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CASANOVA, "BONAPARTE", AND THE LOTERIE DE FRANCE

Stephen M. STIGLER *

RÉSUMÉ

La loterie de France a débuté avec l'aide cruciale de Casanova et ses activités se sont déroulées de 1758 à 1836 avec peu d'interruptions et grand succès. Une source d'information peu utilisée ouvre une fenêtre sur les activités de la Loterie, éclairant la façon dont le risque était ressenti à cette époque. Elle révèle aussi certains aspects des personnages participant à cette Loterie, leur manière de parier et le montant de leurs paris. Cet article discute aussi le rôle de la Loterie dans l'encouragement de l'étude des probabilités et de l'analyse combinatoire.

ABSTRACT

The Loterie de France was begun with a crucial assist by Casanova, and it operated from 1758 to 1836 with few interruptions and great success. An unusual data source opens a window on the operation of the Loterie, shedding light on how risk was viewed in that era. It also reveals some of the characteristics of the people who participated, and how and how much they bet. The role of the Loterie in encouraging the study of probability and combinatorics is discussed.

1. Introduction

Anyone with even a casual interest in the history of probability knows that gambling motivated early interest in the subject. Yet many, I suspect, treat this fact as quaint and slightly embarrassing. They tend to tell the story much as they would tell an anecdote about a long-dead ancestor, one who engaged in colorful but ultimately frivolous behavior and should not be taken as representing our present, more serious purpose. Yes, they might say, we once considered games, but only as simple abstractions; we are really scientists, not deeply concerned with frivolous pursuits.

But were the games our ancestors studied simply frivolous pursuits? Were they only incidental to our history, important only as rhetorical devices that provided a grounding for abstract probability theory? The story of the Loterie de France suggests otherwise. That lottery operated from 1758 to 1836 and it was a truly grand enterprise: over its lifetime it contributed huge

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sums to the state treasury of France, as much as 2% to 4% of the annual national budget at one time, more than postal and customs levies (Leonnet, 1963). By 1811 it ran more than 1000 offices across France. It developed an energetic communications network before the telegraph, with carriages and riders conveying the results of as many as 15 drawings a month to and from Paris. The Loterie probably did more for public awareness of – and education in – the calculus of probabilities than any other state effort, before or since. And it gives us a rare window into early ideas of risk and how its perception was related to the evolving calculus of probabilities.

The story of the Loterie begins, strangely enough, with Giacomo Casanova. Casanova was many things: a Venetian adventurer who (based upon self-reporting) is best known today as a lover. He was also at various times a military officer, a gambler, and a secret agent (Masters, 1969). In his youth he took religious orders and briefly aspired to be Pope. He sought fame and fortune by any means at hand; he even dabbled in mathematics. But the central focus of his life was the pursuit of risk.

2. Casanova

Casanova understood risk. He was born in Venice in 1725, died in Bohemia in 1798, and his multi-volume posthumously published *Memoirs* tell of a long sequence of adventures in which the common element is a calculated awareness of risk. In his romantic adventures, the risk of sexually transmitted disease was willingly accepted: *"I have often remarked that the greatest part of my life was spent in trying to make myself ill, and when I succeeded, in trying to recover my health."* (Casanova, 1930, 2:212-213). He faced risk in battle and diplomatic intrigue, but his own accounts dwell more frequently on his experiences as a gambler. *"That passion was rooted in me; to live and to play were to me two identical things."* (Casanova, 1930, 2: 440). Casanova's favorite game was the card game faro (also known as pharaon or pharaoh), an antecedent to modern casino games that was extremely popular through the 18^{th} century in Europe, and in the 19^{th} century in the American West. Casanova understood the odds at faro, which had already been thoroughly discussed by De Moivre and by Montmort, and he properly preferred to hold the bank. He did not restrict his play to favorable games, however, and he seems to have lost as frequently (or at least as much) as he won. He used a martingale betting system on at least one occasion (Casanova, 1930, 2: 459). Casanova knew sufficient mathematics to analyze simple games and left some work in geometry (Henry, 1882). He also played the lottery. He records that in 1750 he won 3,000 ducats at the lottery at a time when he also had large winnings at faro (Casanova, 1930, 2:85). In 1756 Casanova ran afoul of the Venetian Inquisition, and without formally being charged or sentenced he was placed in Venice's dreaded prison "The Leads," so named for the large slabs of lead that formed the roof. Apparently acting on a tip from one of his enemies, the Inquisitor had raided his home and found cabalistic books Casanova had been reading for amusement. Casanova tells us some of the works they found (various occult books such as Instructions on the Planetary Hours), and these were sufficient for his imprisonment without trial. After 15 months in what were dungeon-like conditions, Casanova executed a daring and clever escape from The Leads, and succeeded in reaching safety beyond Venetian territorial boundaries, from where he made his way to Germany and then to France. He had lived in Paris during 1750-52, and with the assistance of friends he re-established himself there. The international fame he had acquired as a result of his spectacular escape made him a lion of French society. Among others, the king's mistress Madame de Pompadour wanted to hear his dramatic story first-hand, and he gladly obliged.

In Paris as a result of these social contacts he found himself engaged in a project to raise money in support of the new Ecole Militaire, a favorite project of Madame de Pompadour. An Italian, a M. Calsabigi, had proposed that money be raised through a lottery conducted along the lines of the Genoese lottery. Casanova, feigning that he had the same idea himself, joined forces with Calsabigi and was instrumental in convincing a very conservative State Council that the idea was sound.

Casanova argued that the announcement that the lottery was backed by the king, and that the king stood prepared to lose up to a hundred million francs, would dazzle people. The Councilors were taken aback by this prospect, even when Casanova reassured them that before the Crown would lose a hundred millions it would receive at least a hundred and fifty millions. The reaction, according to Casanova's account, was concern. A Councilor replied,

> "I am not the only person who has doubts on the subject. You must grant the possibility of the Crown losing an enormous sum at the first drawing?
>
> Certainly, sir, but between possibility and reality is all the region of the infinite. Indeed, I may say that it would be a great piece of good fortune if the Crown were to lose largely on the first drawing.
>
> A piece of bad fortune, you mean, surely?
>
> A bad fortune to be desired. You know that all the insurance companies are rich. I will undertake to prove before all the mathematicians in Europe that the king is bound to gain one in five in this lottery. That is the secret. You will confess that the reason ought to yield to a mathematical proof?
>
> Yes, of course; but how is it that [Calsabigi] cannot guarantee the Crown a certain gain?
>
> Neither [Calsabigi] nor anybody in the world can guarantee absolutely that the king shall always win. What guarantees us against any suspicion of sharp practice is the drawing once a month, as then the public is sure the holder of the lottery may lose."

