seen the sun or the starry heavens ;but observation of day and night, of months and seasons of the year, of equinox and solstice, has produced our knowledge of numbers, which has conferred on us the notion of time and inspired us to investigate the universe ; whence we have derived philosophy, which is the greatest boon ever bestowed on mortal man by the heavens ...

But, in my opinion, the cause and purpose of vision is this : God invented it and entrusted it to us in order that we should observe the orbits of reason in the heavens and use them to correct the circuits of our own thought which are akin to them, though ours be troubled and they unperturbed, so that — when we learned to know them and to compute them rightly according to nature — we could bring order to our own errant circles by imitating those of God which are perfectly regular. »²

« Does the harmony the human intelligence thinks it discovers in nature exist outside of this intelligence? No, beyond doubt, a reality completely independent of the mind which conceives it, sees or feels it, is impossible. A world as exterior as that — even if it existed — would for us be forever inaccessible. But what we call objective reality is, in the last analysis, what is common to many thinking beings, and could be common to all. That common part can only be a harmony expressed by mathematical laws. It is this harmony, then, which is the sole objective reality, the only truth we can attain. When I add that the universal harmony of the world is the source of all beauty, it will be understood what prize we should attach to the slow and difficult progress which little by little enables us to know it better. »³

« Simple, as is the origin of this law (of reciprocal attraction), which relies only on the relationships between spherical surfaces of different radii, nevertheless its consequences are so rich, as regards the variety of their mutual consistencies and uniformities, that not only does it describe all possible trajectories of heavenly bodies by conical sections, but it does also imply relations of such a kind to obtain between these sections, that no other law of gravitation than that depending on the

¹ [Kant 1783].

² [Plato].

³ [Poincaré 1905a].

inverse square of the distance can be considered appropriate to a world system »⁴

From Plato to Kant

The merits of Jules Henri Poincaré as one of the greatest mathematicians of all times are globally acknowledged. But the value of his conventionalist theory of science is still greatly underestimated — partly due to misrepresentation by leading historians, partly due to unfair criticism propagated by important philosophers — and his contributions to physics as well as to its philosophy have unjustly fallen into oblivion, as compared to the overwhelming fame of Albert Einstein.

In this lecture/paper it is my intention to stress the importance of Poincaré to physical theory and the theory of physics by hailing him as the principal figure in the traditional interplay between classical philosophy and modern cosmology. As an example, I want to install him as the central link in a line of development connecting the main stream of European thought, as represented by Kant's *Critique of Pure Reason*, to two seemingly incompatible non-standard cosmologies : viz. that of Arthur Stanley Eddington, and that of Edward Arthur Milne.

Within the restricted frame of time/space allowed to me at this very special occasion it is of course not advisable for me to dwell at length on historical detail, neither do I feel able to do so without further study. What I want to do is to draw, with colored brush and sweeping gesture, some very broad lines in the history of scientific ideas. These, as I see it, opens some exciting philosophical perspectives which might in the end help to throw light on the present impasse of cosmology. But in order not to pretend too much I shall close these introductory remarks by reminding you of the obvious fact, that science is always in need of bold new ideas. This is one of the reasons why we should not forget about its history.

The story of how Immanuel Kant was disturbed in his dogmatic slumber by the skeptical doubts of an eloquent Scotchman is well-known : finally he was forced to envisage the scandal of

⁴ [Kant 1783].

contemporary philosophy. A century ago, Isaac Newton had won for physics its most brilliant triumph in history — yet philosophy had been unable to account for this unique achievement, let alone to disclose its legitimacy. In spite of Descartes, the ghost of Aristotelianism was still haunting philosophy ; but to Kant, at least, it had become clear that the way of abstraction is blocked : true knowledge can never be obtained by successive approximation. Inspired by the feat of Nikolas Koppernig, innovator of medieval astronomy, Kant set himself the task to effect a Copernican revolution in philosophy.

