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PER HAGE

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A FURTHER APPLICATION OF MATRIX ANALYSIS TO COMMUNICATION STRUCTURE IN OCEANIC ANTHROPOLOGY

PER HAGE[1]

Prof. Yves Lemaitre (1970) shows how the matrix operations of the product (AA), Boolean addition (1 + 1 = 1), the transpose (A^t) and elementwise multiplication (A x B) may be used to find equivalence classes in a communication network. Lemaitre's proposal amounts to an independent invention of certain aspects of graph theory. Thus, his matrices A and S^n correspond to the adjacency and reachability matrices A and R^n of a graph or digraph and his figure on page 104 which joins the equivalence classes found from $S \times S^t$ (which "en quelque sorts 'debrouille' les relations") corresponds to the condensation of a digraph into its strong components (Harary, Norman and Cartwright 1965).

Lemaitre's example is a directed graph of traditional inter-island voyaging in Oceania where the matrix A shows one-step connections between pairs of islands i and j, the matrix S shows multiple step connections between i and j and the matrix $S \times S^t$ shows (in its rows) the maximal subgraphs of mutually reachable islands. The condensed digraph which

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consists of these subgraphs and their adjacency relations provides a simple and clear model of the communication possibilities in the entire area.

Lemaitre's proposal is of areal significance given the renewed interest in the settlement of Oceania, e.g., Levison, Ward and Webb (1973), Finney (1976), and of methodological significance given the increasing anthropological recognition of graph theory as a metalanguage for the analysis of morphological properties of social networks, e.g., Barnes (1969, 1972), Mitchell (1969), Hage (1979).

His analysis suggests a further anthropological application, also from Oceania, in which matrix operations and theorems from Harary (1959, 1969) can be used to elucidate certain cyclic properties in the undirected graph of a communication structure. The example chosen is an inter-household exchange system in a New Guinea village described in Schwimmer (1973, 1974). The matrices shown and used in this analysis were computed with "DIG: a DIGraph program for the analysis of social networks" (Putnam and Hage n.d.).

AN INTER-HOUSEHOLD EXCHANGE SYSTEM

As Nadel (1957), Barnes (1969, 1972), Mitchell (1969) and others have emphasized, the significant feature of network analysis is its concern with the relations between the relations. A network is more than the sum of its dyads and its patterning is not reliably revealed by their number or density. The significance of patterning is well illustrated in E. Schwimmer's (1973, 1974) analysis of a system of gift exchange among the Orokaiva which shows how a set of relations which are limited in number combine in a certain way to create a global structure which unites most of the participants. The analysis which follows generalizes his observation on the significance of circuits or cycles in a network.

Schwimmer describes a system of gift-giving in a Papuan village consisting of 22 households. The gift consists of cooked taro, presented by women but conceived of as mediating relations between households.

> It was my impression that those households which maintained the most frequent informal association with one another were also those who made each other the most frequent taro gifts. These were the households who were deeply dependent on one another in sentiment, in economic tasks, in political support. They would tend to side with one another in village disputes. Thus, taro gifts were a ritual statement in which one household expressed its desire for close association with another (Schwimmer 1973:127-128).

An ethnographically unusual and important feature of Schwimmer's analysis is his definition of "significant social relations" in terms of the frequency of taro transactions:

> In Sivepe the number of exchange partners maintained by a household varies from three to fourteen, with an average of 8.5. The first (i.e. most frequent) partner is involved in an average of 38 per cent of all taro transactions, the second partner in 24 per cent, the third in 12 per cent while the other partnerships account for only 26 per cent of all taro transactions. It is clear that the first and second partners are in a very privileged position in comparison with the others (Schwimmer 1973:129).