In subsequent testimony before the Council, Casanova answered all objections, and after a three hour session that included testimony by the great D'Alembert, the project – with the financial backing of the Crown – was approved. Calsabigi was made superintendent of the lottery and Casanova was awarded a pension and six sales offices, five of which he sold for 2,000 francs (about 8000 Euros) each. He would run the sixth himself, on Rue St. Denis, with his valet as clerk (Casanova, 1930, 2: 701-704, 710-711).

3. Early Lotteries

There were by the eighteenth century two basic types of lotteries. The dominant one has been called a *Blanks* lottery; it was of a type we might now term a sweepstakes or a raffle. A fixed schedule of (usually monetary) prizes would be announced and a predetermined quantity of numbered tickets sold. The drawing would consist of simultaneously selecting tickets at random from two different "wheels of fortune", cylindrical boxes or cages that held the tickets and could be rotated to mix them. One was a wheel containing sequentially numbered tickets representing the tickets sold, and the other was a wheel with an equal quantity of tickets, some of them labeled as prizes to be awarded and the remainder (the majority) blank. In each selection a numbered ticket randomly drawn from the first wheel would be paired with a prize or a blank, randomly drawn from the second wheel, determining the fate of the holder of the numbered ticket. When both wheels were empty the drawing was complete. Many such lotteries, by the State or privately run, were held in England and on the continent from the 16^{th} century on. In France they dated from 1533, well before the arrival of Casanova.

Figure 1 shows an advertising bill for an English lottery in 1811, which was typical of the type of lottery conducted 169 times by the State in England between 1694 and 1826.

In that 1811 English lottery, 2754 of a total of 13,500 tickets would be awarded prizes, and the total value of the prizes offered was £135,000. At the drawing (which could take a week or a month or more) a total of 13,500 pairs of tickets would be drawn. Of the tickets drawn, 2754 would list the various cash prizes, and the remainder (the majority, numbering 10,746) would be blank, hence the name *Blanks lottery.*

With 13,500 tickets and prizes totaling £135,000, the break-even price for a ticket in that 1811 lottery would then be £10. The State would sell the tickets to agencies (*e.g.* Bish) under contract for (say) £17 each, and the agency would sell a single ticket for perhaps £20 to £24, depending upon demand. This was a large sum, beyond the means of most of the betting public. To open the lottery to a larger audience the agency would also offer fractional tickets at proportionally higher prices, the smallest fraction being one-sixteenth for £2. The schedule of prizes and prices was constructed so that if all or most of the tickets were sold, both the agency and the State would profit.

The second type of lottery evolved in Italy and apparently remained there until the middle of the eighteenth century; it has been called the Genoese lottery. It was a close cousin to modern Lotto. Bellhouse (1991) describes its early history. Apparently in the sixteenth century several Italian city states selected their governing councils by lot from a list of those deemed eligible.

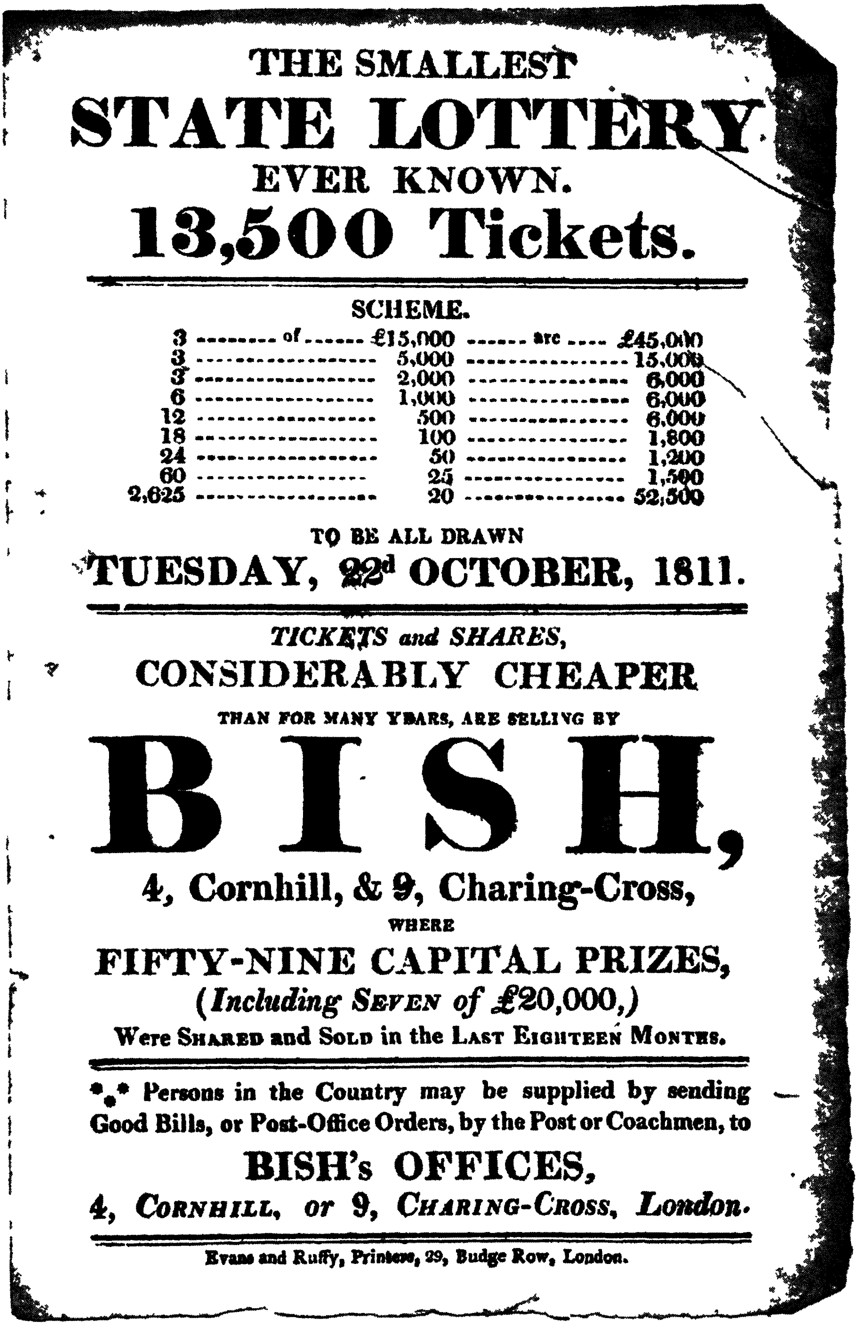


FIG 1 – A lottery bill for the simplest type of "Blanks" lottery

This bill was issued by the Bish agency in London in 1811, and offered prizes totaling £135,000 in value From the author's collection

For example, from among 90 eligible citizens, five names might be drawn at random. These *elections* attracted some betting on the outcome, and by the early seventeenth century an enterprising Italian in Genoa realized that the betting need not be coupled with an election. A set of balls numbered from 1 to 90 would be placed in a round cage, one smaller than those used in the Blanks lotteries but also called a *wheel of fortune*. After the wheel was rotated and the balls well-mixed, five balls were selected at random, without replacement. The citizens of the city would be permitted to bet before the drawing that certain numbers or combinations of numbers would be drawn, and if lucky they would be paid off at pre-specified odds.