In order to ensure the safe progress of science, Kant proposed a distinction between reality to us and reality in itself, i.e. plain phenomena and mere noumena. Of noumena that remain latent we can know nothing. Of phenomena belonging to the realm of experience, which originate from the joint venture of observation and speculation, or the teamwork of sensation and reflection, we can know everything. The inner nature of reality transcends our inborn intellectual capabilities forever ; but apparent nature — the surface of reality — remains transparent to our reason. What we must do, in order to obtain absolute and indubitable knowledge, is only to apply those conceptions, which distinguish our inborn nature as thinking beings, to that manifold of sensations, which is continuously caught in the network defining the structure of our natural intuition — namely, the framework of time and space. True knowledge of apparent nature, reality for us, is then bound to emerge. Knowledge of this kind, albeit occasioned by experience, derives its validity and legitimacy from another source, viz. the collaboration of reason and intuition.

According to Kant, pure reason can collaborate with pure intuition ahead of any mediation of experience, and the result of this activity is pure knowledge *a priori*. Knowledge is *a priori* when it can be derived by strictly transcendental arguments, i.e., formal arguments which hold good independently of any concrete experience. As regards the *a priori* argument, given by Kant, to prove the inverse square law of gravitation, it is clear that his claim — astounding as it is — can be sustained on the assumption that gravitational forces can be described in flat vectorial 3-space : the argument is then on a par with that leading to the so-called ‘Olbers’ paradox’. Although, of course, we always have to wait for such

arguments to be invented, it is nevertheless interesting to speculate how Kantianism might have been received if, *per impossibile*, the Euclidean parallel axiom had eventually been proven.

To Poincaré, the failure of all proofs given hitherto was decisive evidence against the claim of Kant that the structure of space can be demonstrated a priori. Although accepting the possibility of a pure intuition of space, he insisted that this space is devoid of any formal structure, hence definable in negative terms only. This brought him close to the position of Plato who frankly admitted that space — 'the uterus of becoming' — is nothing but an imaginary container, whose dreamlike existence is hard to believe in: formless, and causally neutral, it is next to nothing. According to Descartes, extension is a substantial property, and space is material; but Poincaré rejected the Cartesian aether-hypothesis, just as it had been rejected by Leibniz, and for the same reason: abstract space is relational, not substantial.

It remains for us to point out that the gist of Kantian *a priorism* is not bound up to the problem of geometry and, *a fortiori*, not to the idea of an aether. Its real issue is the active role of the intellect in the reconstruction⁵ of the world. This activity of reconstructing the world is undertaken by a transcendental subject which can be interpreted as a kind of Platonic Demiurge stripped of its mythology. As the Demiurge once created *cósmos* out of *cháos* by applying ideas of geometry and categories of logic to a pre-existing 'ocean' of sensible qualities, in the same way the transcendental subject produces and sustains its 'world' by applying reason to intuition.

The most conspicuous difference between Kant and Plato is, however, that Plato regarded physics as 'the science of the probable' — not *epistéme*, but *dóxa* — whereas Kant insisted on *epistéme* — i.e., a final and absolute knowledge of nature. At this point modern science is clearly much more close to Plato than to Kant. Plato dreamt of a kinship between mind and nature, between concept and reality; he assumed that harmony is inherent in the world of nature and believed that it can be discovered by human

⁵ That the Kantian construction of the universe is indeed meant as a re-construction seems evident from the intent behind his dusky 'disproof of idealism' in the *Critique of Pure Reason*.

reason because it partakes itself in the same harmony. But this is precisely the idea echoed in the so-called 'a priorism' of Kant.

I will elaborate a little further on this idea, as expressed in the language of Poincaré, Eddington and Milne. We shall see that the modern formal equivalent to the ancient idea of harmony is a mathematical one : group theoretical isomorphism. This is also the clue to the art of world-building in modern cosmology.

Jules Henri Poincaré (1854–1912)

« Hence, when we ask to the objective value of science, that does not mean : Does science teach us the true nature of things? but it means : Does science teach us the true relations of things? .. In sum, the sole objective reality consists in the relations of things, whence results the universal harmony. »⁶

In contrast to the theme of this conference, *La science et l'hypothèse*, which refers to the first among the four books on the philosophy of science written by Poincaré, my own reflections will primarily put focus on the second of these, viz. *La valeur de la science*, which I personally consider to be the most important of his published books, the jewel in a quartet of precious stones ; one of my reasons for preferring that book to the other is the priority it gives to time, ahead of space. My exposition of the philosophy of Poincaré is strongly influenced by Giedymin⁷ who describes his aim as follows : to examine the evolution of science and to show that there is real progress in spite of radical changes in scientific theories.

