MATHÉMATIQUES ET SCIENCES HUMAINES

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Parameters in collective decision making models : estimation and sensitivity

PARAMETERS IN COLLECTIVE DECISION MAKING MODELS: ESTIMATION AND SENSITIVITY[1]

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RÉSUMÉ — Paramètres des modèles des processus de formation de décisions collectives

Les modèles de simulation des processus de formation de décisions collectives sont fondés sur des aperçus théoriques et empiriques du processus de décision mais contiennent des paramètres dont les valeurs sont déterminées ad hoc. Certains de ces paramètres, du modèle d'accès dynamique sont discutés et il est proposé de compléter la fonction d'utilité par un terme aléatoire dont la variance serait un paramètre inconnu. Ces paramètres peuvent être estimés en confrontant aux données les prévisions du modèle qui peuvent être des décisions mais aussi des structures relationnelles générées comme un élément du processus de prise de décision. Etant donné la nature stochastique du modèle, l'estimation de ces paramètres peut être obtenue par l'algorithme de Robins Monro. Cet ajustement n'est pas absolument direct. Il faut choisir les statistiques sur lesquelles se fonde l'estimation des paramètres. Il n'est pas certain a priori que l'équation d'estimation admette une solution et que l'algorithme de Robins Monro converge. La méthode est illustrée par des données concernant la restructuration financière d'une grande société.

SUMMARY — *Simulation models for collective decision making are based on theoretical and empirical insight in the decision making process, but still contain a number of parameters of which the values are determined ad hoc. For the dynamic access model, some of such parameters are discussed, and it is proposed to extend the utility functions with a random term of which the variance also is an unknown parameter. These parameters can be estimated by fitting model predictions to data, where the predictions can refer to decision outcomes but also to network structure generated as a part of the decision making process. Given the stochastic nature of the model, this parameter estimation can be carried out with the Robbins Monro process. Such fitting is not completely straightforward: statistics must be chosen on which to base the parameter estimation, it is not certain a priori that there will be a solution to the estimating equation and that the Robbins Monro process will converge. The method is illustrated with data from the financial restructuring of a large company.*

1. INTRODUCTION

In recent years, several formal models have been developed for collective decision making. The primary aim of these models has been to yield predictions for the decision made. Early models were developed by Coleman (1972) and Laumann & Knoke (1987). Building on their approaches, the idea was elaborated that the actors who participate in decision making processes should be regarded as members of *policy networks*. The policy network idea was incorporated in the Two Stage Model (2S model, Stokman & Van Den Bos, 1992; Stokman, 1994; the model also is explained in Degenne & Forsé, 1994) and the Dynamic Access Model

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(DA model, Stokman & Zeggelink, 1996). The last-mentioned model is the focus of this paper, and it is explained below.

The evaluation of these decision making models has focused up to now on the correctness of the predicted decision outcome. The DA model, however, pretends not only to give a reasonably trustworthy prediction of the decision, but also to model the relation forming process associated with the decision making. From this point of view, it becomes important to consider the details of the specification of the decision-making models. These models can be regarded as translations of political insights into mathematical formulae. Diverse insights correspond to alternative formulae and to alternative values of the crucial parameters in the formulae. However, there always remain a number of parameters of which the specification is not dictated by available political insights and theory. In the past, values for these parameters have been determined *ad hoc*, often by trial and error. *In this paper we consider the sensitivity of the models for the values of these parameters, and a method for estimating these values so that certain results of the model* (e.g., network characteristics) *correspond optimally to the empirical observations.* Such an estimation method was proposed for the case of one parameter in Snijders, Stokman & Zeggelink (1996). The present paper elaborates the simultaneous estimation of several parameters for dynamic simulation models. The estimation method was proposed for other network models in Snijders (1996), and can also be used for other computer simulation models. In this way, the collective decision making models obtain a second aim: to predict and explain the influence network that is generated by the decision making process.

The starting point for each decision making model is that one or more policy *issues* are given, on each of which a decision has to be taken. The 2S model can be applied to one or more issues; if several issues are treated, the 2S model is applied to these independently. In this model, it is assumed that each actor has a given *policy position* on each issue, and that he prefers the decision to be as close as possible to this position. The *salience* of the issue indicates how important this issue is to him. The power of each actor is distinguished in three elements: his *resources*, his *access* to other actors, and his *decision or voting power*. The decision process is modeled as a process with two stages. In the first stage (*influence stage*), actors try to convince other actors and influence their policy positions. Whether this is possible depends on the access between the actors; the success of an influence attempt depends, a.o., on the actors' resources and on the salience of the issue for the actors. This stage can be iterated. In the second stage, the *decision stage*, the actors with voting power take the decision, voting according to the policy positions obtained during the first stage. The decision taken depends on the voting rule; e.g., it can be the weighted mean of the final policy positions, weights being proportional to voting power.

To apply the 2S model, it is necessary to have empirical data on the following elements: policy positions, saliences, and voting power of actors on each issue; the resources of the actors; and the access network. The 2S model has been tested extensively. Its explanatory power has been investigated by predicting decision outcomes and comparing these to the real decisions. Predictions have been made on a local level (Berveling & Roozendaal, 1992; Berveling, 1994a), on a national level (Schonewille, 1990; Stokman & Van Den Bos, 1992; Stokman, 1994; Berveling, 1994b) and on the European level (Van Den Bos, 1991).

When applied to decision making about several issues, the 2S model treats the issues independently. It was extended to models for jointly taking decisions on several issues, where "give and take" and mutual influence processes operate between the actors. Such models are the Exchange model (Stokman & Van Oosten, 1994; not treated here) and the Dynamic Access model.

The access network is regarded as fixed in the 2S model. In the DA model, on the other hand, account is taken of the fact that actors actively shape their networks, trying to increase their influence on other actors. Stokman & Zeggelink (1996) developed dynamic simulation models in which actors can establish as well as terminate influence relations. Actors base their influence activities on the limited information and limited foresight they have about the consequences of their actions. Stokman & Zeggelink specified two alternative basic models, each corresponding to a known view on the political process. In the *Control Maximization* (CM) model, actors simply try to create relations with the most powerful actors in the network. In the *Policy Maximization* model, actors are more sophisticated and make a rough estimate about the effects of their possible relations on the expected decision outcomes. The present article presents only the Policy Maximization version of the Dynamic Access model. This version appeared the most promising in Stokman & Zeggelink (1996) and Stokman & Berveling (1996).

