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THE SUBLEXICAL STRUCTURE OF A SIGN LANGUAGE[1]

Lucinda Ferreira BRITO[2], Rémi LANGEVIN[3]

RÉSUMÉ — Structure sublexicale d'une langue de signes
Analyser et transcrire une langue de signes est une tache difficile puisque ce mode d'expression, mouvement des mains dans un espace situé près du corps, complété par des attitudes et des expressions faciales, est a priori moins séquentiel que la parole.

Notre travail vise à compléter les nombreuses tentatives antérieures, et s'appuie en particulier sur le dictionnaire de Stokoe.

En analysant le mouvement d'un repère attaché à une main comme le mouvement d'un point dans $\mathbf{R}^3 \times SO(3)$ *nous parvenons à discrétiser de manière naturelle les gestes les plus fréquents de la [LSCB] ou LIBRAS, Brazilian Cities Sign Language.*

Ce travail permet un premier classement de type alphabétique des signes de la LSCB et donc la constitution d'un premier dictionnaire LSCB → Portuguais par L. Ferreira Brito et son équipe.

SUMMARY — *Analyzing and transcribing a sign language is a difficult task since the mode of expression - hand movements in a space located close to the body, complemented by attitudes and facial expressions - is a priori less sequential than speech.*
Our work aims to complete numerous previous attempts and uses in particular Stokoe's system.
Analysing the movement of a frame attached to the hand as the movement of a point in $\mathbf{R}^3 \times SO(3)$ *we manage to discretize in a natural way the most frequent gestures of the [LSCB] or LIBRAS, the Brazilian Cities Sign Language.*
This work allows a first classification of alphabetical type of LSCB signs. L. Ferreira Brito and her group are writing the first dictionnary [LSCB] → Portuguese using this system.

1. INTRODUCTION

Analyzing and transcribing a sign language is a difficult task since the mode of expression - hand movements in a space located close to the body, complemented by attitudes and facial expressions - is a priori less sequential than speech. Nonetheless, attempts to describe signs date to the beginning of the nineteenth century. As early as 1825, R.A. Bebian was working with movements of lexical items in French Sign Language (FSL). Studies of aboriginal sign languages were also conducted in the late nineteenth century (G. Mallery 1880a, 1880b, 1882 ; J. Axtell 1891 ; and other authors).

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For linguists, the seminal work was Stokoe's dictionary (1965). He and his co-authors proposed parameters that could be used to describe the signs of the American Sign Language (ASL). These have long been used in the descriptions of a number of sign languages.

The following parameters are considered classic in describing signs : (1) handshape, (2) place of articulation, (3) movement, and (4) orientation (Stokoe, Casterline, and Croneber 1965 ; Friedman 1977 ; and Kilma and Bellugi 1979). It should be mentioned, however, that defining orientation as a main parameter of signs has been a subject of some controversy (Klima and Bellugi 1979). B. Bergman (1982) based her system on the articulator (handshape and attitude), the articulation (movement direction and interaction), and the place of articulation, or location.

The mathematician does not feel comfortable with these views, which deal with the possible values of the parameters as if they were of one type - while they are not. There is a small discrete set of values for handshapes, while places of articulation are less clearly definable. Depending on the sign, the place of articulation may be a precise point or a broader region - for example, the tip of the nose or the body. Movement is described with the help of a finite set of adjectives that cannot, by definition, account for the continuum of possible values. Orientation - a word that evokes the idea of a vector or a direction - is generally not precise enough to unambiguously indicate the position of a hand in relation to the signer's body.

More recent theories discuss the classical view. For example, S.K. Liddel and R.E. Johnson (1985) proposed a system called Move-Hold, where these two types of segments are considered prominent and are taken as the basis for applying autosegmental phonology to the study of sublexical sign structure. Like these authors, W. Sandler (1986) also tries to apply autosegmental phonology to the analysis of signs. In Sandler's opinion, however, the Move-Hold model is redundant and does not account for the sublexical structure of signs. Sandler proposes the Hand Tier model, where handshape is more prominent. S. Prillwitz (1989) does not concern himself with phonology but only with the transcription of signs. His system pays closer attention to handshapes and internal movement while other parameters are dealt with as in classical approaches.

It is not our intention to propose another theory. Starting with a practical problem - how to create a dictionary of a sign language (signs → oral language) - we soon realized that to establish an order for listing lexical signs (one which would replace alphabetical order), it would be necessary to establish a reasonably succinct and unambiguous system of transcription. Since sign language relies on movements of the hands in space, a natural way of approaching this task is to attempt to systematically describe the movements of one or two solids. It is interesting to note that robotics would use an approach analogous to the one we propose.

We will start with the notions of handshape and place of articulation, source and target points, and location of movements, and we will take into consideration the groupings suggested by our data. Hand configurations displaying internal movement are assigned to the same level as handshapes. First, we will concern ourselves with the characteristics of signs, which will be listed with their respective symbols. This is feasible when the number of possibilities does not exceed fifty, which is the number of symbols in the Japanese phonetic system.

The most delicate question is how to analyze the orientation of the hand(s) and of the movement. We believe we have arrived at a rather simple system, based largely on the six-dimensional space of space isometries. Reliance on this approach reveals the discrete characteristic of the elements that may be used in a sign language.

This article describes a provisory system of notation. Under a future project, we hope to use the facilities of a computer graphics center to verify the relevance of the choices proposed here (i.e., the segments of Brazilian Cities Sign Language [LSCB] or LIBRAS). Different classifications will be compared and the composition of movements will be synthesized to test the acceptability of reconstructed or invented signs.

2. WHAT ACTS ON WHAT WHERE

In order to have movement, an object and a space are needed. In sign, languages, one or two of the signer's hands constitute the object, while the space in which the movement is performed (the signing space) is an area around the signer's body. Sometimes it is also necessary to take into account the movement of the signer's head and body (see section 6).

2.1. The setting

Let us first define our setting, that is, the signing space. This is an area that contains all the points within the reach of the signer's hands and that moves with the signer. It can be defined by choosing three axes plus and origin attached naturally to the body of the signer. We have selected the navel of the signer to correspond to the origin ω. The ωx axis points forward and away from this origin ω, the ωy axis points to the signer's left, and the ωz axis points up, paralleling the singer's trunk. Together, the origin and these three scaled axes from what we call a frame, using the term in its mathematical rather than in its semantic sense (see Figure 1).

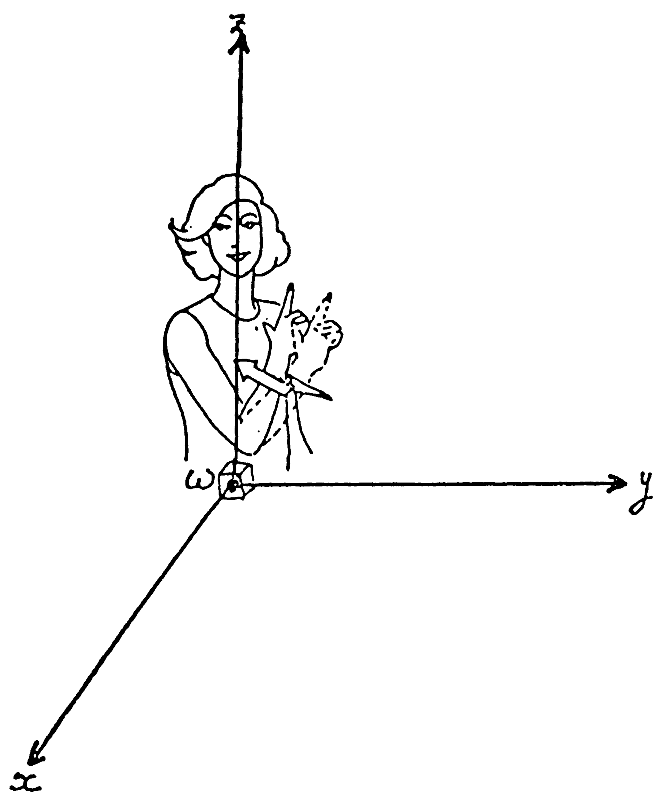

Figure 1. Frame of three scaled axes attached to signer's body

LSCB sign for HAVE [L] *TBm* $K_f MD_5$ *(Y,Z) (xyz, -xy -z)* ⊗ (2)

The three axes correspond to the three degrees of freedom of a movement in space : forward-backward, left-right, and up-down. The signing space is thus contained in a parallelepiped projected on the following segments :