A simple version of this type of lottery had in fact been introduced in China during the Second Han Dynasty, in about 947 AD. It was called the Hua-Hoey Lottery, or the Game of 36 Animals. Thirty-six cards were prepared, each with a different animal depicted. Bets could be placed upon any card, and the gamblers would then assemble to witness one of the cards being selected at random. Payoff odds apparently varied; at one time the operator would wait until all bets were placed and then he himself bet on the animal with the smallest number of bets, the pool of all bets then to be divided evenly among all the winners. At a later time the operator simply paid the winners at 20 to 1 odds (35 to 1 would have been an even bet). The lottery persisted in some Asian locations at least through the 19^{th} century, and it was even imported to Chinese communities in Europe and America. (Charpentier, 1920; Kynnersley, 1885). Might Marco Polo have brought it back to Italy among his other treasures and thus influenced the development of the Genoese lottery? He was, after all, once held prisoner in Genoa, where he famously recounted his Asian adventures.

In any event, it was a version of the Genoese lottery that Casanova and Calsabigi brought to Paris in 1757.

4. The Lottery for the Ecole Militaire

Casanova's Loterie de l'Ecole Royale Militaire began drawing in April 1758 and continued every other month until October. From then until August 1776 (when the king took the Loterie over as the Loterie Royale de France and doubled the frequency of drawings), drawings were held monthly (the only exceptions were December 1758 and two months missed in 1759). Each drawing (a *tirage*) consisted of a sequence of 5 integers drawn without replacement from a wheel of fortune containing a set of tokens numbered 1, 2, 3, ..., 90, the draw being performed in a public room by a small, blindfolded boy wearing a blue suit (a common uniform in foundling homes). For example, a tirage might produce the result 50, 83, 5, 41, 15. The prize structure Casanova and Calsibigi introduced was never altered over the course of the Loterie; the only changes were in the variety of bets offered. The odds did not change, nor did the number of balls in the wheel or the number of balls drawn. The most extensive set of bets and their payoffs are described in Table 1.

TABLE 1. – The bets available on the Loterie, the payoff odds, and the fair odds[1].

Type of Bet	Minimum Bet Allowed	Actual Payoff: Multiple of Amount Bet[2]	Fair Payoff: Multiple of Amount Bet
Extrait simple[3]	5 sous (=25 centimes)	15	18
Extrait déterminé	2 sous (=10 centimes)	70	90
Ambe simple	1 sou (=5 centimes)	270	400.5
Ambe déterminé	1 sou (=5 centimes)	5,100	8,010
Terne	1 sou (=5 centimes)	5,500	11,748[4]
Quaterne	1 sou (=5 centimes)	75,000	511,038
Quine	1 sou (=5 centimes)	1,000,000	43,949,268

[1] Not all bets were available at all times. Initially the Loterie de l'École Royale Militaire offered only the extrait simple, the ambe simple, and the terne, but by the late 1770s all seven bets were permitted. The quine was suppressed at various times, including most years after 1803. The player would pay the Loterie at the time the bet was placed, and then be paid off (or not) a multiple of the bet placed, according to the schedule presented in this table. Starting in November 1797 the minimums were coupled with the rule that the total bet was required to be at least 50 centimes (initially the minimum was one franc). From 1800 on the Loterie also specified upper limits, but only on the amount bet on a quaterne (12 francs) or a quine (3 francs). Parisot (1801, p. 25) says (in reference to an earlier period) that upper limits were usually not enforced.

[2] So the player's net gain is one unit less than this, taking into account the unit paid to the Loterie.

[3] VERY roughly, 5 sous = one Euro.

[4] e.g. Prob match all three = $\binom{3}{3}\binom{87}{2}\Big/\binom{90}{5} = 1/11,748$, or odds of 11,747 to 1. So the player should get 11,747 times the amount bet *plus* the unit initially paid in order for the bet to be fair.

When the Loterie was launched in 1758 only three bets were available: For any day's tirage the player could specify a bet on a single number and would win if it appeared in any position (the bet was called an *extrait* or *extrait simple*). Or the player could bet on two numbers in any positions (an *ambe* or *ambe simple*), or on three numbers (a *terne*). Within a few years the menu of choices was expanded to include four numbers (a *quaterne*), five numbers (a *quine*), and the possibility of choosing either a single number in a *specific* position (an *extrait déterminé*) or a similarly determined pair (an *ambe déterminé*) where both the two numbers and their exact positions were specified. The payoff odds varied with the bet. A bet of one franc on an extrait, that is, one franc being paid to purchase the betting slip, would return 15 francs if successful; a franc bet on a quaterne would return 75,000 francs in the unlikely event it won.

Unlike modern lotto games, all of these bets were separately placed, but multiple bets were permitted involving the same numbers. For example, a player might focus upon five numbers (5, 39, 41, 50, 87) and make several bets: the player could for example bet 3 francs on each of the five possible extraits simples, 1.60 francs on each of the ten possible ambes simples, 1 franc on each of the ten possible ternes, and 60 centimes on each of the five possible quaternes, for a total bet of $3 \times 5 + 1.60 \times 10 + 1 \times 10 + 0.60 \times 5 = 44$ francs. Whichever of the $5 + 10 + 10 + 5 = 30$ bets were found to be winners after the tirage would be paid to the player; the others would be lost. With these bets and the sample tirage 50, 83, 5, 41, 15, the player would win :

3 Extraits (5, 41, 50) [pays $3 \times 3 \times 15 = 135$ francs]

3 Ambes ([5,41], [5,50], [41,50]) [pays $3 \times 1.6 \times 270 = 1296$ francs]

1 Terne [5, 41, 50] [pays $1 \times 1 \times 5500$ francs = 5500 francs]

Total spent: 44 francs. Total won: 6931 francs. Net Profit: 6887 francs.