The structure of the paper is the following. First a basic description is given of the decision making process with respect to the restructuring of the AVEBE company; this case is used as an example to illustrate the proposed methods. Subsequently the DA model is specified in its Policy Maximization version. In Section 4, some alternative specifications are mentioned. These specifications contain parameters of which the value is not dictated by political insights but should be made on empirical grounds. The addition of some random elements is important in one of these alternative specifications. The Robbins Monro process can be utilized in an algorithm for estimating the unknown parameters; this is explained in Section 5. Finally the application to the AVEBE company is elaborated, and conclusions are drawn about the parameter estimation method and the decision making process.

2. EXAMPLE: THE AVEBE

We illustrate the Dynamic Access model with a small empirical example, namely *the financial restructuring of a large Dutch company (AVEBE)*. The explanation of this decision making problem is taken from Stokman & Zeggelink (1996). The issue domain consists of eighteen actors and three decisions. The AVEBE is a cooperative company of farmers in the northern part of the Netherlands that produces potato derivatives. The company, already in financial problems since the mid-Sixties, got serious financial problems in the mid-Eighties. Its survival was of very high importance for the northern part of the Netherlands. Not only did it employ a large number of people, but its financial bankruptcy would have serious financial consequences for the farmers because of their unlimited liability in the cooperative firm. Three main issues were at stake. The company's own capital was almost completely lost and had to be restored. Moreover, its survival required a considerable reduction of its debt. Finally, the company was seriously polluting the environment and high investments had to be made to adapt the pollution to the more stringent norms of the Dutch government. In January 1986, the company asked the Dutch government for an interest-free and redemption-free loan to reduce its debt by Dfl 200 million. Moreover, it asked for a financial arrangement for environmental investments of Dfl 80 million. These matters were delegated to an Advisory Committee for Financial Restructuring, called (after its chairman) the Committee Goudswaard. In actual practice, this Committee had the ultimate power to decide on the matter. The Committee came with its advice in the Summer of 1986.

In our model, the Committee Goudswaard was given full voting power as it was *de facto* the final decision making body. The other data were obtained by interviewing two experts. These data concern the actors with their resources and mutual access relations, and the policy positions and saliences of the actors regarding the three issues. The two experts generated the same list of actors with only one exception. They agreed to a very large extent on all variables

to be used in our models. We therefore use the mean values of the two experts on the variables (see Tables 1 and 2). Clear differences in reports were obtained only in the access networks. As the specific knowledge of the two experts was complementary (one had expertise at the national, the other at the regional level), their access data were combined. This means that we assume an access relation between two actors to exist in the empirical network if at least one of the experts reported such a relation.

3. SPECIFICATION OF THE DYNAMIC ACCESS MODEL

From Stokman & Zeggelink (1996) we repeat the basic definitions for the DA model. Further background motivations are to be found in the mentioned paper.

Starting point of the model is an *issue domain* consisting of a set of n actors $(i, j, k = 1, ..., n)$ and a set of m issues or decisions $(f, g = 1, ..., m)$. The decision process is time dependent, with discrete time parameter $(t = 1, 2, ..., T)$. The final decision is taken at time T. (Time-dependence of variables always is indicated explicitly by the index t.) It is assumed that decisions are made according to a given procedure characterized by the actors' *voting powers*; the voting power of actor i on issue f is denoted v_{if}. If $v_{if} > 0$ for some f, actor i is called a public actor. In most cases there are only few such actors. All other actors are called private actors (although they may be public actors in other meanings of this term). Voting powers are normalized in the sense that for each issue f,

$$v_{if} \geq 0, \quad \sum_{i=1}^{n} v_{if} = 1 \ .$$

The possible decisions on each issue are assumed to be one-dimensional, i.e., they are characterized by real numbers. The most extreme possibilities are represented by the numbers 0 and 1. Actors have preferences regarding the outcomes of the decisions. The preferred outcome of a decision for an actor is called his *policy position*, and is a number between 0 and 1. The initial policy positions of the actors, x_{if}^0, are supposed to be given. Actors influence each other during the decision making process, so that their preferred policy positions are time-dependent, denoted by x_{if}^t. The final decision y_f on each issue is supposed to be the weighted mean of the final policy positions of the actors:

$$y_f = \sum_{i=1}^{n} v_{if} x_{if}^T \ . \tag{1}$$

This also is a number between 0 and 1.

Influence activities of the actors spring from the utility they attach to the various possible outcomes. On each issue f, actors have a *single-peaked utility function*, depending on the distance between the hypothetical outcome y_f and the actor's policy position x_{if}^t. In order to obtain comparability of the utilities for different issues, the model also includes *salience parameters* s_{if} that indicate how important issue f is to actor i. These are not time-dependent. Saliences for each actor are restricted by

$$s_{if} \geq 0, \quad \sum_{f=1}^{m} s_{if} \leq 1 \ .$$

(The total salience for an actor may be less than 1, because his main interests may be outside the issue domain under consideration.) In this article, the utility function is assumed to be

$$U_i^{\ t}(x_f) = -s_{if} \mid x_f - x_{if}^t \mid .$$

The total utility over all m issues for actor i is assumed to be the sum of his utilities over all issues (cf. Baron, 1991):

$$U_i^t(x) = \sum_f U_i^t(x_f) .$$ (2)

The last two formulae are not used directly in the model, but indirectly via the expected utilities defined below in Section 3.1.