According to Poincaré, the search for truth is the sole end worthy of science. Truth must be pursued in a spirit of righteousness, without prejudice and passion. But just as nature in itself is beautiful, so the truth of nature is likewise beautiful ; if it were not, it would not be worth knowing, and life would not be worth living. Although there are worlds of difference between the passionate pursuit of beauty, the dispassionate search for truth, and the unselfish devotion to a higher purpose, these three cannot, and should not, be separated.

⁶ [Poincaré 1908].

⁷ [Giedymin 1982].

As ideal values, they are of the same kind : whosoever loves the one truly cannot help loving the other two as well. The world is one — for this reason art, science and morals belong together.

Poincaré, being more dedicated to science, speaks mostly of scientific truth. In order to reach its goal, science must strive for unity, simplicity, and objectivity. Now experience is the only source of truth and the ultimate arbiter of our theories, but it is we who decide how to search for truth, and when to trust the evidence ; our hypotheses, our criteria and our method are our own choice and responsibility. Elaborating his own version of the widely accepted hypothetical deductive method, Poincaré seeks to steer a middle course between what he considers to be extremes : the pure *a posteriori*, as propagated by the positivistic empiricism of his own time, and the pure *a priori*, as advocated by the transcendental criticism of Kant.

Contrary to the extremes he holds theories to be constructions, or artifacts. The whole enterprise of science is constructive, aiming at a reconstruction of the formal relations which inhere in nature ; in this sense, science is also descriptive. Science never bothers about particular facts, its only concern being kinds of facts ; what it describes is the order or structure of facts, not their essence or substance ; disregarding substance and matter, natural science puts focus on order and form. Science seeks regularity in order to predict ; only repeatable facts can be predicted ; the only facts of relevance to science, therefore, are those which can be repeated ; hence, the first step towards science is a preliminary classification of observables. Between the extremes, he also discusses and rejects the radical conventionalism — close to his own position — proposed by some contemporary adherents of idealism.

The object of the scientific method is the selection and treatment of facts. Scientific facts are merely common-sense facts expressed in the language of science, but the language of exact science is artificial : the formal language of mathematics. The final outcome of the process of scientific research is a scientific theory, and a scientific theory is a mathematical structure summarizing relations between facts. In his theory, the scientist expresses his knowledge of the harmony of the universe. Just as an artist selects those features of his model which perfect his picture of it, his refined

sense for congruity induces him *a priori* to select precisely those facts which conform to his preconceived ideas and hypotheses of the cosmic harmony. But it is a gross misunderstanding to believe that a scientist creates his own facts ; all that is manufactured in a fact is the formal language in which it is enunciated, and it never depends on the scientist whether his prediction of a fact is fulfilled. Therefore empirical reality remains the ultimate test of theoretical speculation.

Poincaré thereby assumes a balanced position — apparently quite traditional, but only apparently — equally far away from all excesses of nominalism or realism. It is customary to describe his position as ‘conventionalism’, although he did not use the term himself ; but this brand of conventionalism is certainly very moderate. I agree with Giedymin that the term ‘constructivism’ may seem more appropriate. It is clear, in any case, that it presupposes an empirical basis amounting to the existence of a kind of observational ‘invariant’ beneath all theoretical conventions. Invariant reality can be known up to the structural isomorphism of rival theories. Changes mostly concern ontologies and metaphors, they seldom affect structures. This enables us to make steady progress towards objective knowledge of nature, but our knowledge remains limited in the sense that objective reality can consist in no more than what can be described by a structure of group transformations.

Both physics and geometry study invariants under transformational groups. Whereas geometry studies the properties of certain ideal spaces, physics studies the temporal changes of relations obtaining between objects described in ideal spaces. Now the passage of time is real, whereas ‘space’ is merely a word that is wrongly supposed to refer to reality : real empty space simply does not exist, and what experience tells us is merely the relations holding between solid bodies. The structure of time & space is not forced upon us by nature, it is we who impose it on nature, not by *a priori* intuition, but because it is convenient. As regards space we have to distinguish between : a) solid bodies whose qualities are manifest to our senses ; b) their quantitative relationships which are measurable relative to standards fixed by convention ; c) the ideal spaces of geometry proper.