Had the player bet the ambe déterminé "50 in the 1^{st} position, 5 in the 3^{rd} position", he would have won 5,100 times the amount bet; any other ambe déterminé based upon these two numbers would have lost with this tirage.

5. Why the Loterie was a Hard Sell

To a modern eye it may be hard to understand why Casanova had difficulty convincing the king's Council of the merits of the plan. The Loterie was simple in conception, the odds were clearly favorable to the king. We would expect that the only question would have been, would anyone buy a ticket at those odds? But the resistance was there, rooted in the long history of lotteries of other kinds and a natural risk aversion by the Council charged with managing the king's accounts.

The earlier and dominant lotteries of that period, the *Blanks* lotteries, had shown themselves susceptible to several problems. They depended crucially upon mounting a successful sales campaign for each and every drawing. If the sales were insufficient the lottery could lose money. Indeed, one of the earliest State lotteries in England, offered under Queen Elizabeth I in 1567-69, failed dramatically for just this reason: A total of 400,000 tickets were offered but less than 40,000 sold, leading to changes in the payoff rules, loss of confidence among the people, and loss of money by the State. By the end of the eighteenth century other problems appeared, including widespread black market sales that were called *insurance*: You could on the black market purchase insurance on a ticket number whether or not you owned it, for or against its appearance on a subsequent draw. It amounted to an unregulated and untaxed derivatives market, and attempts by the State to control it were largely unsuccessful. Other problems existed that had led to scandals in France as well. For example, Voltaire made a fortune of perhaps a million francs on a lottery of this type in 1729, by recognizing with a friend a situation where he could take advantage of poorly written rules (Donvez, 1949, pp. 37-55).

Casanova's Loterie was free of most of these problems. Where the Blanks lottery depended upon a successful sale of a pre-specified large number of tickets, the Loterie Casanova espoused did not. Even if sales were poor, the expected winnings (while lower) were still positive. The Loterie's tirage involved drawing only five numbers, not 13,500 or more, and the setup costs were correspondingly low. It was hard to stage more than one Blanks lottery a year; it was easy to hold several Loteries a month. And with the Loterie there would be no attraction for a black market: Unless the odds offered were substantially better than the king's odds, the assurance of the king's backing would surely be more attractive than trusting to payout by a clandestine operation. Only one problem was evident: Unlike modern lotto where the large prizes are paid from a fixed parimutuel pool and there is a ceiling on the payout, in Casanova's case there was the potential that the bettors would get lucky. If 100 bettors won with 1 franc bets on a quaterne, the king would be bound to pay 100 x 75,000 = 7,500,000 francs. Against this fear Casanova had to convince the Council that the law of large numbers would rule, and with supplementary testimony by D'Alembert and Casanova's appeal to the success of insurance companies, he did just that.

The Loterie flourished from the beginning. Casanova tells of carrying tickets and selling them in social gatherings he was invited to in the salons of Paris. He paid all winners promptly, even when he had to go personally into debt to do so, and he was rewarded by a vigorous business at his sales office, collecting a portion of the proceeds for himself. In September 1776 the *Loterie de l'École Royale Militaire* became the *Loterie Royale de France*, and beginning in October of that year the tirage was made twice a month. Through the early years of the revolution it did not miss a draw, but after the second tirage of November 1793 it was suppressed by an act of the Convention Nationale, on grounds of morality (and presumably because the profits had declined in those increasingly turbulant revolutionary times). In December 1797 the Directoire reestablished it under exactly the same rules as before, to gather revenue (and presumably because after reconsideration the moral problems were not judged so severe, especially since the business of the clandestine selling of tickets on foreign lotteries was flourishing).

Already by the 1780s there were Loterie offices selling tickets in Lyon, Bordeaux, Strasbourg, and Lille, and agents in other cities. Late in 1800 the Loterie began to expand. In November they went to three draws a month in Paris and initiated drawings in Bruxelles, also three per month. In December they expanded to drawings in Lyon, also three per month, and in March and in May 1801 the same plan was implemented for Strasbourg and for Bordeaux, respectively. In August 1814 the Bruxelles Loterie was moved permanently to Lille. The offices in Paris accepted bets on the regional Loteries, and each regional office accepted bets on Paris as well as on the Loterie of its own region. The timing of the draws was staggered and by mid-1801 a bettor in Paris could wager on $3 \times 5 = 15$ tirages a month!

Only the wars of 1814-1815 had a sensible impact: the Paris Loterie went to only twice a month from January-May 1815 and the regional Loteries were suppressed for a few months early in 1814 and again in 1815. Finally, by a Law of May 21, 1836, all Loteries were suppressed for reasons of public morality (and declining profits).

TABLE 2. – Dates of Tirage.

Location	Dates	Each	Month
Lille	1	11	21
Bordeaux	2	12	22
Paris	5	15	25
Strasbourg	7	17	27
Lyon	9	19	29

6. The Statistics of the Loterie

The Loterie was a standard source of simple combinatorial probability problems in the Laplacian era: What was the probability of a terne? A quaterne? A quine? And as such it gets passing mention in nearly all histories of probability. But I did not think seriously about it as a possible object for statistical study until 1994, when I acquired a small book from a French bookseller, *Almanach romain sur la loterie de France*, published in 1834 by M. Menut de St. Mesmin.

This *Almanach* was one of a series of works Menut offered for bettors on the Loterie. He lists over 25 other works, including mathematical tables and a *Répertoire Cabalistique* that may have been a descendent of the book that got Casanova into trouble in the first place! The *Almanach* itself contains all manner of useful information: the rules for the game, the locations of over 150 betting offices in the Paris area, tables of the numbers of possible combinations, and advice as to which numbers were propitious. Menut told his readers which numbers were associated with which seasons, signs of the zodiac, women's names, and which numbers were *sympathetique* with which others. He reported also on the numbers in each regional Loterie had not been drawn in a considerable number of tirages, and which pairs of numbers had not been drawn. But more to the point he gave a considerable amount of data: The winning tirage in every Loterie since the first in April 1758. Menut gave results through the end of 1833, with spaces for the reader to record those for 1834, and the first owner of this book had obliged by recording the tirages for Paris and Bordeaux through October 1834. In all there were 6606 tirages recorded, and since the Loterie was suppressed in May 1836, these constituted the vast majority of all tirages ever held. By cross-checking with some newspaper accounts and a few partial listings (*e.g.* in Parisot, 1801), I was able to verify the substantial accuracy of what was given.