The actors base their influence requests on the expected outcomes of the decisions. Access relations are assumed to be established when one actor makes an access request, and this request then is accepted by the other actor. It is assumed that actors are aware about the current policy positions of all actors, and at time t $(1 \le t \le T)$ the expected outcome for issue f, in accordance with (1), is given by

$$y_f^t = \sum_{i=1}^n v_{if} \, x_{if}^t .$$ (3)

Actors have different capabilities to influence each other in the stage before the final vote. One element of this capability consists of the actor's *access* to other important actors and the *resources* he can mobilize in these relations. The resources actor i can mobilize in the influence stage are denoted r_i $(0 < r_i \le 1)$. Resources can include, e.g., information and funds. Resources often result from long term investments, and therefore are not seen as time-dependent. Access relations, however, are time dependent in our models. They are purposively changed by the actors in the course of the decision process in order to exert influence. How this is done will be explained below. The access network is defined by the matrix A^t with elements a_{ij}^t, with value 1 if actor i has access to j at time t, and 0 otherwise. It is assumed that actor i has full access to himself $(a_{ii}^t = 1)$. The access network is not necessarily symmetric. The access network is assumed to be issue-independent. In practice this means that the issue domain must be sufficiently homogeneous.

The effect of established influence relations depends on the salience of the issue for the influencing actor as well as on his *potential control* (or potential influence) over the target actor. It is assumed that the potential control of actor i over actor j depends on the relative size of i's resources, compared to the resources of others who have access to j :

$$c_{ij}^t = \frac{r_i \, a_{ij}^t}{\sum_{k=1}^n r_k \, a_{kj}^t} .$$ (4)

It is assumed that within each time period, influence requests are made simultaneously. Each actor's policy position is the weighted mean of the positions of those influencing him (including the actor himself), where the weights are the product of the potential control coefficients and the saliences for the influencing actors:

$$x_{if}^{t+1} = \frac{\sum_{j=1}^n x_{jf}^t c_{ji}^t s_{jf}}{\sum_{j=1}^n c_{ji}^t s_{jf}}$$ (5)

Actors, issues, voting power, initial policy positions, saliences, and resources are exogenous elements in our models. Voting power usually is determined by investigation of the formal decision making procedures. The other elements are usually obtained either by interviewing experts or representatives of the actors involved in the issue domain. For this purpose special interviewing techniques have been developed (see Bueno de Mesquita and Stokman, 1994).

Access relations among actors are generated as endogenous elements in the model. In the present versions of the models, the actors start from scratch when requesting access relations. This article assumes that no access relations exist at $t = 0$, but an interesting extension would be to consider certain institutionalized access relations as being given at $t = 0$. Establishment and shifts of access relations and their consequences for outcomes of decisions are the main focal points in the DA model.

3.1. Decisions about access requests

In the influence stage, the actors try to realize a more favorable decision outcome by getting access to other actors and thereby shifting, directly and indirectly, the policy positions of public actors. Qualitatively, the process can be sketched as follows; a more precise description is given below.

The influence stage consists of consecutive rounds (or iterations, or time periods). In each round, some access relations may be established. Each round consists of three steps. In the first step, actors make those access requests that would have the highest utility if accepted and that are most likely to be accepted. Since access requests and access relations require time and resources, actors are able to make only a limited number of access requests to other actors. In the next step, actors evaluate the access requests they received. If the number of requests is larger than they can handle, they accept only some of them. We assume that actors learn through experience. Successes or failures of previous access requests are used to adapt their estimate of the likelihood of acceptance of new requests. The third step is that actors shift their policy positions according to formula (5).

Limitations are set to the number of access requests made, and the number of access requests accepted. Stokman & Zeggelink (1996) gave three considerations that set limits to these numbers: attempted influence requests and established influence should be in balance; influence attempted as well as accepted increases with resources; and public actors must accept more influence when their voting power is higher. To express the last effect, *relative voting power* is defined as

$$\tilde{v}_{if} = \frac{v_{if}}{\max_j v_{jf}} .$$

The upper bound to the number of access requests made in time period $t+1$ depends on the resources and the number of established relations in the previous round. For actor i, the sum of already established outgoing relations and the number of new access requests may not be higher than

$$a_{i+}^{t+1} \text{ (max)} = [0.5 \, nr_i + 0.5 \, a_{+i}^t] . \tag{6}$$

Each actor is required to accept a given number of access requests, if this number of requests is made:

$$a_{+i}^{t+1} \text{ (norm)} = [0.25 \, nr_i + 0.25 \, \tilde{v}_{if} + 0.5 \, a_{+i}^t] . \tag{7}$$

In these formulae, [x] denotes x rounded to the nearest integer. In the applications considered, voting power is not issue-dependent, so the required number (7) does not depend on the issue f (in spite of the f occurring as a subscript).

Of several models for the decisions about which access requests are made, the *Policy Maximization Model* turned out in Stokman & Zeggelink (1996) to give the closest correspondence between predicted and observed decision outcomes. In this specification, actors use their knowledge on others' policy positions and the expected outcome y_f^t of the decision. Due to limitations on the information available to actors and on their capability for predicting the effects of their influence requests, the access part of the model is based on heuristic rules followed by the actors rather than on the assumption that they carry out a fullblown utility maximization. The set of access relations is reconstituted in each round, i.e., access relations that exist in round t are not automatically renewed in round $t+1$.

In round $t+1$, the access relations established in the previous round, given by those (i,j) for which $a_{ij}^t = 1$, have a priority but not an absolute one. Denote by A_i^t the set of actors j for which $a_{ij}^t = 1$. This set has a_{i+}^t elements. In the first step of round $t+1$, access requests are made. If the current out-degree a_{i+}^t is equal to (6) (case A), then actor i does not propose any changes, and makes requests to the actors in A_i^t. If a_{i+}^t is less than (6) (case B), requests will be made to all actors in A_i^t and to some additional actors; if it is more than (6) (case C; this is very rare), requests will be made to a subset of A_i^t. In cases B and C, actor i makes requests to those actors outside (for case B) or within (case C) A_i^t, from access requests to whom he expects the highest utility. The expected utility is given by formula (9) below, and explained as follows.