- 20 cm, + 80 cm, along ox
- 100 cm, + 100 cm, along oy
- 20 cm, + 100 cm, along oz

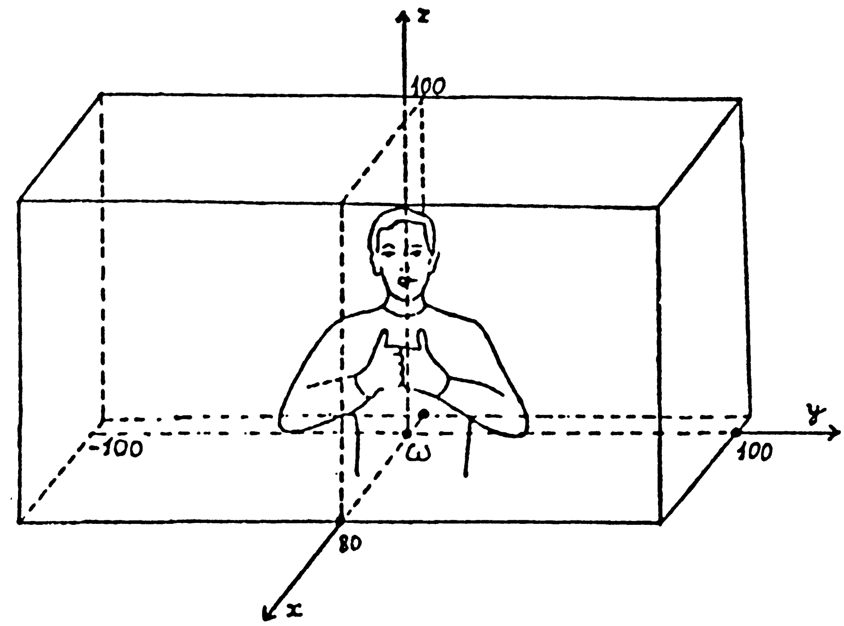

Figure 2. Signing space

A finite and reasonably limited number of places can be recognized within this space and these have been designated places of articulation. Some of these points are very precise, such as "the tip of the nose" ; some are broader, such as "in front of the chest." At times the place where the sign is performed is not relevant. In this case, the place of articulation is called the neutral space.

The following is a slightly modified list of Friedman's places of articulation (1977), divided into four major regions :

C HEAD

O top of head
T forehead
R face
S upper part of face
I lower part of face
Þ ear
O eyes
N nose
B mouth
C cheek
Q chin
A beneath the chin

T TRUNK

P neck
O shoulder
B bust
E stomach
C waist

B ARMS

S upper arm or arm
I forearm
C elbow
P wrist

M HAND

P palm
C back of hand
D fingers
Dp fingertips
Dd knuckles (joining fingers to hand)
Dj knuckles (first joint on fingers)
D_1 little finger
D_2 ring finger
D_3 middle finger
D_4 index finger
D_5 thumb
V interstices between fingers
V_1 interstice between thumb and index finger
V_2 interstice between index and middle fingers
V_3 interstice between middle and ring fingers
V_4 interstice between ring and little fingers

EN NEUTRAL SPACE

We must also employ certain adjectives that locate the places of articulation more precisely :

d = right side
e = left side
m = middle → Referring to the part of the body in question.
in = inner
ex = outer

Other adjectives are needed to describe the horizontal translation of places of articulation as images of a preceding point on the body frame :

l = sideways
f = forwards
a = backwards

Places of articulation are further described using :

p = near
med = medium distance
dist = distant
k = in contact
ki = initial contact
km = medial contact
kf = final contact

When a sign entails two places of articulation, we first indicate the starting place and then, at the end, we indicate the final place.

We have described an ideal signing space, with the signers face to face. There may be occasions when the signing space is totally relocated, thus prompting a reduction in the size of the space (see section 3.4 for a mathematical formulation). For example, if speaker *A* is signing to speaker *B*, who is in the window of a house, or to speaker *C*, who is in back of *A*, or if *A* is concealing his hands as he sign the signing space will be shifted. What is important to understand in signing is that the relative positions of places of articulation are what matter.

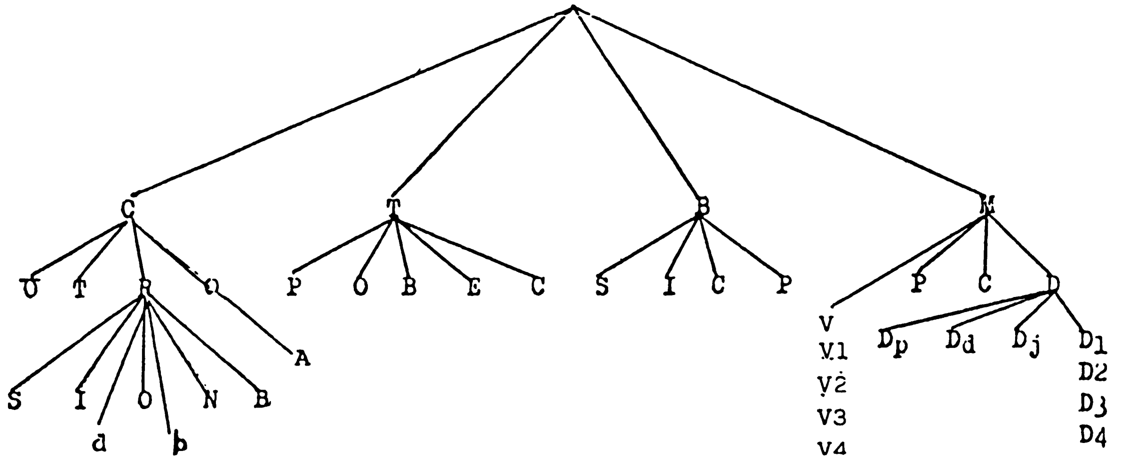

Figure 3. Places of articulation

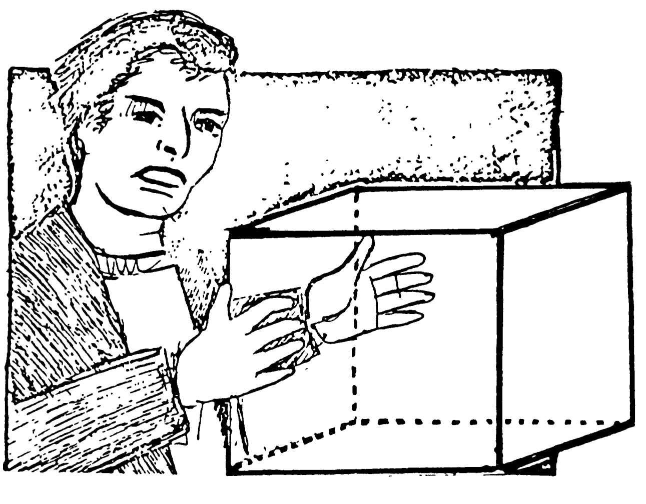

Figure 4. Signing space relocated or reduced

2.2. The object

So far we have identified a set of forty-four hand configurations in LSCB (see Table 1).