A significant and pervasive aspect of social interaction in Orokaiva generally and in this particular system is the making of requests not only between preferential partners but also between widely separated households joined by a series of mediating links:

> The Orokaiva are wary of relations with persons with whom they are not intimate. They do not like to make even simple requests of such persons but prefer to make the request through intermediaries who are intimate with both sides. In that case it is possible, if the request cannot be met, for both sides to pretend that it was never made. No hard words are spoken, nobody is overtly humiliated. There is a customary blank, expressionless style which is used by intermediaries for the transmitting of requests--symbolic gestures of neutrality. The mediator avoids the risk of endangering his own alliance while still making what effort he can to satisfy the party seeking the benefit. If he is successful, the beneficiary is to some extent indebted to him (Schwimmer 1974:232).

It appears that mediating chains may be of considerable length:

> The procedure followed is that A and B use one or more persons to mediate between them. In the simplest case, where A and B are separated by only one link in the chain, they share the same intimate associate who can act as mediator. If they do not share an intimate, i.e. if they are separated by two or more links in the chain, an accommodation may be reached through a series of mediators who act vicariously on behalf of the principal parties (Schwimmer 1973:134).

Schwimmer's graph analysis shows two structural features of the system: the "cluster," defined as "intimate one-degree links between all the members" which would correspond to a maximal complete subgraph and the "circuit," which unites households through intermediaries, which would correspond to a cycle in a graph. The circuit he notes "... is close to what graph theorists call a Hamilton circuit (Harary and Ross

1954; Flament 1968) and would be consistent with an almost total absence of stratification" (Schwimmer 1974:231). Schwimmer enumerates two such circuits (cycles): "The largest circuit or near-circuit connects the great majority of households [19 of 22] of all clans and may thus be considered a unifying principle for the village as a whole. The second much more strongly constructed, links 11 households of the Jegase-Sorovoi clans, but includes not a single Seho household" (Schwimmer 1973:132).

While neither of these cycles is actually Hamiltonian, that is, they are not spanning cycles or ones which pass through every point of the graph, Schwimmer's remarks about the association of such cycles with structural unity and a relative absence of stratification raise a general question about the significance of cycles *per se* in graphs of communication structure. While it is difficult to determine whether a graph is Hamiltonian, i.e., whether every point lies on a single, complete cycle of G since there is no unique criterion for such a graph (Harary 1969), it is relatively easy to determine whether every *pair* of points lies on a *common* cycle of G. The latter condition is true of any cyclic block which may or may not be Hamiltonian. In many situations it may be useful and sufficient to simply know this. If G is a cyclic block than certain conclusions can be drawn about the integrity, the flow of information and the political character of the system insofar as this character is morphologically determined.

ARTICULATION POINTS AND BLOCKS

Every pair of points lies on a common cycle of G if G is a cyclic block, that is if G has at least three points, is connected and contains no articulation points (points whose removal disconnects a graph or component of a graph). These properties are given by the adjacency, reachability and distance matrices of a graph, A(G), R(G) and N(G).

Figure 1 shows the adjacency matrix of the graph G of the taro exchange system. It was constructed from Table VI/2 in Schwimmer (1973) which gives the preferential partners of each household, by symmetrizing the directed graph of the first and second choices (i.e., i and j are adjacent if either is a first or second partner of the other: $(A + A^t)\#$). (This would appear to be empirically justified since no distinctions are made as to the direction of the gift relation and since in all cases except one, if i chooses j as a first or second partner, then j chooses i at least as a third partner.) Figures 2 and 3 show the reachability and distance matrices constructed using the following theorems from Harary, Norman and Cartwright's Structural models (1965) for digraphs which also apply to graphs. (The matrices R(G) and N(G) are partitioned to show the components of G--the maximal connected subgraphs):

THEOREM 5.7. _For every positive integer n,_

$$\underline{R_n = (I + A + A^2 + \dots + A^n)\# = (I + A)^n\#}$$

COROLLARY 5.7a

$$\underline{R = (I + A + A^2 + \dots + A^{p-1})\# = (I + A)^{p-1}\#}$$

(Harary et al. 1965:122)

THEOREM 5.19. _Let N(D)_ = $[d_{ij}]$ _be the distance matrix of a given digraph D. Then,_

1. _Every diagonal entry_ d_{ii} _is 0,_
2. $d_{ij} = \infty$ _if_ $r_{ij} = 0$_, and_
3. _Otherwise,_ d_{ij} _is the smallest power n to which A must be raised so that_ $a_{ij}^{(n)} > 0$_, that is, so that the i, j entry of_ $A^n\#$ _is 1._

(Harary et al. 1965:135).