It is assumed that actor i evaluates the direct effect of a succesful access request from i to j on j's policy position as $s_{if}(x_{if}^t - y_f^t)(x_{if}^t - x_{jf}^t)$. This is positive if moving x_{jf}^t into the direction of x_{if}^t would also move y_f^t into the direction of x_{if}^t, and negative otherwise. The effect on the final decision also depends on j's total control c_{j+}^t and his voting power v_{jj}. These effects are combined in the factor $(c_{j+}^t + v_{jj})$. Further, the perceived utility of the access request is weighted by the control c_{ij}^t of i over j. Together, this leads to the perceived utility given by

$$PU_i^t(a_{ij}) = c_{ij}^t \sum_{f=1}^{m} s_{if}(c_{j+}^t + v_{jj})(x_{if}^t - y_f^t)(x_{if}^t - x_{jf}^t). \tag{8a}$$

There is one exception to this formula, namely for actors whose policy position is equal to the expected outcome on all issues. If this exception were not made, such actors would not make any access requests at all. It is assumed that these actors wish to obtain support for the decision by obtaining access to extreme actors, hoping to moderate their preferences. This is realized by the modification

$$PU_i^t(a_{ij}) = c_{ij}^t \sum_{f=1}^{m} s_{if}(c_{j+}^t + v_{jj}) |x_{if}^t - x_{jf}^t| \quad \text{if } x_i^t = y^t. \tag{8b}$$

The expected utility of an access request, p_{ij}^t is the product of the perceived utility and the perceived likelihood of success. Actor i estimates the likelihood of acceptance of an access request by actor j at time $t = 0$ as follows:

$$p_{ij}^0 = \max \{0.1 \, [10 \, (1 - \frac{1}{m} \sum_{f=1}^{m} \sqrt{|x_{if}^0 - x_{jf}^0|})], 0.1\} \ .$$

Note that possible values of this formula are $0.1, 0.2, ..., 1.0$. Further it is assumed that actors learn through experience. If an access request of actor i to actor j is not accepted by actor j, actor i will reduce p_{ij}^t in the next iteration step by 0.1 until p_{ij}^t reaches its lower bound of 0.1. In all other cases, $p_{ij}^t = p_{ij}^{t-1}$ for $t > 0$.

As a conclusion of these considerations, the expected utility of an access request is perceived by actor i to be

$$EU_i^t (a_{ij}) = p_{ij}^t PU_i^t (a_{ij}) \ . \tag{9}$$

The rule for accepting access requests is simpler. At iteration step $t+1$, if the number of new requests plus the number of incoming relations at time t is less than the norm (7), all incoming requests are accepted. If this sum is higher than (7), the actor accepts the requests made by those whose policy positions are closest to his own. The various issues are combined by the function

$$\sum_{f=1}^{m} s_{if} (1 - \sqrt{|x_{if} - x_{jf}|}) \ . \tag{10}$$

Actor i accepts the new access requests made with the highest values on this function, up to the number of new requests that he has to accept, a_{+i}^{t+1} (norm) - a_{+i}^t .

Both in making and in accepting access requests it is possible that there occur ties in the values (9) and (10) that determine which requests are made and accepted. If ties occur, a random selection is made among the tied values. This gives a small degree of randomness to the model.

4. ALTERNATIVE MODEL SPECIFICATIONS

The DA model as presented in Section 3 is based on political insights and theory, but also contains elements that are based on *ad hoc* choices. This article considers the following alternative specifications:

i. The coefficients in the bounds (6) and (7) are allowed to vary.
ii. A further random element is included in the models for making and accepting access requests.

These specifications introduce additional parameters in the model that are *a priori* unknown and have to be estimated from data about the final outcome and of intermediate outcomes of the decision making process. The most relevant intermediate outcome is the access network that has formed at the end of the process. The unknown parameters are denoted by θ_h.

In Snijders, Stokman and Zeggelink (1996), extension (i) was elaborated, allowing only of a one-dimensional parameter θ . Parameters are included in (6) and (7) because the values of the coefficients in these bounds were determined in Stokman & Zeggelink (1996) in a rather arbitrary way. The present article allows several unknown parameters, but it is not advisable to make their number too large. This would lead to imprecise parameter estimates and to a risk of overfitting (i.e., of finding parameter estimates that reflect unimportant peculiarities in

the data rather than the structure of the decision making process). In view of this restraint, one parameter is included in each of formulae (6) and (7). The following bounds are considered:

$$a_{i+}^{t+1} \text{ (max)} = [\theta_1 \ (nr_i + a_{+i}^t)] \, , \tag{11}$$

$$a_{+i}^{t+1} \text{ (norm)} = [\theta_2 \ (0.5 \ nr_i + 0.5 \ \tilde{v}_{if} + a_{i+}^t)] \, . \tag{12}$$

Of course, every actor can make and accept no more than n-1 access requests. Therefore, if the right hand sides of (11) and (12) are greater than or equal to n-1, further increase of θ_1 or θ_2 will not lead to any real changes in the specified model.

Extension (ii) is based on the idea that formulae (9) and (10) capture only incompletely the determinants of the access process. Other, unknown factors can also play a role. These other factors can be represented as random terms added to (9) and (10), respectively. This leads to a model in the spirit of *random utility models*, cf. Maddala (1983, chapter 3). Of course, (9) and (10) are not utilities in a strict sense, but they do guide actors' behavior at intermediate steps in the decision making process and therefore may be viewed as "intermediate utilities". Those access requests are made which have the highest positive values of

$$p_{ij}^t \ PU_i^t \ (a_{ij}) + V_{ij}^t \, , \tag{13}$$

up to the maximum given by (6) or (11). Here V_{ij}^t are random disturbances. Further, those access requests are accepted which have the highest positive values of

$$\sum_{f=1}^{m} s_{if} (1 - \sqrt{|x_{if} - x_{jf}|}) + W_{ij}^t \, , \tag{14}$$

up to the maximum given by (7) or (12). The W_{ij}^t also are random disturbances. If the disturbance terms have probability distributions that can assume arbitrarily large values, the effect of adding the disturbance term is that any of the access requests could be chosen, but the probability of choosing them is an increasing function of the value in (9) or (10), respectively. It will be assumed that the random terms V_{ij}^t for different i, j, and t, are independent and identically distributed with an expected value of 0. The same is assumed for the W_{ij}^t. If the variance of V_{ij}^t, or of W_{ij}^t is large compared to the variability of (9) or (10), then there are important elements in the access process not captured in (9) or (10), respectively. The precise form of the distribution of V_{ij}^t and W_{ij}^t will not have very great consequences for the modeling results. One possibility is to assume normal distributions with means 0 and variances θ_3 for V_{ij}^t and θ_4 for W_{ij}^t. Values $\theta_3 = \theta_4 = 0$ yield the original model with access requests made and accepted according to (9) and (10). Very high values for θ_3 and θ_4 mean, respectively, that making access requests, or accepting them, happens at random.