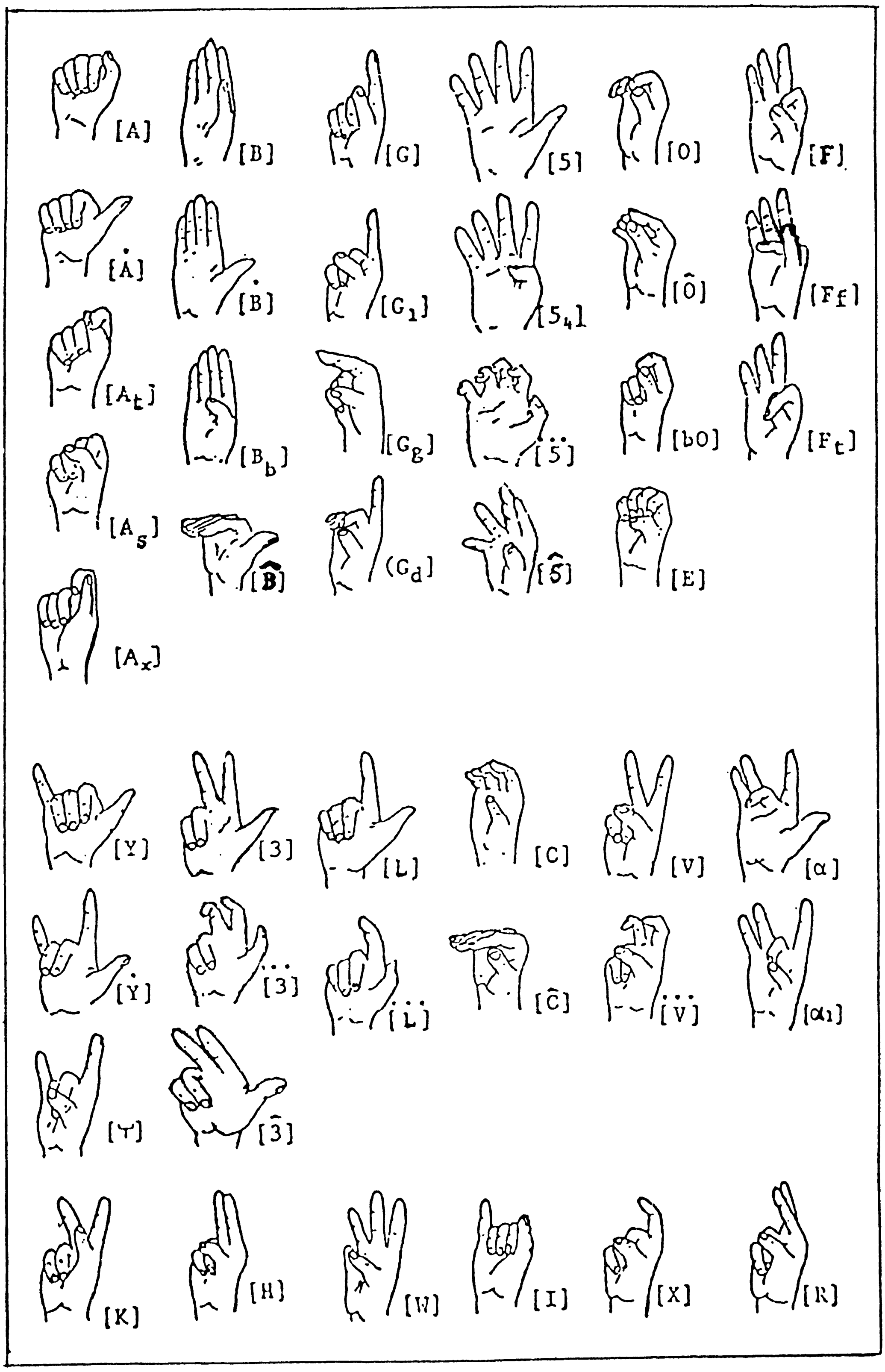

Table I. Forty-four hand configurations in LSCB

2.2.1. Axes and hand configurations

Let us now see what it means to define the position of a signer's hand in relation to his body. From now on we will suppose that we are dealing with the right hand. The main problem in defining the movement of a completely non-symmetrical object, like a dipper with a spout and a handle or a hand, is that it is not as simple as defining the movement of a dot, a marble, or an arrow.

Let us proceed in the same fashion as we did with the body, attaching a frame to the right hand (see Figure 1). Its origin O is situated at the bottom of the palm, near the wrist and in the middle. The direction OX extends perpendicular to the arm and parallel to the outstretched thumb. The direction OY extends toward the fingers of the open hand and parallel to the palm. The direction OZ extends perpendicular to the palm, forming right angles with a line running parallel to the outstretched palm/ forearm.

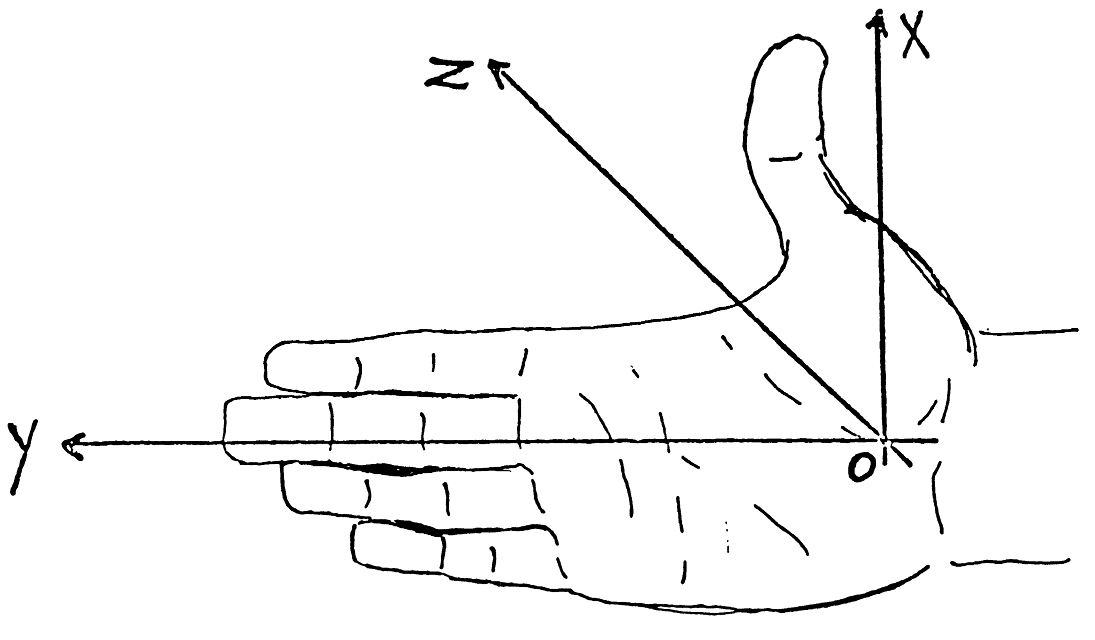

Figure 5. Axes of the hand

Note that the two frames (ωx, ωy, ωz and OX, OY, OZ) have the same orientation. According to Maxwell's rule, this means that a simple T-shaped corkscrew pointing in the direction ωz (or OZ), with the handle turning from ωx to ωy (or from OX to OY) pushes in the corkscrew, resulting in a positive rotation. Turning it from ωy to ωx (OX to OY) will "pull out" the corkscrew, producing a negative rotation.

If the hand were a point or a ball, it would not be necessary to rely on a frame to describe the location of the hand ; knowing the position of 0 would suffice. And in the case of symmetry of revolution, as occurs with a pen or an arrow, one axis would be enough to describe the position of the object. But since a hand is a completely non-symmetrical object, its position can only be described using the location of 0 and its three attached axes.

The shape of the hand may stay the same throughout the performance of a sign or it may change. In the latter case evolving from one static configuration to another. Movements of the fingers do not interfere with the positions of axes OX, OY, OZ, which are rigidly attached to the palm of the hand. The possibility that such internal movement will occur does not justify building a specific system to describe such finger movements.

Before going on to describe the movement of the right hand, we must be able to unambiguously define its position. We claim that the set of all possible positions of the right hand is a six-dimensional space - for the mathematician, a piece of the group $\mathbf{R}^3 \times SO(3)$ of affine isometries.

A static configuration can be defined by reference to all points of the right hand in the frame *OX*, *OY*, *OZ*. Thus, given a certain configuration, we can ascertain the position of the hand if we know the position of the frame *OX*, *OY*, *OZ* vis-à-vis the frame ωx, ωy, ωz.

It is easy to define the position of 0 in relation to the frame ωx, ωy, ωz. The set of all possible positions of 0 is simply the three-dimensional space $\mathbf{R}^3$, as previously described.

Let us first locate the *OX* axis. Imagine a sphere *E* of radius 1, centered on 0. The *OX* axis cuts the sphere *E* at exactly one point. We can thus say that the set of possible orientations of the axis *OX* is a sphere. If the position of *OX* is fixed, *OY* and *OZ* may rotate around *OX*. This means that *OY* cuts a circle (for example of radius 1) centered on 0 and contained in the plane perpendicular to *OX* through 0. The set of orientations of pairs of perpendicular axes (*OX*, *OY*) therefore has three dimensions. We will call this space *SO*(3). We can obtain a geometric picture of this by attaching to each point *m* of a sphere (for example, of radius 1) a circle (for example, of radius 1 or smaller) centered on the origin *x* of the tangent plane and contained in this same tangent place.