A(G)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
2	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
4	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
6	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
8	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0
12	0	0	0	0	0	0	0	0	0	0	1	0	1	1	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	1	1	0	1	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0
17	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1
19	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
20	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0
21	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0

Figure 1. Adjacency matrix A(G) of the taro exchange system

R(G)

	1	2	3	4	5	6	7	11	12	13	14	15	16	17	18	19	20	21	22	8	9	10
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
11	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
12	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
13	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
14	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
15	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
16	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
17	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
18	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
19	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
20	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
21	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
22	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1

Figure 2. Reachability matrix R(G) of the taro exchange system

N(G)

	1	2	3	4	5	6	7	11	12	13	14	15	16	17	18	19	20	21	22	8	9	10
1	0	1	2	3	4	5	5	6	5	5	4	3	2	1	2	4	6	5	3	*	*	*
2	1	0	1	2	3	4	5	6	6	6	5	4	3	2	3	5	5	4	4	*	*	*
3	2	1	0	1	2	3	4	5	5	5	4	3	2	1	2	4	4	3	3	*	*	*
4	3	2	1	0	1	2	3	4	5	6	5	4	3	2	3	4	3	2	4	*	*	*
5	4	3	2	1	0	1	2	3	4	5	5	5	4	3	4	3	2	1	4	*	*	*
6	5	4	3	2	1	0	1	4	5	6	6	6	5	4	4	2	3	2	3	*	*	*
7	5	5	4	3	2	1	0	5	6	7	7	6	5	4	3	1	4	3	2	*	*	*
11	6	6	5	4	3	4	5	0	1	2	2	3	4	5	6	6	1	2	7	*	*	*
12	5	6	5	5	4	5	6	1	0	1	1	2	3	4	5	7	2	3	6	*	*	*
13	5	6	5	6	5	6	7	2	1	0	1	2	3	4	5	7	3	4	6	*	*	*
14	4	5	4	5	5	6	7	2	1	1	0	1	2	3	4	6	3	4	5	*	*	*
15	3	4	3	4	5	6	6	3	2	2	1	0	1	2	3	5	4	5	4	*	*	*
16	2	3	2	3	4	5	5	4	3	3	2	1	0	1	2	4	5	5	3	*	*	*
17	1	2	1	2	3	4	4	5	4	4	3	2	1	0	1	3	5	4	2	*	*	*
18	2	3	2	3	4	4	3	6	5	5	4	3	2	1	0	2	6	5	1	*	*	*
19	4	5	4	4	3	2	1	6	7	7	6	5	4	3	2	0	5	4	1	*	*	*
20	6	5	4	3	2	3	4	1	2	3	3	4	5	5	6	5	0	1	6	*	*	*
21	5	4	3	2	1	2	3	2	3	4	4	5	5	4	5	4	1	0	5	*	*	*
22	3	4	3	4	4	3	2	7	6	6	5	4	3	2	1	1	6	5	0	*	*	*
8	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	0	1	1
9	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	1	0	1
10	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	1	1	0

Figure 3. Distance matrix N(G) of the taro exchange system

G has 22 points and contains two components, c_2 which consists of households 8, 9, 10 and c_1 which consists of the remaining 19. C_2 is obviously a block; the status of c_1 can be determined by checking for articulation or cut points using two theorems in Harary (1959) on N(G) and then the delete point provision of DIG.