Geometry is nothing but the study of groups. It is based on assumptions, chosen with respect to their fruitfulness and

appropriateness in our description of the phenomena of physics. As definitions, postulates, and rules of inference, these premises lead to consequences derived by means of exact *analysis* (cf. Leibniz). Nevertheless, the inventiveness of mathematical construction depends on imaginative intuition, which is our only guide to fertile synthesis (cf. Kant). Geometrical space is taken to be continuous by convention and, as a continuum has no intrinsic metric, the concept of metrical congruence depends on convention. On the whole, therefore, the assumptions of geometry are founded on convention ; this proves their status to be neither empirical, nor synthetic, nor analytic.

Experiments teach us the relations of bodies to other bodies ; but they tell us nothing about the relations between bodies and space, nor about the relations between different parts of space. According to Poincaré, the only relations existing in nature are the non-metrical relations of order which are expressible in topology. Having dismissed the Kantian position, that the geometry of space is derived by synthesis *a priori*, Poincaré upheld *a priorism* as regards the foundation of analysis and did not interpret the axioms of arithmetic as implicit definitions of primitives. Thus he did not extend his geometrical conventionalism to arithmetics which he assumed to depend on strict intuition *a priori*, presupposing the whole numbers and the principle of mathematical induction ; on the contrary, he claimed that the consistency of geometry should be evaluated relative to arithmetics.

From the point of view of Poincaré it seems natural to regard geometries as linguistic frameworks rather than as experimental conjectures ; as such frameworks they cannot be put to test, but that does not imply that they are unchangeable. From the point of view of syntax a geometry is nothing but a formalized language ; such formal languages may treat of very different objects — points, lines, planes — yet they may be identical in their structure, due to group theoretical isomorphism. The choice of a geometry to correlate experiential facts is an opportunistic affair : does it provide us with the best means to solve the central problems of physics? According to Poincaré, physical reality is knowable merely up to the observational equivalence of alternative theoretical systems and their structural isomorphisms. The question therefore is : can we avoid

falsification of our physical ideas simply by constructing a new language which is formally translatable to the old one?

According to Giedymin, Poincaré subscribed to a generalized version of the so-called Duhem-Quine thesis : falsification is possible only with respect to systems of hypotheses expressed in a fixed language. Instead of blaming one or more of our hypotheses in face of contrary evidence, we may blame the experimental evidence, or we may avoid falsification by changing our language. The language of science is not fixed forever, but may be changed in response to experiment and observation. Changing the lexicon is merely a subterfuge tending to conceal the real problems. Changing the syntax of the language of science goes much deeper. It is not fruitful, however, to change our language merely in order to avoid falsification. Sometimes we have to accept facts as final : if we do not, we condemn science to barrenness. But on the whole, the language of science is based on conventional decision.

Poincaré also extended conventionalism to his analysis of the measurement of time and of the principles of physics ; this is the reason for applying the term 'conventionalism' to his epistemology as well as to his whole philosophy of science. According to Poincaré, geometrical space is invented to ensure consistency in our reasonings about solids and their relations ; spatial positions are not real properties of bodies, and bodies do not exist in real space, we merely reason as if they did. And he added : temporal dates are not real properties of what happens to bodies, but signifies merely the sum total of relations between events associated to bodies. He further exposed the simultaneity of instants and the congruence of durations to a trenchant analysis : distant simultaneity, e.g., is neither a datum of observation nor a consequence of the temporal continuum — it simply depends on convention. His conclusion was that the *metrics* of time and space are equally *amorphous*.

Since the enunciation of physical laws varies with the conventions adopted, and since alternative conventions modifies even the natural relations of these laws, it may be doubted whether there are among these laws any that can play the role of a universal invariant which is completely independent of linguistic conventions. However, if we introduce only fictitious beings having senses analogous to ours and admitting the principles of our logic, it appears

to be a plausible conclusion that their language, however different from ours, will always be capable of translation. But the very possibility of translation implies the existence of something invariant ; to translate between two different languages is precisely to disclose this invariant. The invariant laws, then, are those relating the ordinary facts, while the relations between scientific facts remain always dependent on certain conventions.