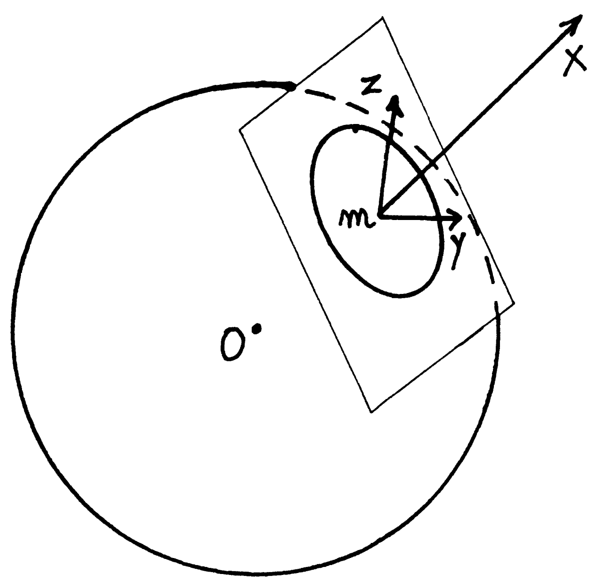

Figure 6. Sphere E.

It might be natural to wonder why we need a six-dimensional space configuration $\mathbf{R}^3 \times SO(3)$ to describe the position of the right hand, when just a few words would seem to suffice. We do indeed need this space because the loop through configuration space $\mathbf{R}^3 \times SO(3)$ allows us to model the possible positions of the hand by means of a discrete and rather small, simple set.

We have shown that it is enough to choose two axes of the frame *OX*, *OY*, *OZ* and state their positions in relation to the frame ωx, ωy, ωz in order to define the position of the hand. In order to make the position of an axis discrete, we will select the directions generated by combinations with a coefficient of +1, 0 or (-1) of the three unitary vectors, which produces the oriented axes *x*, *y*, *z*. Figure 7 shows part of the possibilities (i.e., a coefficient of +1 or 0 in *x* and in *z*). For example, *x* means that a points in direction *x*, *y* means that a points in direction *y* ; and *z* means that a points in direction *z*, while *-x* means that a points in the direction opposite to *x*, and so on.

The other possibilities are combinations of the earlier ones. For example, *x+y* is a direction that lies in the horizontal plane, pointing to the left of and in front of the body, while *x-y* is a direction that also lies in the horizontal plane but to the right and in front of the body. Linguists decided to omit the + and write *xy* instead of *x+y*.

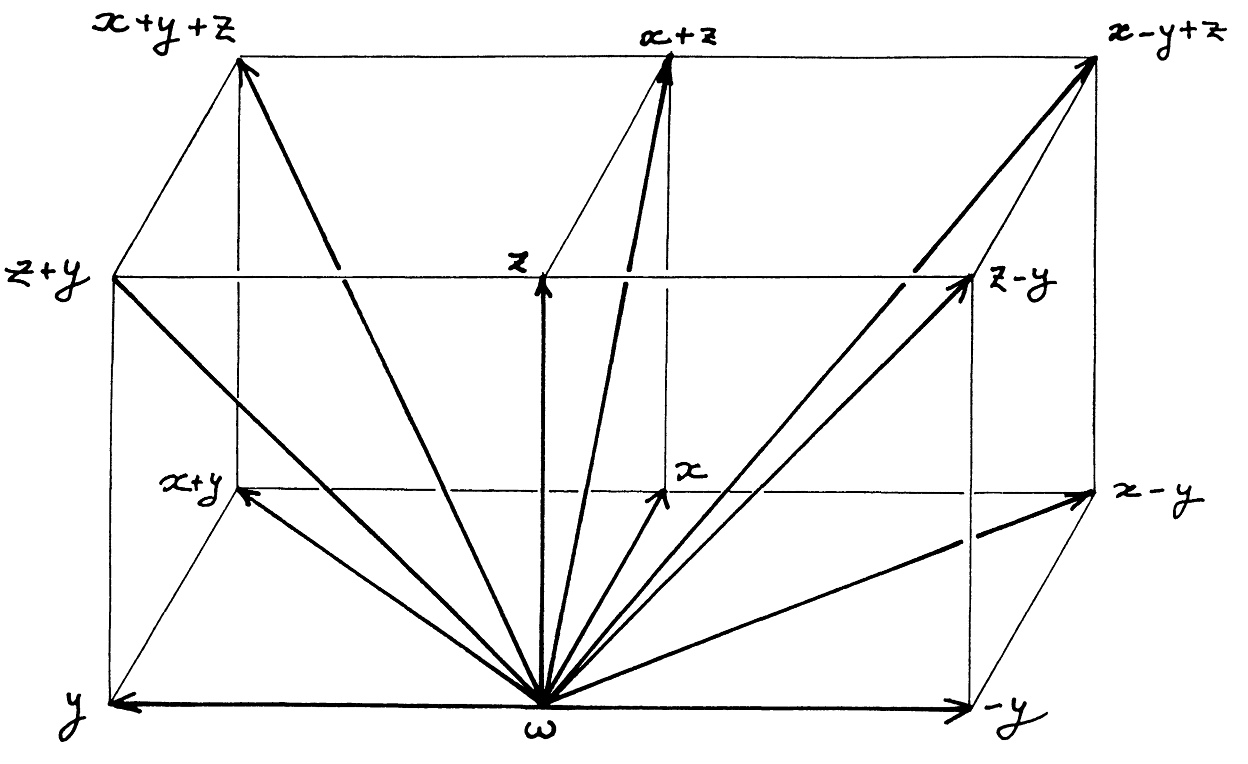

Figure 7. Some directions

In figure 7, we show all possibilities in which a plus sign may occur before *x* and *z*, when those axes are present. (In our transcriptions, the plus sign is frequently omitted).

It may be convenient as well to chose an axis attached to the hand configuration of the same sort as *x, y, z*.

In order to determine the position of the solid, it suffices to choose two non-aligned axes of the frame *ωx, ωy, ωz* and describe their positions in relation to the frame *OX, OY, OZ*. We enclose these first two axes in parentheses - for example (*X, Y*) (*z, x*) - to indicate that *X* is parallel to *ωz* and *Y* to *ωx*.

When the hand movement is solely a translation, we write the position of (*OX* and *OY*). When a rotation occurs, we first describe the axis around which this rotation takes place. The possible axes of rotation of the hand during a sign take on discrete values, as, for example, *x, -x, x-y, -x+y-z*, etc..., in the same way as we proceeded earlier (see Figure 7). The choice of the form of the transcription will depend on the axis around which rotation takes place.

For the moment, a sign may admit a few equivalent notations. For example,

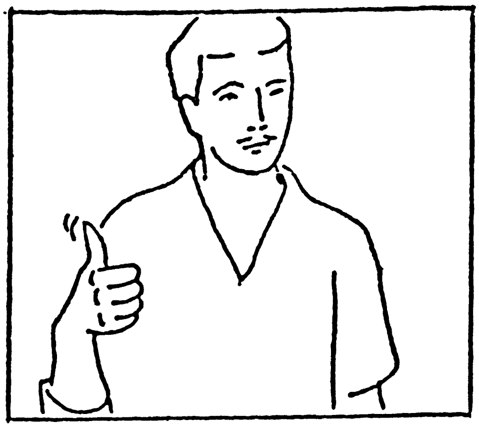

Figure 8. GOOD WELL

GOOD, WELL can be transcribed :

$$[\dot{A}]\ \text{EN}\ (Y,Z)\ (y, -x)\ \odot\ \perp_f$$

or

$$[\dot{A}]\ \text{EN}\ (X,Z)\ (z, -x)\ \odot\ \perp_f$$

So far, we have purposely skipped over one additional structure of space $\mathbf{R}^3 \times SO(3)$, dealt with in section 3.4.

2.2.2. Internal movements

The hand - that is, essentially the fingers - may move in relation to the frame *OX, OY, OZ*. We have adopted the traditional terminology to describe this : internal movement. It seems to us that this internal movement is as important to the meaning of a sign as is the movement of a solid configuration. We have observed that groups of signs with related meanings display similar internal movements. Our transcription always begins with a description of the configuration of the dominant hand, which we suppose to be the right.