4.1. THE GENERATED ACCESS NETWORK

In each iteration step t from 1 to T, a new access network is generated. There are several possibilities for defining the total network generated by the process (the stochastic "predicted" network). One possibility is to take the last: $a_{ij} = a_{ij}^T$. Another possibility is to take the

union of all step-dependent networks: $a_{ij} = \max_t a_{ij}^t$. This can be interpreted in the following way. Contacts are being established during each step in the decision making process because actors try to influence each other, and a number of these access requests are accepted. Each time that an access request is succesfully made, this contributes to the relation between the two actors involved. However, in order to obtain correspondence with earlier applications of the DA model to the AVEBE data, this paper follows the first possibility.

The data collection may be formulated in such a way that the observed network is necessarily symmetric, i.e., $a_{ij} = a_{ji}$ .This was also the case for the AVEBE data. This can be reflected in the model by symmetrizing the predicted network. Therefore, the final matrix generated by the dynamic process is defined here as $a_{ij} = \max \{a_{ij}^T, a_{ji}^T\}$. It should be noted that this symmetrization does not play a role in the modeled influence process.

5. PARAMETER ESTIMATION

The extension of the Dynamic Access model presented above has four unknown parameters. The original specification of the DA model corresponds to $\theta_1 = \theta_2 + 0.5$, $\theta_3 = \theta_4 = 0$. These parameters can be estimated from available data; if the data is not informative enough, some of the parameters could be given fixed values while the others are estimated. In this article we treat *moment estimation*, based on a set of *selected statistics*. The idea is that if k parameters are to be estimated the researcher selects k statistics observed for the decision process. These can be outcomes on one or more issues as well as characteristics of the access network that has been formed during the decision making process. If several decision making processes have been observed that are so similar that it may be assumed that they are governed by the same parameters (note that this is indeed assumed by the original DA model), then the statistics can be averaged over the various decision making processes. The parameters are estimated so that a good fit between prediction and observations is obtained with respect to these statistics. More specifically, indicate the statistics by Z_1 to Z_k, with observed values z_1 to z_k. The distribution of the vector $Z = (Z_1, ..., Z_k)$ depends on the parameter $\theta = (\theta_1, ..., \theta_k)$. The choice of the vector of statistics Z will be discussed below.

One of the basic methods for constructing statistical estimators is the method of moments (see, e.g., Bowman & Shenton, 1985). The moment estimate for θ is the value of θ that solves the equation

$$E_\theta Z_h = z_h \qquad (h = 1,...,k) . \qquad (15)$$

This estimated value is denoted $\hat{\theta}$. Under the assumption that the specified model correctly describes the decision making process, an approximate expression for the covariance matrix of $\hat{\theta}$ can be derived using the delta method (see, e.g., Bishop, Fienberg & Holland, 1973, section 14.6) together with the implicit function theorem. The result is

$$\mathrm{Cov}(\hat{\theta}) \approx (D_\theta^{-1})' \Sigma_\theta D_\theta^{-1} , \qquad (16)$$

where

$$\Sigma_\theta = \mathrm{Cov}_\theta (Z) ,$$

$$D_\theta = \frac{\partial}{\partial \theta} E_\theta Z = (\partial E_\theta Z_i / \partial \theta_j)_{ij} .$$

5.1. Robbins-Monro process

The problem with this moment estimator is that $E_\theta Z$ cannot be calculated in a straightforward way, because Z_h is the stochastic result of a simulation model. However, the Robbins-Monro method (1951) (for an introduction see, e.g., Duflo, 1990 or Ruppert, 1991), or variants of it, can be used to compute approximations to the moment estimates in arbitrary precision. This was proposed for other dynamic network models by Snijders (1996). To explain the Robbins-Monro method, first consider the case where no stochastic element is involved. If our simulation model were a deterministic one, we could write $Z(\theta)$ for the value of Z for a given parameter value θ, and we would have to solve the equation $Z(\theta) = z$. This could be done by the well-known Newton-Raphson method, starting at a suitable starting value $\theta^{(1)}$ and with iteration step

$$\theta^{(N+1)} = \theta^{(N)} - \{D(\theta^{(N)})\}^{-1} (Z(\theta^{(N)}) - z) ,$$

where $D(\theta) = \partial Z(\theta) / \partial \theta$. In the Robbins-Monro method, the exact value $Z(\theta^{(N)})$ is replaced by a random variable $Z_N(\theta^{(N)})$ which has the distribution of Z for parameter value $\theta = \theta^{(N)}$. The randomness is dealt with by a factor $1/N$ in the iteration step. The iteration step is defined as

$$\theta^{(N+1)} = \theta^{(N)} - \frac{1}{N} D_N^{-1} (Z_N(\theta^{(N)}) - z) . \tag{17}$$

The matrix D_N is a suitable matrix. The optimal choice is $D_N = D(\theta^{(N)})$. It is discussed below how D_N can be chosen in practice.

As a good approximation for the moment estimate, one can use $\theta^{(N)}$ for a value of N when convergence may be assumed. Because of the random nature of the Robbins-Monro process, a more efficient approximation is the average of $\theta^{(M)}$ to $\theta^{(N)}$, supposing that the iteration process has been "circling around" the true value from at least the Mth step onwards, for a sufficiently large value of $N - M$.

5.2. Selection of statistics

For the efficiency of the moment estimator and good convergence properties of the Robbins-Monro process it is important that statistics Z_h $(h = 1, ..., k)$ be chosen that carry information about the parameters θ_h $(h = 1, ..., k)$. With an incorrect choice of the statistics, it is possible that the moment equation (15) does not even have a solution. Sufficient statistics as defined in mathematical statistics are impossible to find in this model. Even proving, e.g., that $E_\theta Z_h$ is an increasing function of one of the parameters θ_k, when the other parameters have fixed values, will often be next to impossible. The selection of statistics must be made in a heuristic manner.