Poincaré distinguishes between 3 kinds of hypotheses : a) formal principles, b) inductive generalizations, and c) realistic interpretations. A formal principle is always a convention ; as such it is *a priori* in a relative sense, its status being reminiscent to that of a real definition in the sense of Leibniz. An inductive generalization can be regarded as an empirical law, hence subject to permanent revision, but may at a later stage be promoted to the status of principle ; such a law expresses a relation between two terms, a conceptual and a factual. If a law is elevated to the status of a principle, a third term is introduced to mediate between the first two, whereby the first relation is split up into two other relations : a theoretical one between two conceptual terms, and an empirical one between a conceptual and a factual term. A realistic interpretation of the terms is neutral, if it does not affect the formal relations between the terms, although the terms themselves be changed : the same geometry may result, whether we begin with points, or lines, or planes.

The physics of our own time is the physics of the principles, said Poincaré. Thus any law can be broken up into an *a priori* principle and an *a posteriori* law, and in this way the number of scientific principles has increased and still increases, while a conceptual structure is taking form in relative independence to experience. However far the partition be pushed, there will always remain laws which need to be tested by experience ; if not, science as we know it will be brought to an end. But we cannot satisfy all possible principles simultaneously, in face of the experimental evidence, and if a principle ceases to be fertile, experiment will have condemned it without contradicting it directly. It is useless to heap up hypotheses. The obvious reason is that if a principle is wholly exempt from being contradicted by experience, it ceases to be informative : we can infer nothing from it.

In contemporary physics, empirical generalizations are still being upgraded to theoretical conventions. From an epistemological point of view, the difference between geometry and physics is that, while all the principles of geometry are conventional, only some of the principles of physics are. But the role of principles in science appears to be growing, whereas our ability to obtain experimental results that can discriminate between theories seems to be diminishing. It is interesting to compare this view of Poincaré to the well-known position of Niels Bohr, that the description of the experimental apparatus must always be given in classical terms, even in quantum physics. Although the exact cut between object and subject, or between reality and apparatus, is made by arbitrary decision, a cut must be made. In contrast to Bohr, Poincaré seems open to a revision of classical physics.

As regards the contribution of Poincaré to Special Relativity, my own view conforms to the assessment of Keswani⁸ : Poincaré indeed had the whole theory, and he had it before Einstein, the only important difference being one of emphasis. The fundamental problem, posed by Poincaré, was this : Will not the principle of relativity, as conceived by Lorentz, impose upon us an entirely new conception of time and space, thereby forcing us to relinquish our most cherished conclusions? How could Poincaré describe the principle of relativity as being unfalsifiable, and yet consider to give it up in face of apparently negative experimental results? There is a simple answer to this question : he did not want to heap up hypotheses! But he never accepted that SR could force him to renounce his conventionalism : even space-time is conventional, and nothing like a real framework, or real entity. As a continuum, space-time has no natural metric, i.e., it is amorphous.

Poincaré was well acquainted with the work of Einstein, but did not credit him for the invention of SR. After emphasizing that the simultaneity of two events as well as the equality of two durations should be so defined that the natural laws may be as simple as possible, he credited Lorentz for having saved relativity by means of his ingenious idea of local time. Lorentz later, paying tribute to Poincaré for his great contributions to physics, praised him for having stated the relativistic transformations in their most convenient form,

⁸ [Keswani 1964 / 65].

ahead of Einstein and Minkowski. Lorentz had claimed that the forces of physics should be defined in a way which would make them invariant to his transformations. Poincaré, accepting this claim, tried to modify Newton's law of gravitation and ended up with a Lorentz-invariant action-at-a-distance theory. In the opinion of North⁹ : had the general sympathy not been so decisively in favor of a field theory of gravitation, Poincaré's memoirs might well have become a turning point in the history of the subject.

It has been pointed out by Stump¹⁰ that the burden of conventionalism is to explain the conventionality of the basic principles of science in a relationist way without relying on any arguments of underdetermination ; in order to be consistent conventionalism must explain the relational origin of both gravitation and inertia. In fact, Poincaré believed acceleration to depend merely on the external relations between bodies ; velocity and acceleration being on a par, both has to be relative. As he felt obliged to find a solution in terms of bodies and forces acting upon them, he did not consider the possibility of reducing gravitation to space-time structure. But, according to Stump, acceleration cannot be relativized without introducing universal forces, and the Einstein space-time theory of gravitation seems to have effectively disproven the possibility of a pure relationist framework.