Here is a list of hand configurations involving internal movement :

$[A \rightsquigarrow 5]$	gradual extension of fingers beginning with index finger
$[As \rightsquigarrow 5]$	gradual extension of fingers beginning with little finger
$[As \rightarrow \hat{5}]$	simultaneous opening of fingers
$[\hat{5} \rightarrow As]$	simultaneous closing of fingers
[L → *b*0]	pinching (with index finger and thumb)
[5 + mouv.]	drumming movement with fingers
$[\ddot{5}$ + mouv.]	drumming movement with fingers
$[54 \rightarrow G]$	gradual closing of all fingers except index and thumb
$[5 \rightsquigarrow \dot{A}]$	gradual closing of all fingers, except thumb
$[\dot{B} \rightarrow \hat{B}]$	bending the hand, with fingers extended out
$[V \rightarrow \ddot{V}]$	"scratching" with the index and middle fingers
[*V* + mouv.]	drumming movement with fingers
[*V*.mouv.]	scissoring

$[As \rightarrow \dot{A}]$	extending the thumb
$[As \rightarrow L]$	extending the thumb and index finger simultaneously
$[\dot{B} \rightarrow \dddot{V}]$	abrupt closing of all fingers except index and middle, which are flexed
$[As \rightarrow 3]$	simultaneous extension of thumb and index and middle fingers
$[As \rightarrow 5_4]$	simultaneous extension of all fingers except thumb
$[As \rightarrow 5]$	simultaneous extension of all fingers
$[\dot{A} \rightarrow L]$	extension of index finger
$[\dot{A} \rightarrow 3]$	simultaneous extension of index and middle fingers
$[\dot{A} \rightarrow 5]$	simultaneous extension of all fingers, with thumb already extended
$[G_1 \rightarrow X]$	bending of index finger several times
$[As \rightarrow V]$	extension of index and middle fingers
$[As \rightarrow I]$	extension of little finger
$[\dot{B} \rightarrow \dot{A}]$	bending all fingers together, except thumb
$[\hat{5} \rightarrow \hat{O}]$	bringing fingers and thumb together
$[\dddot{5} \rightarrow As]$	closing fingers into a fist
$[5 \rightarrow \alpha]$	shaking the middle finger while it is bent
$[\alpha_1 \rightarrow \hat{5}]$	flicking middle finger against thumb (marble shooting)
$[0 \rightarrow \hat{5}]$	opening all fingers simultaneously
$[b0 \rightarrow Ax]$	sliding index finger against thumb
$[\mathrm{A} \leadsto 5]$	opening all fingers one by one

3. MOVEMENT

3.1. Where is the movement ?

When a marble moves, it is not very difficult to describe this movement. We need to know its path and all the points through which the marble has passed ; we also need a clock to register the time(s) when the marble passed through each point along its path. We would proceed in much the same way with a round bullet in movement through the air.

What we have discussed in the previous section allows us to describe the movement of a highly non- symmetrical object : the hand. The only difference is that the path is found in space $\mathbf{R}^3 < SO(3)$. For example, put your right hand is shape B and in position (Y, X) (x, y) (i.e., with the fingers pointing away from you and the thumb pointing to the left). Now push your hand forward, at the same time rotating the thumb clockwise, from your viewpoint. During the movement, the point 0 performs a translation along the x axis, while the Y axis maintains the same direction and the X axis rotates.

Let us draw the path of 0, y and x in space $\mathbf{R}^3 \times SO(3)$.

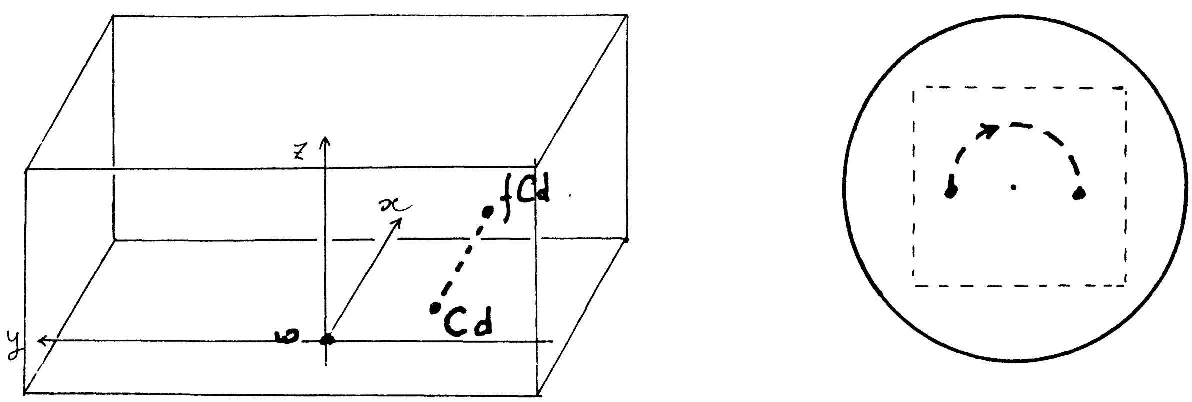

Figure 9

Both the point of intersection of OY and the sphere, as well as the circle of possible positions of x, are located behind the sphere.

It would be more complicated to draw the path of $(0,X,Z)$ in space $\mathbf{R}^3 \times SO(3)$ as both axes X and Z move simultaneously. But we are fortunate in that most movements in sign languages can be subdivided into small segments of movements where each segment can be assigned an axis. We have in fact found this to be true of all signs transcribed in our system to date.

Of course, a description of the path is not in itself enough to describe the movement. We also need a clock. In our example, we would need to know the position $0(t)$ of point 0 at all times : similarly, we would need to know the positions $Y(t)$ and $X(t)$. As we have already seen it is sufficient to know the movement of only two of the axes to describe the movement of a solid. In short, we will represent the movement of frame F as :

$$F(t) = (0(t), X(t), Y(t), Z(t)).$$

3.2. Speed

Another characteristic of movement is its speed. Normal speed is expressed as a ratio of distance to time. In this case, we must account for distances not in one-dimensional space (the position of O) but rather in space $\mathbf{R}^3 < SO(3)$ (the position of F). We also need to rely on other information not only on speed as a rate of displacement but also on the specific direction of the movement. When we talk about the speed of a body in a specified direction, we are talking about a vector that we will call the speed vector.

To describe the movement of a point in space, one can define :

$$v(t) = \lim_{h \to O} \frac{0(t+h) - 0(t)}{h}$$

The above formula means that we approach the limit of the ratio displacement time as we consider shorter and shorter intervals of time. The situation is slightly more complex for the movement of $F(t)$ since space $\mathbf{R}^3 < SO(3)$ is not a vector space.

Speed, which is a given rate of displacement independent of direction, is written $|v(t)|$. (The absolute value $|v(t)|$ is the length of the vector $v(t)$).

Let us define three characteristics of movement that depend on speed $|v(t)|$, or, more precisely, on variations in speed when the sign is performed : tension, holding, continuity. The notation we have chosen is :

tense ʌʌʌ
hold $\perp$
continue ∼

If tension or hold occur at the end we can add an index f (final) : $ʌʌʌ_f$, $\perp_f$

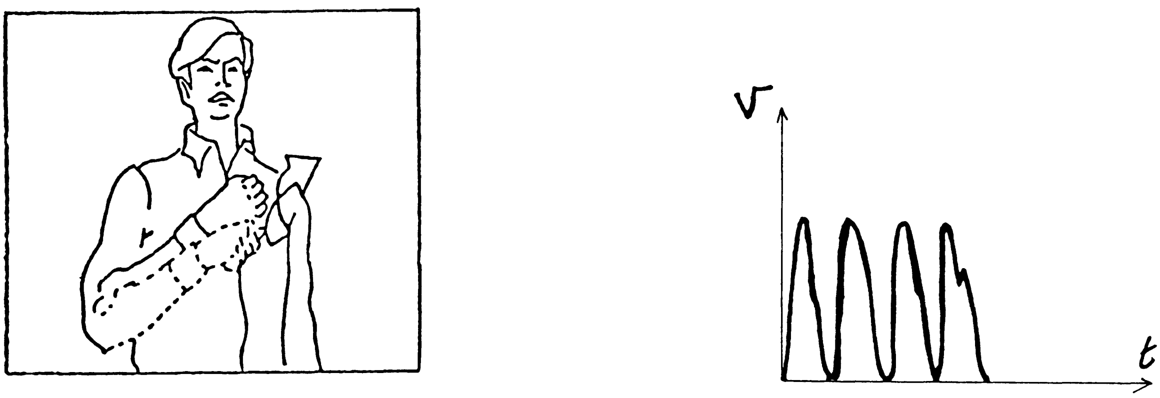

Figure 10. HATE (tension)
$[As]TBm\ K_{\mu}MP(Y,Z)\ (yz,\ y\text{-}z)$ ↷ ⟐ ʌʌʌ ⌒ ⌒