In this article we do not use the decision outcomes for estimating, but only the structure of the generated access network. Two global network characteristics stand out as candidates: the density,

$$d_A = \frac{1}{n(n-1)} \sum_{\substack{i,j=1 \\ i \neq j}}^{n} a_{ij} ;$$

and the variances of the in- and the out-degrees (cf. Snijders, 1981),

$$V_{A,in} = \frac{1}{n} \sum_{i=1}^{n} (a_{+i} - (n-1) d_A)^2$$

$$V_{A,out} = \frac{1}{n} \sum_{i=1}^{n} (a_{i+} - (n-1) d_A)^2$$

(Note that the definition of the density implies that the mean in-degree as well as out-degree is given by $(n-1)d_A$. However, other statistics can be used that may be closer to the roles of the parameters θ_h in the model. Formulae (11) and (12) indicate that parameters θ_1 and θ_2 moderate the degree to which resources r_i and voting power v_i set limits on influence obtained and influence accepted. This suggests that the following covariances, or the corresponding correlation coefficients, based on the components of (11) and (12), can be used as statistics for estimation:

$$C(a_{i+},r) = \frac{1}{n} \sum_{i=1}^{n} r_i(a_{i+} - (n-1) d) ;$$

$$C(a_{+i},r) = \frac{1}{n} \sum_{i=1}^{n} r_i(a_{+i} - (n-1) d) ;$$

$$C(a_{+i},v) = \frac{1}{n} \sum_{i=1}^{n} v_i(a_{+i} - (n-1) d) .$$

(The voting power here is assumed to be issue-independent).

With respect to θ_3 and θ_4 , the amount of "randomness" in the access network is important; this notion of randomness refers to not being determined by the "input variables" consisting of resources r, voting power v, saliences s, and initial policy positions x^0. Except for the density, all mentioned statistics reflect this randomness in some measure: the degree variances and the various covariances all may be expected to become lower when the amount of randomness in the system (measured by θ_3 and θ_4) increases. Other statistics that may be viewed as reflecting randomness are the degree covariance (in view of the third term in (12)),

$$C(a_{i+},a_{+i}) = \frac{1}{n} \sum_{i=1}^{n} (a_{i+} - (n-1) d_A) (a_{+i} - (n-1) d_A) ;$$

the graph autocovariance between the access network and the initial policy positions, measured by a Geary-type statistic (cf. Sprenger and Stokman, 1989, chapter 18),

$$\sum_{i,j=1}^{n} \sum_{f=1}^{m} a_{ij} (x_{if}^0 - x_{jf}^0)^2 ;$$

and the degree to which the powerful actors get what they want, measured by the inner product between resources and utilities,

$$\sum_{i=1}^{n} r_i U_i (y) .$$

The available network data for the AVEBE access network are symmetric. The networks predicted by the simulation model are therefore symmetrized, as described in Section 4.1.

This implies that in-degrees are identical to out-degrees and that some of the statistics mentioned above coincide. Further, only one actor has voting power, namely, the Committee Goudswaard. The covariance between degrees and voting power therefore is of limited value, and it will not be used in the analysis presented below. We explore the use of the following three statistics for parameter estimation: the density d_A, the degree variance V_A, and the covariance between degrees and resources, $C(a_{i+},r)$.

In Section 6.1 we will explain that preliminary simulations suggested, however, that these statistics are not directly appropriate. Since the set of possible values for the degree variance V_A is highly related to the density d_A, and since the covariance between degrees and resources $C(a_{i+},r)$ on its turn depends on the degree variance V_A, normalized versions of these statistics seem better candidates. Instead of the degree variance V_A, we use V_A - $EV(d_A)$, where $EV(d_A)$ is the expected value of the degree variance in a random graph with n vertices and density d_A. Snijders (1981) shows how this value can be computed. Similarly, we use the correlation $r(a_{i+},r)$ between degrees and resources instead of its corresponding covariance.

5.3. Practical implementation of the Robbins Monro algorithm

For the practical use of the Robbins Monro method, a good starting value is important and a good choice of the matrix D_N is crucial. The derivative matrix $D(\theta)$ would be the optimal choice for D_N Since this matrix, however, is unknown, D_N has to be determined adaptively on the basis of provisional knowledge.

For a preliminary and approximative reconnaissance of the function $E_\theta Z$, it is advisable to determine a region of likely values for θ and define a grid of θ-values in this region. For each θ-value in this grid, the simulation model is run once or several times. The outcomes of the statistics Z_h are averaged over the runs for each fixed value of θ. These averages serve as approximations of $E_\theta Z_h$ and yield a preliminary impression of the behavior of $E_\theta Z$ as a function of θ. Inspection of these results can indicate whether indeed a solution to the moment equations (14) is to be expected within this region of values for θ, and whether $E_\theta Z$ is a sufficiently smooth function of θ. If it is not a reasonably smooth function, then Z may be an unsuitable statistic for the method of moments.

The matrix D_N may be chosen as follows. Suppose that the reconnaisance indicated above resulted in a k--dimensional interval

$$\{ \theta = (\theta_h) \mid \theta_{h0} \le \theta_h \le \theta_{hl} \quad \text{for all } h \}$$

for which it is provisionally assumed that the solution to the moment equations lies in this interval. Let e_h be the h'th unit vector and define $\delta_h = \theta_{hl} - \theta_{h0}$. Now define the vectors θ $^{(0)} = (\theta_{h0})$ and $\theta^{(h)} = \theta^{(0)} + \delta_h e_h$ for $h = 1, ..., k$. Run the simulation model M times for each of the parameter values $\theta^{(h)}$ $(h = 0, ..., k)$ and calculate the average outcomes of the statistics Z; these averages are denoted $z^{(h)}(h = 0, ..., k)$. Then for the first steps of the Robbins Monro algorithm, D_N is chosen as the corresponding matrix of difference quotients d_{hj}, defined by

$$d_{hj} = \frac{z_h^{(j)} - z_h^{(0)}}{\delta_j}.$$

With this value of D_N , a number of steps of the form (17) are taken. When it may be supposed this has led to a smaller k-dimensional interval of which it may be assumed that it contains the solution to the moment equations, a new value of D_N can be similarly determined, and further steps of the form (17) are taken. This process is repeated a number of times.