Now Roxburgh & Tavakol¹¹ have written an important paper disclosing some hidden affinities between the gravitational theories of Poincaré and Milne. They see the great value of Poincaré's action-at-a-distance theory in the fact that it has led to the discovery of a whole family of consistent theories which cannot be geometrized in a Riemannian manifold, but only in the more complex framework constructed by Finsler. The cosmological solutions for these theories can be derived by means of a generalized version of the kinematic technique invented by Milne. Whittaker¹² has issued a warning which it is tempting to quote in this context : 'It may be unwise to accept a

⁹ [North 1965].

¹⁰ [Stump 1989].

¹¹ [Roxburgh & Tavakol 1975].

¹² [Whittaker 1953].

theory hastily on the ground of agreement between its predictions and the results of observation in a limited number of instances'.

Arthur Stanley Eddington (1882 - 1944)

« We do not accept the Kantian label — but, as a matter of acknowledgment, it is right to say that Kant anticipated to a remarkable extent the ideas to which we are now being impelled by the modern development of physics. »¹³

The scientific career of Eddington was very special : he took several degrees ; he was appointed Plumian professor of astronomy at Cambridge when he was 31 ; he made great contributions to astrophysics for which he was deservedly famous ; and he wrote a lot of books, scientific and popular, which were much acclaimed. At his height he enjoyed a public authority almost second to none ; but the style of his books, though exceedingly well written, was also provocative and earned him much opposition ; he exposed himself and became a favorite target of criticism for positivist philosophers armed with heavy irony, but with scant sense of humor. What arose the hostile feelings of many of his scientific colleagues was his insistence on the possibility of mapping the structure of the universe *a priori*.

Whittaker¹⁴ credits Eddington with the following principle : It is possible to calculate the exact values of all pure numbers, summarizing timeless relations between the basic constants of nature, by *a priori* mathematical deduction from epistemological principles. By scientific knowledge *a priori* Eddington understood knowledge prior to actual measurement, but not prior to exact specification of the operational procedures of measurement. He claimed to have expressed in symbols of mathematics what the physicist thinks he is doing when he is measuring things. Archimedes, by calculating π , the ratio of the area of a circle to its squared radius, assumed qualitative geometry in order to deduce quantitative geometry. Whittaker describes Eddington as a modern Archimedes : he allowed himself to use everything in physics except the numerical values of the constants

¹³ [Eddington 1939].

¹⁴ [Whittaker 1958].

of nature, which he claimed to deduce mathematically from epistemological principles in analogy to π .

According to Galileo, the aim of natural science is: 'to measure what can be measured and to make measurable what cannot yet be measured'. Science focuses on the quantitative aspect of nature by effecting a reduction of quality to quantity. To say that science is based on experiment and observation performed by means of apparatus is to say that it is based on counting and on the readings of pointers. Among pointer-readings, the primary readings give the intensities of the qualities to be measured, while the secondary readings give their location in time and space; the experimental setting or context is given by pointer-readings of a tertiary order. Pointer-readings, marking the coincidence of events in space-time and referring to intersections between world-lines, are quantities produced by our own operations. Thus lengths and durations are not properties that inhere in the external world: they are the relations of things in the external world to a particular observer.

All variety in the world, all that is observable, stems from the diversity of relations between entities; therefore, when we consider the intrinsic nature of the entities related, nothing is left but sameness; there is nothing in the external world which can force us to split it up in identical units, but this is our way of thinking. Eddington's theory implies that there exists only one kind of fundamental particle, of which all the rich variety of elementary particles is a manifestation in disguise. Now it is possible for a group of sensations in a mind to have the same structure as a group of sensations in another mind. It is also possible for groups of entities to display the same structure though their true nature be unknown to us and they be connected by relations unknown to us. So we can obtain structural knowledge of 'things outside ourselves'. The recognition that all physical knowledge is structural makes obsolete the dualism of mind and matter, Eddington claims.