The first theorem is based on the concept of the relative peripherality of the points in a graph:

> The associated number of a point of a connected graph is the greatest distance between this point and all other points. A peripheral point ... is a point whose associated number is maximal. A point c is relatively peripheral from point b if the distance between b and c is equal to the associated number of b. We are now able to state two sufficient conditions, one of which assures us that a point is not a cut point and the other that it is.
>
> THEOREM: All relatively peripheral points are not cut points. (Harary 1959:390).

In the distance matrix of a graph the relatively peripheral points are those with the largest number in each row. In N(c_1) points 1, 6, 7, 11, 12, 13, 14, 15, 16, 18, 19, 20, 21, 22 are not articulation points.

The second theorem is based on the concept of a point at a unique distance from another point:

> THEOREM: If b is a point of a connected graph and c is the only point at a certain distance from b less than the associated number of b, then c is a cut point (Harary 1959:390).

N(G) reveals no such point in c_1, i.e., no rows in which there is a unique occurrence of a number less than the associated number of P.

The status of the remaining points can be determined by deleting and then restoring each one and noting in each case whether or not R(G) remains universal. The result for c_1 is that none of the remaining points are articulation points, so c_1 is a block.

To say that a connected graph of three or more points has no articulation points is equivalent to saying that every pair of points lies on a common cycle. The following theorem from Harary (1969:27-28) gives the full implications of this statement and thereby the conclusions which may be drawn about the structure of c_1. (The analysis is based on the example for digraphs in Harary et al. 1965:249-250.)

THEOREM 3.3. Let G be a connected graph with at least three points. The following statements are equivalent:

(1) G is a block.

(2) Every two points of G lie on a common cycle.

(3) Every point and line of G lie on a common cycle.

(4) Every two lines of G lie on a common cycle.

(5) Given two points and one line of G, there is a path joining the points which contains the line.

(6) For every three distinct points of G, there is a path joining any two of them which contains the third.

(7) For every three distinct points of G, there is a path joining any two of them which does not contain the third.

(Harary 1969:27-28)

The application of Theorem 3.3 reveals a number of structural properties of c_1, the large group which comprises all but three households in the village. By condition (1) there is no household whose disappearance or demise will disrupt communication within the group and by condition (4) if any one household loses or falls out with a preferential partner any two households will still be able to communicate. The system is therefore structurally invulnerable. By condition (2) every pair of households has (at least) two alternative paths by which to make requests of each other. Further, by condition (5) any particular link may occur in a mediational chain between any two households and by conditions (6) and (7) any particular household may occur in a chain but no particular household must necessarily occur. The communication structure is therefore flexible in the alternative channels it allows. And it is egalitarian or unstratified to the extent that relative power depends on the ability of any one household to uniquely control the flow of information between any two households or sets of households.

This analysis does not of course exhaust the empirically significant features of the taro exchange system. Thus, certain households may communicate more frequently, more effectively and so on. But it does serve to operationalize concepts such as "egalitarian," "flexible" and "invulnerable." It reveals certain global properties of the system and thereby defines the constraints within which actual communication must operate.

Two further general observations may be made on the empirical significance of block structures in social organization.

1. Blocks and group size. In a complete graph kp which has p(p-1)/2 relations the communication structure is invulnerable and flexible. If a group expands in size while the number of relations remains the same or does not increase commensurately (assuming some limitation on the quantity of energy, e.g. taro gifts, at the disposal of each unit to communicate) these properties may still be preserved if the structure remains a block, which may have as few as p relations. All that is required is the addition of a limited number of new relations strategically

placed and/or the redistribution of some previous relations. A block is a minimal means for the creation of a large cohesive structure.

2. Blocks and norm enforcement. In a block there is provision for flexible communication since A may use <u>either</u> path 1 or path 2 in communicating with B and also for redundant communication since A may use <u>both</u> paths 1 and 2 to transmit the same message to B. Mitchell has observed that,

> The sociological significance of the notion of reachability lies in the way in which the links in a person's network may be channels for the transmission of information including judgements and opinions especially when these serve to reinforce norms and bring pressure to bear [on] some specified person (Mitchell 1969:17).