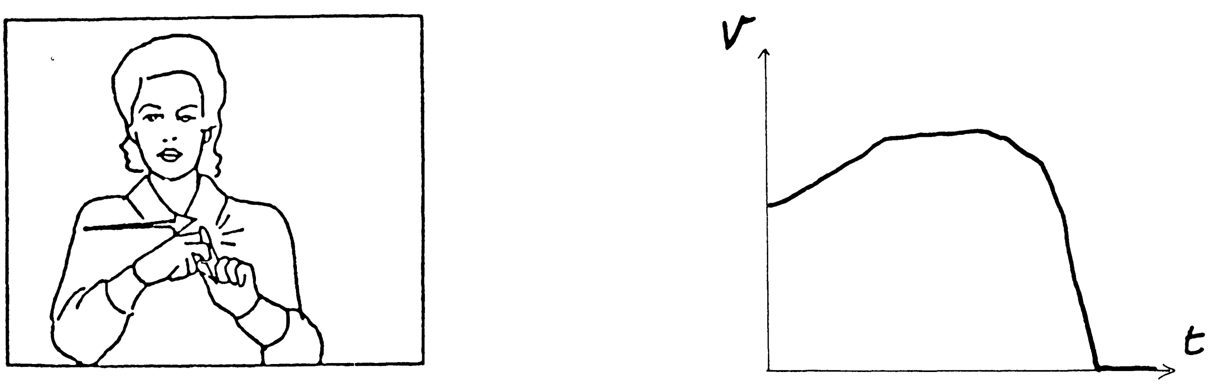

Figure 11. ACCIDENT (holding)
$[\dot{B} \rightarrow \ddot{V}]MD_4KfV_2(Y,Z)\ (xy,\ -z)$ ⊙ 6 $\perp_f$
$M[G_1]TB_e med(Y,Z)(z,x\text{-}y)$

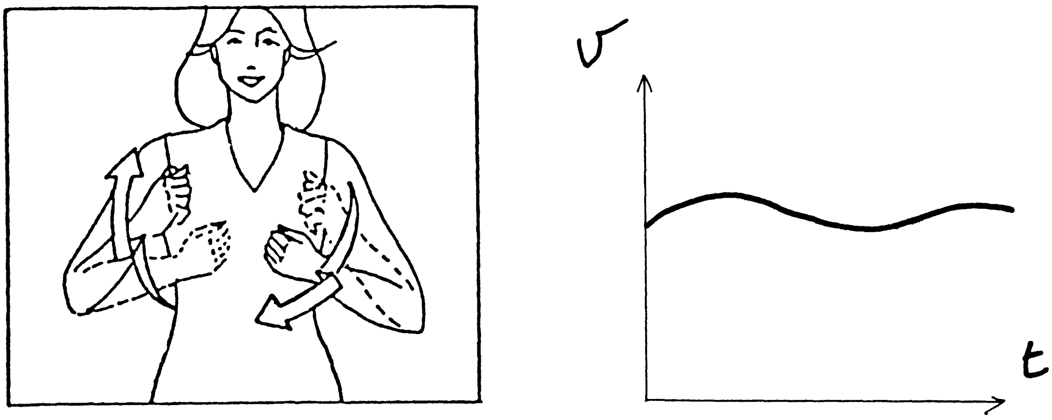

Figure 12. CAR (continuous)
[*As*]*Tf med* (*Y,Z*)(*xy, y-x*)[↻](2) ~ *So*

There is a fourth possibility - a rebound or, as it is classically called, restraint. In this case, |*v(t)*| does not display discontinuity but the direction of *v(t)* changes suddenly. The notation we have chosen is V↗

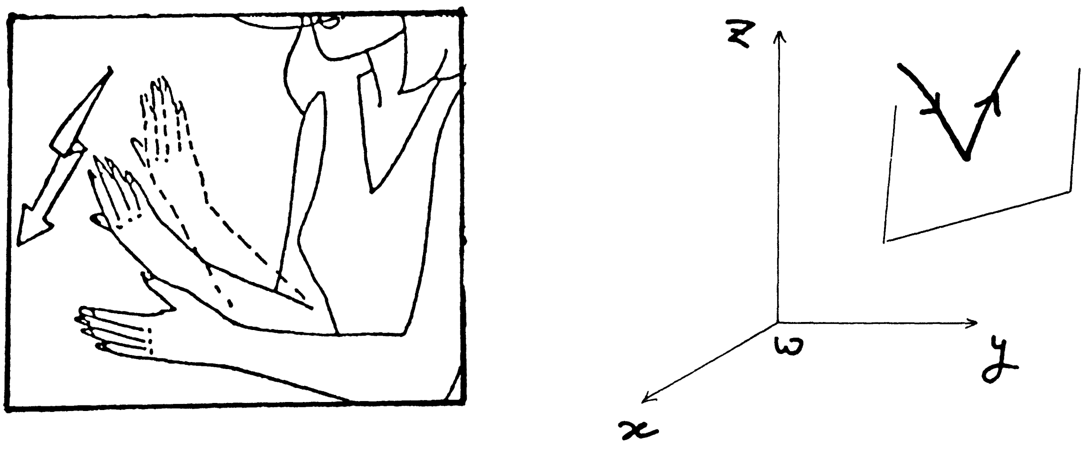

Figure 13. SCHOOL (restraint)
[B] *MP KfMC* (*Y,Z*)(*xy,z, -x-yz*) ⊙ 5 (2) V↗

3.3. Specific movements : translations and rotations

When the three axes of F do not change position during a movement, we call the movement a translation. To illustrate, when you move a glass of water without spilling a drop and without rotating the glass, this movement constitutes translation (however awkward the path may be). In this case, the initial position of the frame and the movement $0(t)$ of the origin fully describe the movement of the frame.

Most of the translations that appear in LSCB consist of rectilinear movements. Their directions belong to a discrete set, which we describe below.

3.3.1. Translations restricted to the plane $y\omega z$

Here the direction of a translation is indicated by eight numbers equally spaced in clock-like fashion around the edges of a circle (the person performing the sign is looking to the clock).

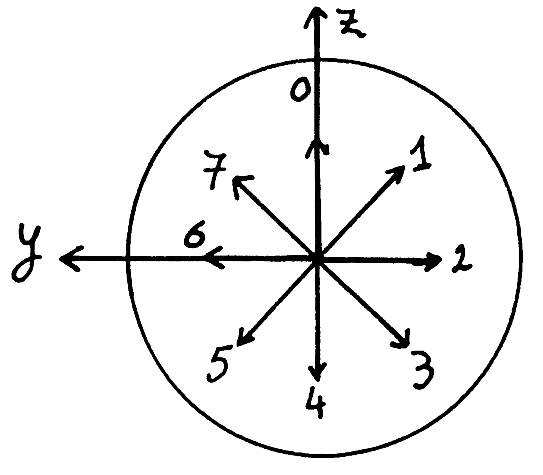

Figure 14. Directions of translations contained in a vertical plane

The symbol • appearing immediately before the number (direction) means that the translation vector does not have an x- component.

3.3.2. Translations displaying a positive component (i.e., forward movement) along the ωx axis

The symbol ⊙ indicates that the translation moves forward along the ωx axis, while the same symbol preceding a direction 0, 1, 2,...7 indicates that a component found in the $y\omega z$ plane is added to the positive component along the ωx axis.

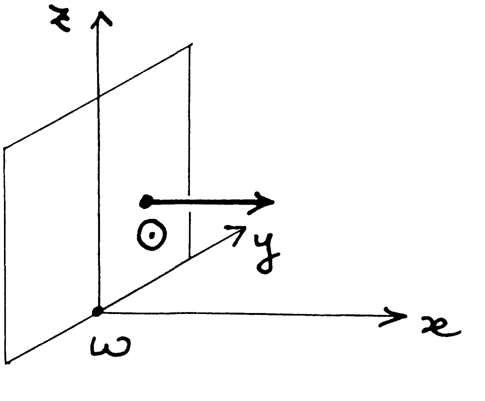

Figure 15. Forward translations

3.3.3. Translations displaying a negative component (backward movement) along the ωx axis
The symbol $\otimes$ indicates that the translation occurs backward along the ωx axis. The $\otimes$ placed before a direction 0, 1, 2,...7 indicates that a component found in the plane $y\omega z$ joins the negative component along the ωx axis.