The first question that comes to mind is, were the draws fair? This was, after all, a century before the development of statistical tests of fit, and the chance to be the first to apply modern technology to uncover possible subtle dependencies and biases was irresistible. The data set, partially handwritten and all in older fonts, was formidable, but with the indispensable aid of a student, Teresa Ging, they were put in machine readable form.

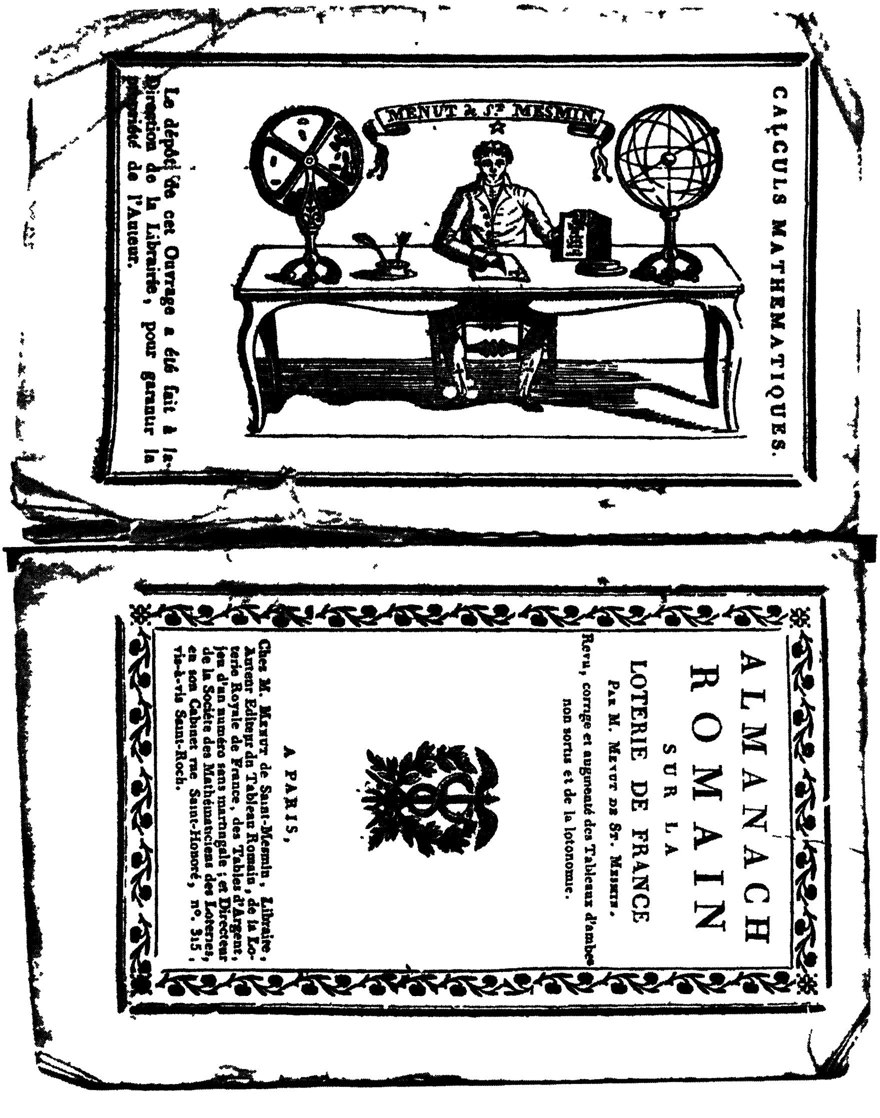


FIG 2. – The title and facing page from Menut (1834), showing a wheel of fortune.

For the Loterie to be fair – for the tirages to be true random samples from the 90 numbers available – it would be necessary to test if all possible selections were equally probable. Clearly this would not be feasible: Even neglecting the order of the numbers drawn, there were nearly 44 million possible tirages, and only 6606 data values, a ratio of 1.5 to 10,000. At that level only the most flagrant discrepancy could be detected; it would be necessary to focus upon more limited features of the data. Figure 3 shows the frequency distribution for the extrait simple, for the aggregate of the $5 \times 6606 = 33{,}030$ numbers drawn over the 76 years covered by the data.

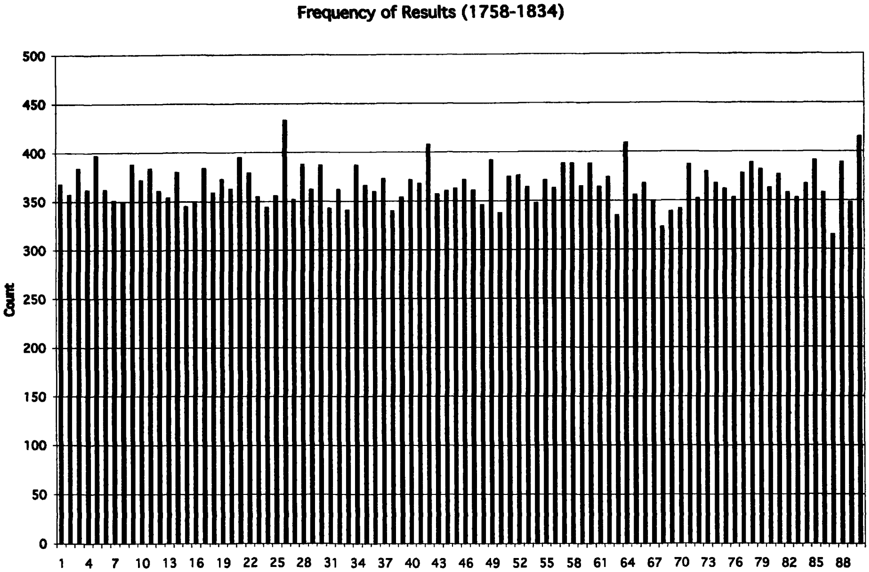


FIG. 3. – The frequency distribution for a total of 6606 tirages over the years 1758-1834, comprising 33,030 single numbers drawn in batches of 5 without replacement. Based upon data from Menut (1834).

The expected number of occurrences for each of the 90 possible extraits would be 33,030/90 = 367, and no marked discrepancy meets the eye. Because the drawing is made without replacement the standard Chi-square test is not appropriate. However McCullagh and Nelder (1989, p. 191-2) and Joe (1993) note that if the usual Chi-square statistic is multiplied by 89/85 it can be properly compared to a Chi-square distribution for 89 degrees of freedom. As Table 3 shows, this value, $\chi^2 = 100.07$, is far from statistical significance at any conventional level.