In blocks such pressure may be multiplied by the transmission of the same message which arrives at a person through multiple channels (as exemplified in one meaning of the expression, "he was getting it from all sides").

DISCUSSION AND GENERALIZATION

The preceding analysis shows how matrix operations and theorems from graph theory may be used to elucidate the communication structure of a group, with particular reference to blocks in undirected graphs. Related potential anthropological applications are contained in the following concepts and references.

<u>Clusters in signed graphs</u>

A signed graph S is one in which the lines have positive or negative values which represent antithetical relations. An illustration comes from the data in M. Young's (1971) monograph which describes a system of competitive food exchange on Goodenough Island in Melanesia. The system consists of nine patrilineal clans or clan segments each of which has traditional food friends, fofofo, and food enemies, nibai. Nibai make competitive gifts of food, niune, to each other (which the recipients do not consume but pass on to their respective fofofo). In confrontations between groups, those connected directly and indirectly by fofofo links support each other. The system may be represented by the s-graph in Figure 4 in which positive (fofofo) relations are shown by solid lines and negative (nibai) relations by broken lines.

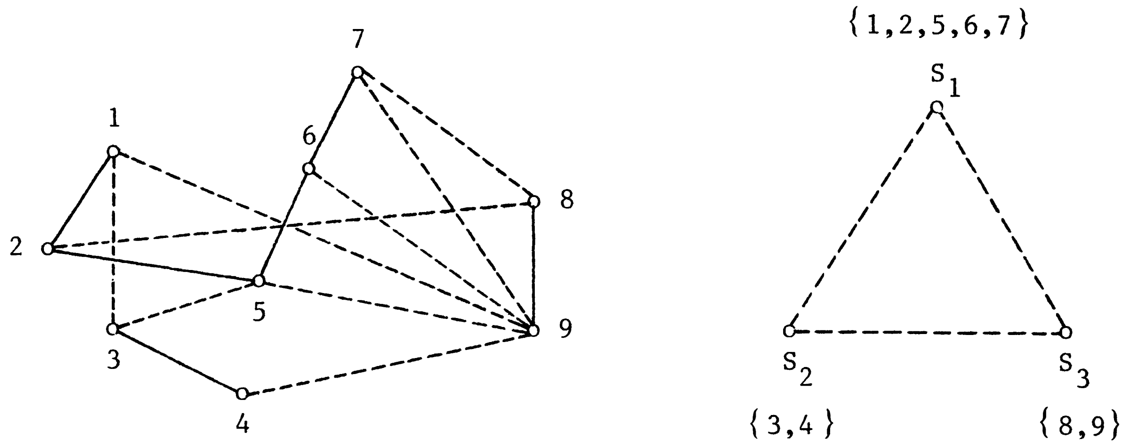

Figure 4. A signed graph S of inter-clan competitive food exchange in Kalauna

An empirical prediction about structures consisting of antithetical relations is that they will contain clusters such that all positive relations join units within subgroups (clusters) and all negative relations join units in different subgroups (Davis 1967). Figure 4 for example contains three clusters (S_1, S_2, S_3). If the structure of a group is clusterable, there is no contradiction such that an enemy of a friend is a friend (or such that a friend of a friend is an enemy or that a friend of an enemy is a friend). (If a group is polarized into two clusters then it is also true that an enemy of an enemy is not an enemy.) Such structures are presumably stable and it is significant that Young describes the clan alignments as corresponding to a "general balance of power." One criterion for a clusterable s-graph is in terms of the cycles of a graph:

> THEOREM 1. Let S by any signed graph. Then S has a clustering if and only if S contains no cycle having exactly one negative line (Davis 1967:181).