Implemented in this way, the Robbins Monro algorithm consists of several series of iteration steps, each series having the same value for D_N. Convergence must be determined by monitoring moving averages of the generated values $Z_N (\theta^{(N)})$ and deciding whether these are sufficiently close to the target (observed) value z.

This description of the implementation is admittedly not very precise. Convergence properties of adaptive versions of the Robbins Monro algorithm are difficult to give in a way that is completely satisfying for practical purposes (see, e.g., Pflug, 1988). The implementation still needs some "trial and error" by the investigator. However, experience has shown that a practical implementation is indeed possible.

6. PARAMETER ESTIMATION FOR THE AVEBE DATA

In this section, parameters are estimated for several alternative specifications of the DA model, applied to the AVEBE example. The data were collected by Schonewille (1990), and are summarized in Tables 1 and 2. The model was used with $T = 10$ iterations.

Actors	Resources	Voting Power
Executive board AVEBE	0.18	0
Supervisory board AVEBE	0.18	0
Farmers	0.20	0
Farmers workgroep Veenkolonien	0.04	0
Regional trade union	0.04	0
Board of Employees AVEBE	0.06	0
Farmers bank RABO	0.76	0
National Investment bank	0.24	0
Province Groningen	0.10	0
Ministry of Agriculture	0.34	0
Ministry of Economic Affairs	0.12	0
Ministry of Finance	0.12	0
Ministry of Environment	0.18	0
Second Chamber Cie Agriculture	0.08	0
Second Chamber	0.08	0
Committee Goudswaard	0.60	1
Political Party CDA	1.00	0
Green movement	0.80	0

Table 1. Actors and Resources AVEBE Policy Domain

Actors	Policy position			Salience		
	own capital[a]	reduction debt[b]	investment environm.[b]	own capital	reduction debt	investment environm.
Executive Board AVEBE	30	200	71.25	0.36	0.38	0.25
Supervisory Board AVEBE	30	200	71.25	0.36	0.38	0.24
Farmers	10	300	37.5	0.24	0.40	0.32
Farmers workgroup Veenkolonien	20	700	75	0.32	0.36	0.12
Regional Trade Union	40	200	75	0.32	0.36	0.14
Employees AVEBE	27.5	175	56.25	0.36	0.30	0.22
Farmers Bank RABO	35	175	unknown	0.30	0.24	0.12
National Investment Bank	35	175	unknown	0.06	0.12	0.12
Province Groningen	20	250	95	0.20	0.26	0.30
Ministry of Agriculture	35	150	78.125	0.28	0.26	0.14
Ministry of Economic Affairs	35	150	65.625	0.20	0.18	0.14
Ministry of Finance	35	150	65.625	0.20	0.18	0.14
Ministry of Environment	35	150	95	0.20	0.18	0.24
Second Chamber Cie Agriculture	30	150	56.2	0.32	0.32	0.24
Second Chamber	30	150	56.2	0.32	0.32	0.24
Committee Goudswaard	35	150	65.63	0.16	0.18	0.12
Pol Party CDA	40	200	75	0.16	0.20	0.12
Green Movement	nil	nil	110	0.32	0	0.40

[a] Percentage of whole capital

[b] Million Dfl

Table 2. Policy positions and Saliences in AVEBE Policy Domain

The *empirical values* of the three statistics are

$$d_A = 0.50, \quad V_A - EV(d_A) = 6.6, \quad \text{and } r(a_{i+}, r) = 0.09.$$

The *simulated values* of these statistics in the *standard DA model* (corresponding to $\theta_1 = \theta_2 = 0.5$, $\theta_3 = \theta_4 = 0$) are

$$d_A = 0.21, \quad V_A - EV(d_A) = 10.1, \quad \text{and } r(a_{i+}, r) = 0.50.$$

This shows the standard DA model generates access networks that are less dense than the observed network, and much more centralized in the sense that the relations are predominantly with the actors who have voting power and/or many resources. The

expected values of these statistics, which occur in the moment equation (15), are denoted $E_\theta d_A$, $E_\theta(V_A - EV(d_A))$, and $E_\theta r(a_i, r)$. First the estimation of θ_1 and θ_2 is discussed, and next the estimation of θ_3 and θ_4 .

6.1. Bounds for making and accepting access requests

The first alternative model contains parameters θ_1 and θ_2 that play a role in the upper bounds (11) and (12) to, respectively, the number of access requests made and accepted.

We mentioned in Section 5.2 that our initial choice of statistics was inappropriate. For illustrative purposes we show now how we came to this conclusion, and why we decided to take normalized versions of the statistics. To explore the possibilities of estimating the parameters θ_1 and θ_2 , each combination of two out of the three initial statistics d_A , V_A , and $C(a_{i+}, r)$ was considered as a possible basis for the moment estimators. The original DA model has $\theta_1 = \theta_2 = 0.5$. Plausible values for these parameters are considered to be values between 0.5 and 1.5. First a rough investigation was made of the dependence on θ of the expected values of the three statistics. A grid of values of (θ_1, θ_2) was considered with step sizes 0.1 in the square with boundary values 0.5 and 1.5. For these parameter values the model was run once, and the statistics were calculated. It turned out that for values of θ where the simulated density is less than 0.8, the simulated degree variance and degree resource covariance are much too large, while the latter two mean values quickly drop to 0 when the simulated density approaches 1. This implies that it is doubtful whether a solution to the moment equations exists at all for these statistics, and if it exists it might lie in a region of θ values where the derivative matrix $D(\theta)$ has large and fast changing elements. Such an instability of the matrix $D(\theta)$ is inauspicious for the Robbins Monro algorithm. Indeed it appears that the algorithm does not converge when it is based on any two out of these three statistics.