Other paths of movement found in LSCB that limit themselves to translation are :

in the plane (y,z)

a half or quarter turn in the plane (x,z)

a half or quarter turn in the plane (x,y)

a half or quarter turn in the plane $(x + z, y)$

helicoidal translation where the axis is indicated by relying on the convention described above

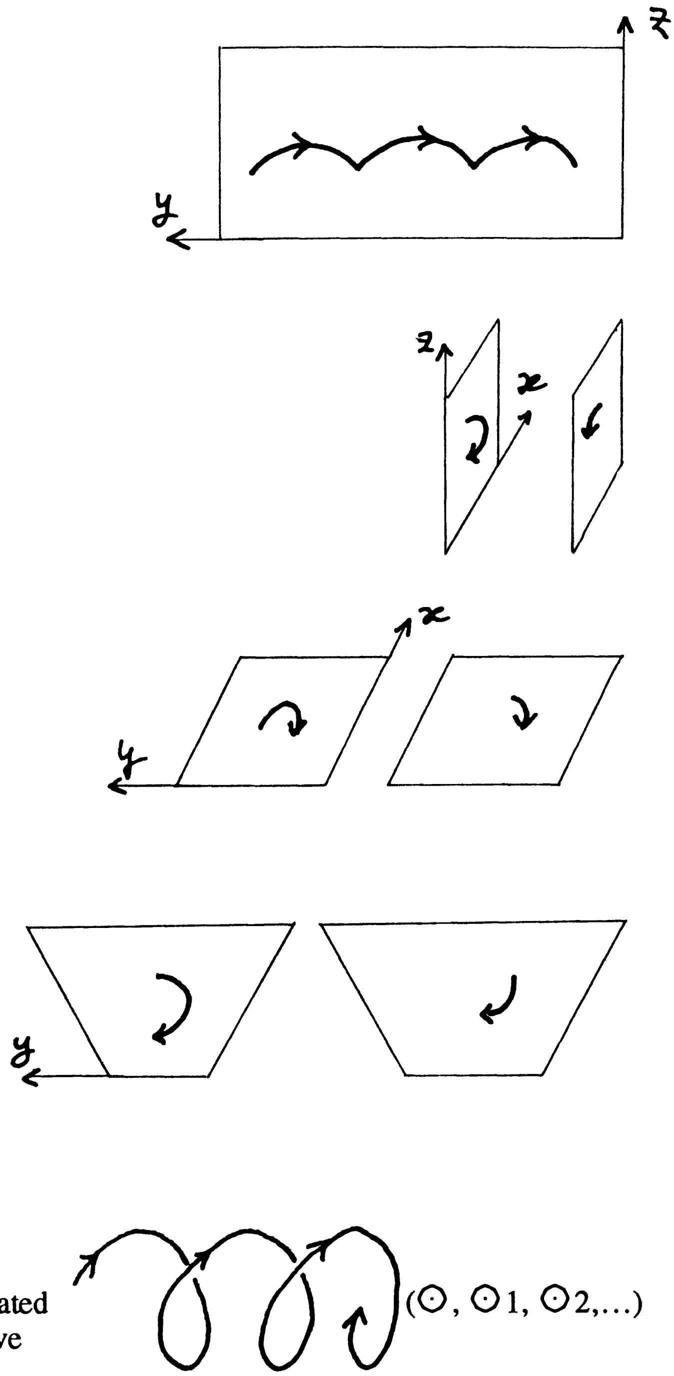

Figure 16. Possible non-rectilinear translations.

We call the other specific type of movement rotation. This refers to a movement $F(t)$ such that the origin O of the frame remains fixed. We can decompose any movement $F(t)$ of a frame into a movement of translation $T(t)$ plus a movement of a rotation $R(t)$:

$$F(t) = R(t) \circ T(t),$$

where the o indicates composition.

It can be shown that in any rotation one axis remains fixed. Although this is not a priori true for the family $R(t)$, in all signs that we have observed the rotations generally do have one fixed axis, and when they do not, the sign can nonetheless be decomposed into smaller segments where in the rotational part of the segment of movement has a fixed axis.

The rotations observed generally consist of a half or quarter turn in the plane perpendicular to the rotational axis. We will indicate this rotation as if the plane perpendicular to the axis were the plane of the sheet you are reading. The corkscrew convention allows us to distinguish a positive rotation (one that pushes the corkscrew in, or forward) from a negative one which pulls the corkscrew out, or backward). Note that our convention does not coïncide with the standard orientation of the circle adopted by mathematicians. (Z,X) (x,y) means that the OZ axis, parallel to the ωx axis, constitutes the rotational axis of a half turn in a positive direction. (By convention, the rotational axis is seen from behind). For example,

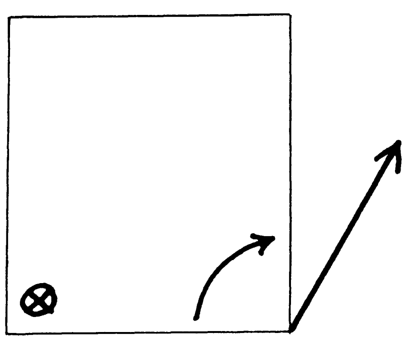

Figure 17. Rotational axis and rotation.

Figure 18. Of course, obvious, evident
$[\dot{B}]$ *TEd med*$(Y,Z)(x, -z)$ ↷ $\perp_f S$

3.4. Mathematical remarks

As we saw in section 2, the idea of decomposition has further applications. Let us start with the articulation frame F_0. Applying an isometry F_1 to F_0, F_0 reaches position F_1. Applying an isometry F_2 to F_0, F_0 reaches position F_2, while F_1, which we suppose to be rigidly linked to F_0, will reach a position that can be written $F_2 \circ F_1$. Note that order matters. Similarly, if we have two movements $F_1(t)$ and $F_2(t)$, we can compose them, repeating this construction for each t.

Let us look at it from another angle. Imagine a complete setting attached to the first frame ωx, ωy, ωz. We will make the ωx axis coincide with OX, ωy with OY, and ωz with OZ. This is our setting. We have thus defined an isometry of $\mathbf{R}^3$, a transformation that does not modify the relative distances of two points attached to the frame ωx, ωy, ωz. In Ferreira Brito and Langevin (1990) we show that the composition of a movement away from the body with the movement of a sign itself may constitute the negative form of that sign.

There are (infinitely) many ways of decomposing a movement into two components, but such a decomposition will only be meaningful if the two resulting components are simpler than the original movement. This is obviously the case in the decomposition $F(t) = R(t) \circ T(t)$.

Applying to speed the same derivation process that we used to define the speed of the movement, we obtain a new vector : acceleration. Considering only the length of the acceleration vector, we may, by studying acceleration as a function of time, reveal some patterns whose complexity may be characteristic of languages.

4. REDUPLICATION, SYMMETRIES, AND REPETITIONS

We have noticed that reduplication (repetition in time) and different types of repetition in space are used in similar ways by sign languages. For this reason we have chosen to deal with these in the same section of this paper and to locate them at the same place in our transcriptions.

4.1. Reduplication

The movement of a sign may be reduplicated (in time). We write the number of repetitions in parentheses following the symbol for translation and/or rotation.

4.2. Symmetry in relation to a plane

Our body admits a plane of symmetry $(\omega x, \omega z)$, and symmetry in relation to a plane is also an isometry. However, as the reader can verify by associating a frame to his left hand - as we did with the right hand in section 3 - symmetry in relation to a plane makes a direct frame an indirect frame. The corkscrew moves back along the third axis.

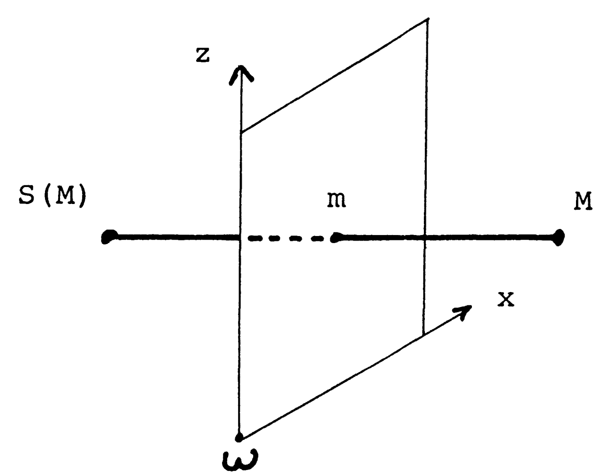

Figure 19. Symmetry in relation to a plane

The left hand can likewise describe the movement $S \circ F(t)$, symmetrical to the movement of the right hand. We indicate the symmetrical repetition of a sign (in relation to plane wx, wz) by placing an S at the end of the transcription.