Additionally and equivalently, because "S has no negative line joining two points in the same positive component" (Cartwright and Harary 1968: 85, Theorem 1).

The clustering of Figure 4 is apparent by inspection. For large s-graphs Gleason and Cartwright (1967) have shown how matrix operations may be used to determine (1) whether S is clusterable, (2) what its clusters are and (3) whether the clustering is unique. Also, Peay (1970) provides a method for determining whether there is a statistically significant tendency towards clustering in an s-graph. These concepts and methods may have wide application in Melanesian ethnography where tribal structure often involves traditionally defined friend/enemy relations existing among a large number of groups (Strathern 1969, Berndt 1964, Hage 1973). We may note in passing that Ryan's (1959) concept of "clan cluster" in the social structure of the Mendi in Highland New Guinea would easily be treated as a special case of clusterability in s-graphs--in the example Ryan gives, of unique 2-clustering.

Connectedness in directed graphs

One matrix of particular interest with respect to Lemaitre's method is the connectedness matrix C(D). In Lemaitre the equivalence classes are found by the elementwise multiplication of the reachability matrix and its transpose, $S \times S^t$. A more general matrix, given in Harary et al. (1965), is the connectedness matrix, C(D) which is based on the partitioning of D into its weak components and the addition of R and R':

> THEOREM 5.18. <u>For any digraph D, the connectedness matrix $C = [c_{ij}]$ is obtained from the reachability matrix $R = [r_{ij}]$ as follows</u>:
>
> 1. <u>If v_i and v_j are in the same weak component, $c_{ij} = r_{ij} + r_{ji} + 1$.</u>
> 2. <u>Otherwise, $c_{ij} = 0$</u> (Harary et al. 1965:133).

C(D) provides the following information about the structural properties of D based on the entries 0, 1, 2, 3: (1) the connectedness of D, C_0 (disconnected), C_1 (weak), C_2 (unilateral), C_3 (strong)[2] (given by the minimum entry 0, 1, 2, 3 in C); (2) the connectedness of every pair of points (given by the entry in c_{ij}); (3) the strong components of D (given by the entries of 3 in the rows of C). In the example of inter-island communication, one would know from this matrix the reachability relation for both the entire digraph and for every pair of islands as well as the subsets of mutually reachable islands. This matrix, in turn, could be exploited to gain new information. For example, the structural significance of each island in the network of communication could be determined by deleting it and seeing how its presence or absence affects the relation of reachability in the network (i.e. the connected-

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2 D is weak if every pair of points is joined by a semipath, unilateral if for every pair of points at least one can reach the other and strong if every two points are mutually reachable (Harary et al. 1965).

ness category of D). (See the discussion of strengthening, weakening and neutral points in Harary et al. 1965:230-241.)

Path strength in valued graphs

A valued graph is one in which numerical values are assigned to the lines, which may represent the strength of a relation between a pair of points. Doreian (1974) has shown how standard matrix methods may be modified to determine the strength of a reachability relation between every pair of points. This procedure might be useful in a case such as Schwimmer's which distinguishes thresholds of social relationships (first, second and third preferential partnerships) or in one such as the recent computer simulation of drift voyages in Oceania (Levison, Ward and Webb 1973) which shows digraphs of inter-island contact similar to those in Lemaitre except that the lines are drawn in different thicknesses to show different categories (from two to five) of the likelihood of contact.

CONCLUSION

This paper provides a further application of Lemaitre's matrix methods for the analysis of equivalence classes in an inter-island communication network by showing how, in conjunction with a set of theorems from Harary, they may be used to elucidate cyclic blocks in an inter-household exchange system. Lemaitre's methods can be subsumed under the more general mathematical theory of graphs. This theory contains additional matrices based on the same operations and distinguishes different types of graphs --directed, undirected, signed and valued graphs--each of which may be coordinated to particular types of communication and social structures found in Oceanic ethnography and presumably generally.

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