These numerical problems, however, hide conceptual and statistical problems. The degree variance and degree-resource covariance are strongly related to the density. E.g., they tend to 0 when the density approaches 0 or 1. It seems conceptually preferable to transform the statistics so that they are not so strongly related. This was done for the degree variance by subtracting the expected value for a random graph with the given density, and for the degree-resources covariance by using instead the correlation. In both cases this is only one of the possible transformations, and other transformations will have to be studied in future research.

Accordingly, we now consider d_A , $V_A - EV(d_A)$, and $r(a_{i+}, r)$. It still appears impossible to obtain convergence when d_A and $V_A - EV(d_A)$ are used as the pair of statistics. For very small or very high values of $E_\theta d_A$, $E_\theta (V_A - EV(d_A))$ gets close to the empirical value 6.6 of $V_A - EV(d_A)$, but this statistic is much too large when d_A approaches its empirical value of 0.50. Apparently, even when playing around with the parameter values of θ_1 and θ_2 in the restrictions (11) and (12), the whole model is such that the simulated network, when it has a density close to 0.50, still is much more centralized than the empirical one.

We do obtain results close to convergence, however, for the two other combinations of the statistics. With $\theta_1 = 1$ and $\theta_2 = 0.5$, we obtain:

$$E_\theta \, d_A = 0.17, \quad E_\theta \, (V_A - EV(d_A)) = 6.1, \quad \text{and} \quad E_\theta \, r(a_{i+}, r) = 0.12.$$

This is about as close as we can get to the empirical values of $V_A - EV(d_A) = 6.6$ and $r(a_{i+}, r) = 0.09$. It is especially difficult to obtain small values of the correlation between degrees and resources. The expected density with these parameter values is far below the observed density.

With $\theta_1 = 2$ and $\theta_2 = 0.7$, we obtain

$$E_\theta \, d_A = 0.50, \quad E_\theta \, (V_A - EV(d_A)) = 12.6, \quad \text{and} \quad E_\theta \, r(a_{i+}, r) = 0.11.$$

This is also as close as we can get to the empirical values of $d_A = 0.50$ and $r(a_{i+}, r) = 0.09$. It is again difficult to obtain smaller values of the correlation between degrees and resources. The expected value of $E_\theta \, (V_A - EV(d_A))$ is far above the empirical value.

Preliminary conclusion and interpretation

It is impossible to obtain the empirical values of degree variance and density simultaneously given the specification of the restrictions (11) and (12), even when we allow the parameters θ_1 and θ_2 to vary. For a simulated network with the empirical density value, the simulated networks are always over-centralized. Our models produce a too strong positive relationship between voting power, resources, and the number of access relations.

The observed low correlation between degrees and resources can be reproduced rather closely (but not perfectly), combined with the density and also combined with the degree variance but not with both together. The reason that a better correspondence is not produced, is that our model assumes that there is an important relationship between degrees and resources, not only in the restrictions (11) and (12), but implicitly also in the behavioral rules determined by (8). This might be improved upon when the assumptions on the behavioral rules are relaxed, and random disturbances are included in (11) and (12) just as in (13) and (14).

6.2. Randomness in making and accepting access requests

The second alternative model contains parameters θ_3 and θ_4 in (13) and (14). These represent the variances of random disturbances in the perceived utilities of making and accepting access requests, respectively.

A number of unforeseen problems arise, so that more can be reported about sensitivity than about estimation. With the introduction of a random disturbance in making access requests, the behavior of most actors in the first rounds differs dramatically from that in the standard DA model. In the latter model, actors do not make access requests to actors if they expect zero utility from such a request. Since no actor initially has any control over other actors, the only actors to which access would have any utility are those actors with voting power (see (8)). Since in this example the Committee Goudswaard is the only actor with voting power, this is the only actor to which actors initially like to have an access relation. This initial situation is highly influential for the further process of the formation of access relations. However, with the addition of a random disturbance to the expected utility of an access relation (13), actors expect positive utility of access relations to many more actors (on average half of the total) than just the Committee Goudswaard. As a result, the process deviates too much from the process of the standard DA model, even for the smallest values of θ_3.

The expected values $E_\theta \, d_A$, $E_\theta \, (V_A - EV(d_A))$, and $E_\theta \, r(a_{i+}, r)$ increase with the introduction of a positive θ_3. Therefore it is even more difficult to reproduce the small

observed value of $r(a_{i+}, r)$. Intuitively, one would first expect lower values of the correlation between resources and degrees after the introduction of random disturbance in the expected utility. However, as soon as behavior gets more random, the restrictions (11) and (12) determine more and more strongly the structure of the resulting simulation network. By definition, these restrictions entail high correlations between degrees and resources unless the density is extremely low or extremely high. A similar story holds for the increasing value of $E_\theta(V_A - EV(d_A))$.

With regard to the introduction of a random disturbance term θ_4 in the determination of which access requests to accept, in our experience up to now, little happens to the network parameters. This probably means that restrictions on accepting requests are less important than restrictions on making requests. Actors whose requests are initially not accepted if $\theta_4 = 0$, may now initially be accepted, but for $\theta_4 = 0$ they might as well have become accepted in later rounds.

Given these obstacles, it is understable that we did not obtain any convergence for the estimation of the parameters θ_3 and θ_4 for any combination of the fitting statistics.

6.3. Conclusion

It appears feasible to modify the DA model in such a way that networks are generated that are more realistic than the extremely centralized networks predicteo by the DA model specified in Stokman & Zeggelink (1996). However, the standard DA model appears to be so rigid, and especially dominant in its restrictions for requesting access, that it is very difficult to produce simulated networks that resemble the empirical access network, by the adaptation of parameters as reported in this paper.

Future research activities shall concentrate on the relaxation of the definition of the restrictions (11) and (12). Moreover, in the determination of the expected utility of access relations to other actors, the initial dominant effect of voting power should be attenuated. Finally, more empirical data should be used, in particular, decision situations with more than one actor with voting power.

The method of moments combined with the Robbins Monro algorithm is promising for the estimation of parameters in extensions of the DA model, but the behavior of the algorithm is (of course) sensitive to the choice of network statistics for the fitting process. In our initial attempts, the complicated association between the network density on one hand, and the degree variance and degree-resource covariance on the other hand, led to non-convergence of the algorithm. Further investigations about the choice of suitable fitting statistics are necessary.

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