4.3. Translation and repetition in space

A movement may simply be repeated in space by the left hand, that is, the left hand may duplicate the movement of the right. This means that a translation T occurs, remitting the point selected on the right hand to the homologous point on the left hand, so that the left hand's movement $F'(t)$ is defined as a function of the right hand's movement $F(t)$:

$$F'(t) = T \circ F(t).$$

The vector of translation most often follows the y axis. We indicate the occurrence of this type of repetition by placing an equal sign (=) at the end of the transcription.

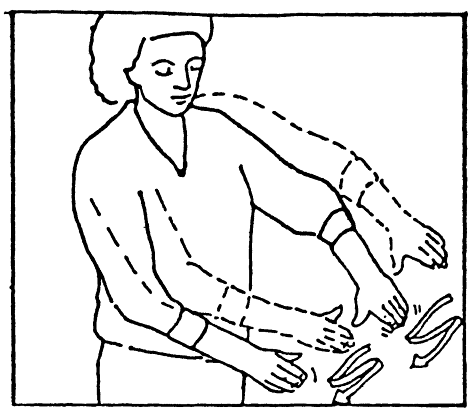

Figure 20. SKATES, TO SKATE in LSCB
[$\dot{B}$] *TEf* dist (*Y*,*Z*) (*x*, -*z*) (2) ~ =

There are two more families of symmetries to examine : symmetry in relation to a point and symmetry in relation to a straight line. However, we have not yet detected any example of symmetrical movement in relation to a straight line other than planar symmetrical movements in relation either to the point of intersection of the axis of symmetry or to the plane of movement.

4.4. Symmetry in relation to a point 0

The points M', M and 0 are aligned, with 0 located halfway along M, M'. We can write $M' = S_0(M)$.

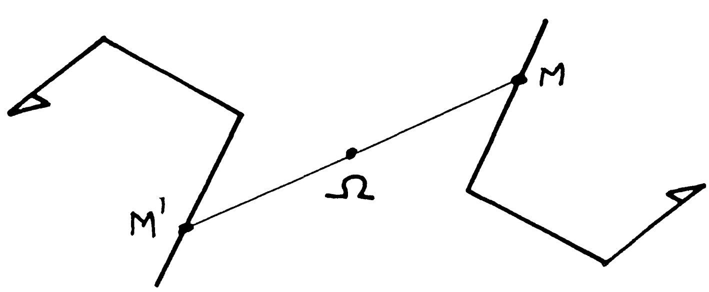

Figure 21. Two planar figures symmetrical in relation to a point

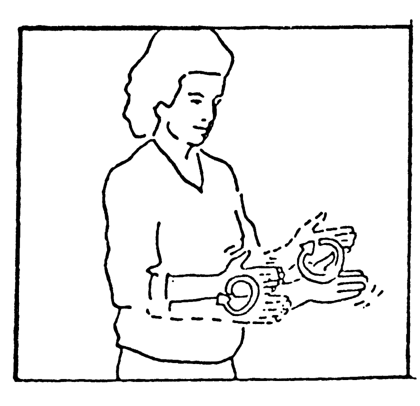

Figure 22. TRAIN (SP)}
[$\dot{B}$] *TEf* med (*Y*,*Z*) (*x*,*y*) (2) ~ S_0

Two movements, $F(t)$ and $F'(t)$, are considered symmetrical in relation to 0 if $F'(T) = S_0 \circ (F(t))$. We represent this symmetry by writing S_0 after the transcription of the hand movements.

4.5. Symmetry in relation to a vertical straight line

Let Δ be a vertical straight line (axis of symmetry) and M, a point that does not belong to Δ. The plane perpendicular to Δ passes through M and cuts Δ at m. The point $M' = S_\Delta (M)$ is the symmetry of M in relation to m. It should be kept in mind that the point m depends on M.

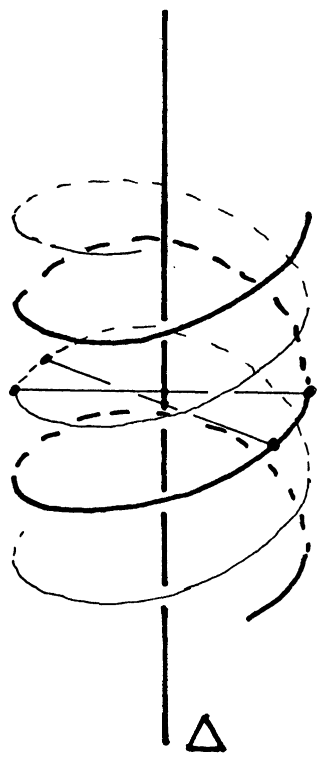

Figure 23. The figure formed by two helices is symmetrical in relation to Δ

The movement $F'(t)$ is obtained from the movement $F(t)$ by the symmetry of the axis Δ if $F'(t) = S_\Delta \circ (F(t))$.

It should be noted that the composition of isometries is once again used in cases (*b*), (*c*) and (*d*).

Lastly, we have observed that the movement of the non-dominant hand sometimes does not perform a movement exactly symmetrical to the movement performed by the dominant hand but instead performs a more simplified movement. The sign FREVO, as used in Recife, Brazil, illustrates this partial symmetry (the idea of partial symmetry was suggested to us by C. Cuxac).

5. SPEED AND ACCELERATION

Our study of movement has so far relied essentially on the paths of a frame. Now we must describe those characteristics of a movement that depend on speed and acceleration. Once again we will not be concerned with the intrinsic values of speed (the speed of T in meters per second, for example) but with the relative values of speed within one movement.

We represent speed with a positive number. We have defined four possible characteristics of a sign that depend on the type of speed : tension, holding, restraint, and continuity (see Figures [10, 11, 12 and 13]).

6. NONMANUAL EXPRESSIONS

According to Baker (1983), nonmanual expressions (movements of the face, the eyes, the head, or the trunk) serve two purposes in sign languages : marking syntactical form and acting as a lexical component. The nonmanual expressions marking yes-no, *wh-*, and rhetorical questions, conditions, relative clauses, or topicalizations serve a syntactical function, while those expressions that serve as a specific reference or as a pronoun reference, a negative particle, an adverb, a modifier, or a mark of aspect all constitute lexical components.

Again according to Baker, there is a natural tendency in sign language to transfer nonmanual behavior to the hands. She illustrates this hypothesis with the ASL sign for THINKING, which was originally accompanied by a head movement that disappeared with time and was replaced by a circular movement of the hand in *G*1, formerly immobile and touching the right temple. Despite this tendency, nonmanual expressions still often accompany the manual performance of signs, and some signs are even constituted solely of nonmanual expressions - for example, the LSCB sign for SEXUAL RELATIONS and a number of other signs likewise considered taboo.

Our primary goal is to create a dictionary. We have limited ourselves to studying the nonmanual behavior that is part of a lexical item, including therein those expressions that introduce interrogatives (who, when, etc...).

Based on Baker (1983). We can identify the following nonmanual expressions in LSCB :

FACE

upper portion

- ⁄ ⟍ knitting the eyebrows
- ô opening eyes wide
- < glancing
- ⌒ ⌒ raising the eyebrows

lower portion

- *db* puffing cheeks out
- *bd* sucking cheeks in
- = pouting lips and knitting eyebrows
- *lb* running the tongue along the lower inside part of the right cheek
- *b* puffing out right cheek only
- ◡⌒ raising upper lip
- *x* wrinkling nose

HEAD

+ nodding up and down (yes)
- shaking sideways (no)
ɾ/ tilting forward
/ tilting sideways
/ɾ tilting back

FACE AND HEAD

wh head tilted forward, eyes slightly closed, eyebrows knitted (e.g., What, When ?, How ?, Where ?, Why ?)
wô head tilted back and wide-opened eyes (e.g., Who ?)

TRUNK

—| forward
|— backward
ΛS_0 shrugging shoulders alternately
ΛS shrugging both shoulders simultaneously
Λ shrugging one shoulder

Two nonmanual expressions can occur simultaneously. For example, in LSCB, interrogation and negation can be expressed together by shaking the head sideways (no), knitting the eyebrows and moving the trunk forward, all followed by tilting the head back. Of course, some combinations are impracticable, such as shaking the head sideways while nodding.

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