A number of experimental observations in physics can be termed structural; it is the aim of Eddington is to construct their theory, and the important result of such a theory is an extensive unification between the different branches of physics. Eddington does not search for new laws, he wants to explain those already known, and it is the invariance under different circumstances of elementary

particles with few attributes that provides him with the ultimate numerical standards of nature. The pure numbers of nature arise as ratios of the numbers of dimensions of certain phase spaces and the task is to calculate the numbers of dimensions of such spaces. To solve this task Eddington invented his famous calculus of *E*-numbers, which is a generalized version of the even more famous Hamiltonian algebra of quaternions. Now Hamilton originally interpreted his quaternion algebra in Kantian terms as a 'science of pure time'. According to Kant, arithmetic maps the structure of our intuition of time, whereas geometry maps the structure of our intuition of space. Eddington, by analogy, viewed *E*-algebra as 'the science of space-time'.

According to Yolton¹⁵ the modern edifice of natural science has developed so far that most of the relevant data in many fields have already been collected ; the remaining task is to unify them and to formulate them in a deductive system. When new data emerge, it is sometimes found that another system is required ; however, temporary set-backs cannot block the general trend towards unification. So detailed laws governing the quantitative results of observation are inferable solely from our operational specification of the relevant observational procedures, and fundamental hypotheses can often be replaced by epistemological principles. What cannot be foreseen are those facts which distinguish the actual universe from other possible universes obeying the same laws ; they are not given once and for all, but are being born continually ; hence the actual course of events is unpredictable. Yolton opines that Eddington has made no 'real' *a priori* deduction : the codes of empirical science are not violated by putting focus on its theoretical aspects.

Eddington has had quite a number of important followers joining his search for a deductive explanation of the strange numerical coincidences of the universe : Paul Dirac, Pascual Jordan, Erwin Schrödinger, Hermann Bondi, Peter Landsberg. A small society, ANPA, standing for *Alternative Natural Philosophy Association*, has been formed by scientists devoted to the quest for explaining these numbers, and Eddington has recently been conferred a posthumous honorary membership. Ted Bastin & Clive Kilmister, prominent members of ANPA, have written a series of papers on

¹⁵ [Yolton 1960].

Eddington's *Fundamental Theory*, which has since been rounded off by the monograph of Kilmister & Tupper¹⁶ on *Eddington's Statistical Theory*. Assuming the quantitative aspects of the universe to be finite and discrete in a fundamental sense, David McGoveran of ANPA has recently used binary algebra and computer theory to improve the combinatorial hierarchy of F. Parker-Rhodes, now accepted as the common ground and point of departure for further research.

Edward Arthur Milne (1896 - 1950)

« The so-called principle of induction .. has no content ... It is a piece of outmoded furniture, and in fundamental investigations it had better be scrapped »

« There is no entity 'physical space' ; there is only the abstract space chosen by the physicist as a structure in which to plot phenomena ; and some choices give simpler theorems than others (thus making the laws of nature look simpler). »

« The essence of scientific freedom is the right to come to conclusions which differ from those of the majority. »¹⁷

As a scientist Milne never attained the fame or prominence of Eddington, nor did he become victim of so bitter and fierce an opposition ; but that in itself does not make him less interesting, nor less important. His feat as a cosmologist was to construct an exceedingly simple model of the universe implying the uniform expansion, in accordance with a cosmological principle demanding a specific type of cosmic symmetry, of a primary substratum of so-called fundamental particles. He further showed that the superposition of a secondary set of arbitrarily moving accidental particles on this substratum must give rise to spontaneous accelerations. In this way he fulfilled the relationalist program of Poincaré.

The main idea of Milne is that the laws of nature can be deduced rationally, starting from an individual observer's awareness

¹⁶ [Kilmister / Tupper 1962], [Bastin / Kilmister 1995].

¹⁷ [Milne 1951].

of a temporal sequence of events and his tempo-spatial coordinates as defined by means of radar-signals and clocks. His central hypothesis, that the laws of nature are akin to geometrical theorems, places him nearly on line with Eddington ; but Milne did not depend on Eddington. His claim, that it is possible to deduce the inverse square law of gravitation, together with the sign of gravitation, *a priori* from some very simple premises that effectively reduce gravitation to inertia, sounds shocking to most scientists. What arguments did he adduce in support of this startling point of view?