Harry Joe (1993) has also developed a corresponding test for the frequency of occurrence of pairs of numbers (i.e. for ambes) and these results are given in Table 3 as well. Whether the data are tested in aggregate, or they are tested separately for regional lotteries and early stages, no serious question of unfairness is indicated. The closest any test comes is that for single numbers in the Loterie de France (1776-1793), where $\chi^2 = 115.1$ is significant at the 5% level but not the 3% level; however in view of the number of tests performed this is unremarkable. Subsequent to the presentation of these results in Spring 2002, Christian Genest has confirmed these findings using a different test he developed with Richard Lockhart and Michael Stephens (see Genest, Lockhart, Stephens (2002), which also gives earlier references to corrections to the Chi-square test for lottery data).

This leaves open the possibility that there were interactions of higher order than two, *nombres sympathiques* in the words of Menut. As mentioned, an omnibus test of the full frequency distribution is not feasible, due to the relative shortage of data, but another possibly exists. We may ask if there were too many instances among the 6606 tirages where the same five numbers were drawn (without regard for order), or where two or more tirages contained four numbers in agreement.

The question of five numbers agreeing in two or more tirages is actually an instance of the classical *Birthday Problem*, where the usual question is, for a group of N people, what is the probability that two or more share the same birthday? The standard observation is that if N is at least 23 and all birthdays are equally likely, then the chance is above a half that two or more will agree. It is usually not emphasized that too many birthday coincidences can be taken as evidence that birthdates are not equally likely (which is in fact the case). Here we have 43,949,268 *Birthdays* (the number of *possible* tirages, unordered).

The standard calculation tells us that the chance of finding at least one matching pair would be just above a half with 7806 *people*; we have N = 6606, and for that number the chance of one or more matching pairs of tirages is 0.39. In fact, there is exactly one pair of tirages that match (although the numbers were drawn in different orders). Thus we judge the Loterie to pass this test as well, although the fact that both of these tirages occurred in Lyon in 1820, seven months apart, might raise an eyebrow. [5] If we look for fourth-order agreements, pairs of tirages that agree in four of their five numbers, we find 233 matches. The expected number of matching pairs for N = 6606 is 211, and Aldous's Poisson Heuristic gives us the approximate standard deviation as 15. The number of matches is high, suggesting some clumpiness that was not picked up in the tests of lower order interactions, perhaps, but the tendency if real is quite slight.

What other tests might be made? The possibility of serial dependence was addressed by grouping the numbers 1,..., 90 into thirds, and into decades, and in each case testing for Markov dependence within tirages. In both cases no sign of dependence was found.

Of course biased draws were not the only problem the Loterie had to worry about; there was always the possibility of fraud. The most obvious possibility is that a bettor could acquire a winning ticket after the draws were made, by counterfeit or by conspiracy. The Loterie's accounting scheme would have effectively prevented counterfeits, but conspiracy was another matter. It appears that collusion between Loterie agents and bettors did occur and that the Loterie was on the watch for it. In November 1798 the official newspaper *Moniteur Universel* (An VII 6 Brumaire) reported that one bettor named Bodin claimed a 814,000 prize for a quaterne, but that after investigation

5. The two draws were (Feb. 9) 33, 19, 78, 9, 46, and (Aug. 29) 33, 78, 9, 19, 46. The quine was not allowed as a bet at that time, but a second eyebrow might be raised by learning that at the August 29 Lyon tirage some *lucky* bettor in Paris won 42,224 francs on a quaterne by betting on 9, 19, 46, 78, and that in the year 1820 there were 5 quaternes in all drawn in Lyon between February 19 and September 19, four with Paris bets winning, while the average number of quaternes drawn per year at Lyon between 1801 and 1834 was only 27/33 = 0.82, including these five. Suspicious, but insufficient for conviction, considering the time span and the number of tirages involved in all Loteries.

TABLE 3. – Chisquares for testing for uniformity in the tirages, by single number and by unordered pair.

Lottery	Singles χ^2 (89 df)	Singles Exp. Count	Pairs χ^2 (4004 df)	Pairs Exp. Count
All together (6606 tirages; 33030 numbers)	100.07	367	3884	16.5
Ecole Militaire (215 tirages; 1075 numbers)	89.1	11.9	4105	.54
Lot. de France (413 tirages; 2065 numbers)	115.1	22.9	3942	1.03
Paris (1289 tirages; 6445 numbers)	74.9	71.6	3879	3.2
Strasbourg (1161 tirages; 5805 numbers)	90.6	64.5	3985	2.9
Lyon (1171 tirages; 5855 numbers)	109.8	65.1	4101	2.9
Bruxelles/Lille (1170 tirages; 5850 numbers)	84.3	65	4042	2.9
Bordeaux (1187 tirages; 5935 numbers)	79.6	65.9	4028	3.0

χ^2 5% point =	112.0	4151
χ^2 3% point =	115.7	4172
χ^2 1% point =	122.9	4212

he and the agent Commeau in Angers who had sold him the pre-dated ticket were awarded instead 20 years in irons. The Directoire, it was announced, was taking new measures to prevent such frauds in the future. Indeed, when the next set of instructions was issued to the bureaus (in November 1799) they were much more detailed on procedures, including stipulations on exactly how numerals were to be written to avoid ambiguity, as well as safeguards employing supervised duplicate paperwork to try to prevent frauds.

For all practical purposes the tirages of the Loterie were true random samples from the available 90 numbers. It is not hard to see how this could be true. Because the Loterie paid off at fixed odds and made no adjustment to reduce the payoff in the case of multiple winners (as the modern lotto parimutuel system does), there was a strong incentive for the Loterie to avoid bias in the draws. They would have been watching carefully for irregular frequencies, as would the gamblers. Indeed Menut's book included tabulations that would have helped with this, giving the frequency of all 90 extraits in all five Loteries, announcing that the table could reveal lucky numbers (*Ce tableau est pour trouver les numéros heureux.*) Menut also gave a table of the age each of the 90 numbers, the number of draws since it had last appeared before December 1833. [6]

7. Risk and Revolution: A Random Sample of Bets!

Let us accept that the tirages were each a true random sample of the 90 numbers in the wheel of fortune. The data pass all tests, and the only anomaly, the rash of winners in Lyon in 1820, was, if not a simple fluke, the result of a fraud that did not compromise the randomness of the selections. Now, Menut's book also included a list of all the big winners from the reinstitution of the Loterie in 1797 until the end of 1833: One quine, some 327 winning quaternes, and 3 large ternes. For each he gave the date of the tirage, the numbers that won, the amount won, the location of the Loterie, and the number and location of the bureau where the bet was placed. If the tirages were a true random sample, and to all appearances they were, then these are a true random sample of the bets placed, at least of the bets on quaternes.

This is a somewhat subtle point, since (as we shall see) the bettors did not spread their bets uniformly about the numbers available for wagering. But on every tirage, each bet on a quaterne has the same probability of being sampled (1 *in* 511,038). Popular numbers will then have a proportionally larger chance of being selected. The sample size (327 quaternes, 331 large wins) may be regarded as random, but all bets which are capable of yielding a win on a quaterne, a quine, or a terne over 130,000 francs, are equally likely to be included. Of course these were not the only bets placed, but since the available evidence (a few surviving losing tickets) suggests that people who bet on extraits and ambes also included a quaterne in their list of bets, we may hope to learn from this sample about the approach the entire gambling public took to risk at this turbulent time.

6. There was an interesting recent instance of a gambler noticing a bias in a lottery. On June 9, 1998, an Arizona woman noticed that the number "9" had not occurred in the State of Arizona's Pick 3 game that had been introduced a month earlier on May 4. She had bet her son's September birth date (9-0-7) daily (In Pick 3, three numbers from 0 to 9 are selected with replacement.). The embarrassed State discovered that the computer doing the draws had inadvertently been programmed to avoid "9". They went back to a mechanical scheme for drawing numbers (*The Arizona Republic* June 12, 13, 20, 1998; Kaigh, 2001).

8. The Numbers

The easiest question to answer is, what numbers were the bettors choosing? Experience with modern lotto (where typically six numbers are chosen from 1 to 49 or 52) would lead us to expect a markedly non-uniform distribution of choices. Kadell and Ylvisaker (1991) and Henze and Riedwyl (1998) report data showing a strong preference for low numbers (for example, birthdates and other anniversaries) and simple arithmetic progressions. Even in lotto games where over 50% of the bettors' choices were made as *quick picks* – random selections by computer, an option not available in the Loterie – nearly 1% of the choices were one of seven such progressions, with one of the most popular being 1, 2, 3, 4, 5, 6. Menut's sample of n = 331 winning bets is too small to reveal much about such patterns (the chance that a simple progression would be drawn among 331 tirages is very small, as the bettors in the lotto games in Henze and Riedwyl's data base no doubt discovered). But we do get information on one-dimensional margins, on the frequency of single numbers. Figure 4 shows the distribution for the sample, and as expected there is a preference for smaller numbers, also for the jumeaux (the pairs 11, 22, 33, etc), for 90, and for the sets 76/67 and 63/36.

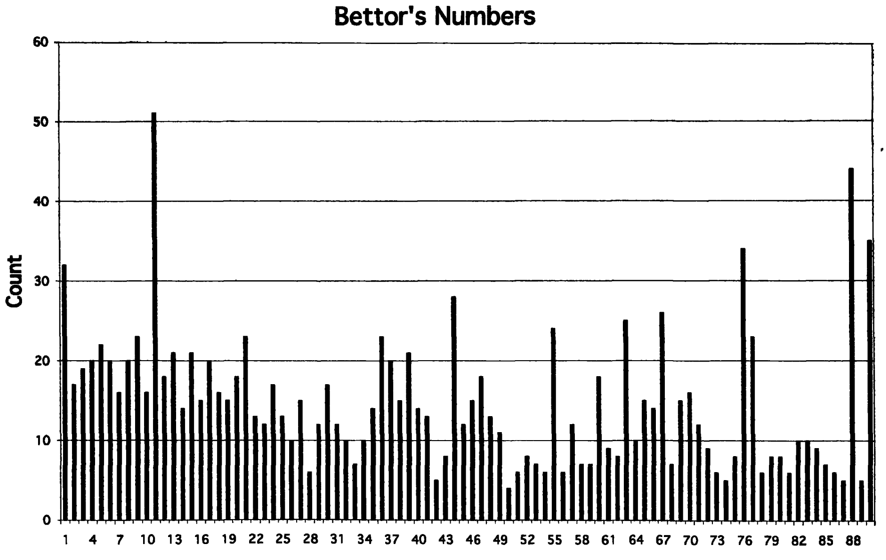


FIG. 4. – The frequency distribution of a sample of numbers chosen by winners of 331 large prizes over the years 1798-1833. Based upon data from Menut (1834).

We get limited information on the joint distribution of numbers selected in a single bet from the clumping of winners – the frequency of occurrence of more than one quaterne in a single tirage. Even with the nonuniform distribution of numbers shown in Figure 4, if the bettors were choosing single numbers independently from the distribution the chance of two or more quaternes in a single tirage would be negligible. Actually, there were 32 instances of multiple winners, 26 where two won, 5 with three winners, and on one day in March 1802 there were 20 quaternes on one tirage, earning the 20 holders in cities from Paris to Toulouse a total of 277,244 francs! The tirage that day was 67, 76, 11, 88, 63, and while all the winners included 76 and 88 in their choices, all three of the remaining possible choices (11, 63; 11, 67; 63, 67) were represented, 10, 8, and 2 times respectively. Inspection of the other multiple wins show a tendency for them to occur in tirages with jumeaux or reverse pairs (63, 36; 67, 76 etc. Several partially arithmetic sequences appear (e.g. 6, 24, 36, 48, or 5, 25, 35, 65).

Trois Citoyennes consulte le Cabaliste Cagliostro, sur la Loterie nationale de France il leur presente cinq N°. extrais de sa Colonne

FIG. 5. – Cagliostro advises his clients [7].

7. From "Cagliostro" (1803), a book that capitalized on a famous name and sought to give advice to bettors on the Loterie The Comte de Cagliosto (true name Joseph Balsamo) was an adventurer/swindler who lived from 1743-1795, and while apparently innocent played a minor role in the Affair of the Necklace, where a swindler attempted to sell a fabulous necklace to Marie Antoinette He was played by Christopher Walken in a recent movie about that Affair

Where did the numbers come from? Clearly some (*e.g.* the jumeaux) came from the same lack of imagination that makes 1, 2, 3, 4, 5, 6 popular today, and others came from books like Menut's, chosen by the bettor as congenial to the season, the sign of the zodiac, or the name of a loved one. Professional *experts*, such as *Cagliostro* (*see* Figure 5), provided more personalized service.

An article in the *Moniteur Universel* on December 28, 1797 suggests another source. It explains that a citizen bet a total of 1,008 francs on all 28 ambes (24 francs each) and all 56 ternes (6 francs each) corresponding to the numbers associated with the name *BONAPARTE*, with $A = 1$, $B = 2$, etc. He won on one terne and 3 ambes, for a net gain of 51,432 francs, a considerable fortune! The article concludes with a short notice that General Bonaparte was received the day before as a member of l'Institut National, suggesting that a press agent may have been involved.