At the dawn of history, the theorems of geometry were considered to be principles of nature. All we know of Egyptian mathematics, at least, is consistent with the view that the Egyptians regarded regularities like those summed up in the Pythagorean theorem as laws of nature. These regularities were discovered by drawing up different triangles, measuring them, and experimenting with them ; as observational laws they were nothing but brute facts. The Greeks, by deducing these laws from combinations of simpler statements, postulates, or axioms, later turned geometry into an exact science wherein everything depends on pure theory. They thereby showed the possibility of eliminating brute fact from science.

In modern presentations of geometry, the axioms are neither brute facts nor statements which may be true or false ; instead they are definitions, i.e., minimal descriptions of what we are talking about ; they delimit the subject of investigation. The theorems which are derived from the axioms are valid precisely if the process of inference contains no flaw ; their truth, then, does not depend on verification. The tendency of all exact sciences is to pass from the Egyptian inductive phase to the Greek deductive phase ; the only question is how far this can be carried out. The extent to which the process can be carried out is simultaneously our measure of the degree to which we can regard the universe as rational, says Milne.

The laws of geometry are derivable by pure deduction ; this is evident to all. Why not assume that the laws of dynamics are also derivable by pure deduction? Whether strict deduction is possible is a question that cannot be decided *a priori* ; one cannot begin by stating a program of this kind, and then just carry it out, it derives from the a posteriori experience of pushing deduction as far as possible. When we introduce fully operational definitions, a just-sufficient description

of the relations obtaining between real entities is provided ; further appeal to brute fact is unnecessary because our principles cannot be verified by induction, but experience is needed to test whether a specific model of the universe is realized in nature.

Together with McCrea, Milne undertook to revive the classical cosmology of Newton in a climate completely dominated by the ideas of Einstein¹⁸ ; how far this development can be taken has been shown by Landsberg & Evans¹⁹. More interesting to our discussion, however, are the brilliant attempts of Walker and of Schutz to transform the kinematics of **SR** into an exact deductive science. Their investigations have in a convincing way disclosed the unique significance of the method of radar-signaling to enlighten problems of modern relativity theory ; but to achieve the same for dynamics as for kinematics is a great step, indeed.

Further, it is a question whether the results obtained by Walker and Schutz have benefited sufficiently from the ingenious conventionalistic ideas of Poincaré. What they have obtained is a mapping of the topology of current relativity theory — what remains to be done is to expose the conventionality inherent in the metric. This reflects, for instance, on the standard definition of simultaneity at a distance : I believe that Poincaré would have welcomed an attempt to show that Einstein's dissolution of distant simultaneity is not an inevitable consequence of topology, but depends on his conventional choice of space-time metric.

Milne's technique of radar-signaling has been further developed and refined by Whitrow²⁰ and Walker²¹ as well as Törnbohm²² and Schutz²³. A popular version of the same method is found in the so-called k-calculus of Bondi. Lucas²⁴, referring to Whitrow & Milne, approves 'their transcendental derivation of the Lorentz transformations' as probably the best of all possible ways in which

¹⁸ See [North 1965].

¹⁹ [Landsberg / Evans 1977].

²⁰ [Whitrow 1961].

²¹ [Walker 1935] & [Walker 1948].

²² [Törnbohm 1957] & [Törnbohm 1963].

²³ [Schutz 1973].

²⁴ [Lucas 1973].

co-existing Leibnizian monads could re-establish their lost harmony. According to North²⁵ the independent derivations, by Robertson and by Walker, of the so-called **RW**-metric are based on assumptions inspired by Milne.

In cosmology, we push the deductive aspect as far as possible. Now it seems as if Milne was able to make his mathematics yield more than he had put into it : his output appears to exceed his input. If this is right, then it is no longer true that only synthetic propositions contain new knowledge ; analytic propositions may also do that when they add to their premises the leaven of the deductive process. If we assume that the universe is rational, it need not be in vain to hope that the constants of nature can be deduced *a priori*, as contended by Eddington.

The hard-baked, or hard-boiled, scientist will traditionally hold that science and religion, whilst on nodding terms, have no immediate bearing on one another. On the contrary, Milne says one cannot study cosmology unless one has a 'religious attitude' to the universe. Cosmology presupposes the rationality of the world, but it can give no reason for it except the origin of nature being a Rational Creator. To Milne, the Creation of our universe remains the ultimate irrationality.

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²⁵ [North 1965].

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