There are eight distinct *Bonaparte* numbers (2, 14, 13, 1, 15, 17, 19, 5), and the question arises, did others imitate the lucky citizen? With a random selection these numbers should occur about 9% of the time, but of course they are also among the favored small numbers, so it is not surprising that over the years 1798-1814 they occurred as about 11% of the choices. Interestingly, in the years after the Battle of Waterloo this increased to 14%. Choosing numbers from names in this manner was clearly quite limiting (to the numbers 1 to 26) and an entrepreneur named J. B. Marseille (who billed himself as a "mathématicien") responded with an extremely complicated cryptological scheme that could yield no end of sets of numbers based upon the same name (Figure 6).

9. The Bets

Since the winnings were in principle a constant multiple of the amount bet, we can work backwards from Menut's data to infer the amounts bet. I say *in principle* since the sums reported are only approximately consistent with this supposition; apparently there were varying agency fees extracted from the winners. Figure 7 shows the frequency distribution of payoffs, and indicates that with few exceptions the wagers were quite small – usually only a sou or two.

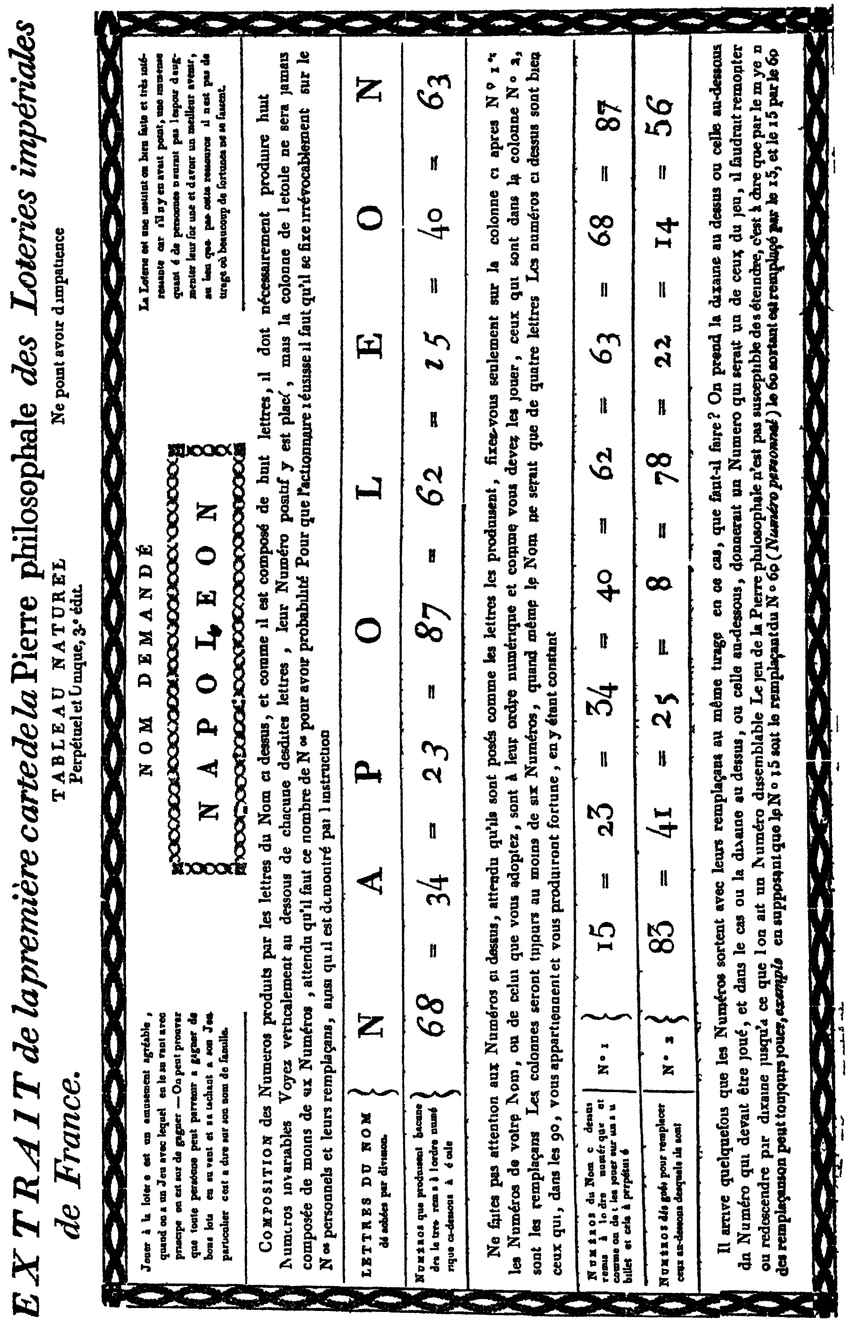
The largest recorded payoff, 689,620 francs, was awarded to a bettor in Marseille who bet the quaterne for the tirage held August 9, 1810, at the Loterie of Lyon. Presumably the bet was for 10 francs, an extraordinary sum for such long odds, and some agency fees account for the discrepancy. Nearly as large a sum (683,000 francs) was won on October 9, 1820 by a Parisian bettor, also betting on the tirage at Lyon. However that bet was on a terne at more favorable odds, and the sum wagered would have been about 125 francs.

Clearly most gamblers bet only small stakes, but some were willing to risk enormous sums. Who were they – where were they? Table 4 lists all of the

TABLE 4. – Who was winning.

Loterie de :	Bureau à								
Bordeaux	Bayonne	Lyon	Arles	Paris	Blois	Paris	Marseille (7)	Paris	Rhodès [Rodez]
Bordeaux	Bordeaux (11)	Lyon	Le Puy	Paris	Bordeaux (5)	Paris	Meaux	Paris	Rochefort
Bordeaux	Paris (10)	Lyon	Bagnois	Paris	Boulogne-sur-mer (2)	Paris	Melun	Paris	Rouen (5)
Bordeaux	Poitiers	Lyon	Besançon	Paris	Caen (5)	Paris	Metz (2)	Paris	Rouffac
Bordeaux	Rochefort	Lyon	Bêziers (2)	Paris	Calais	Paris	Montpellier	Paris	Salins
Bordeaux	La Rochelle	Lyon	Châlons-sur-Saône (2)	Paris	Colmar	Paris	Mülhausen	Paris	Sedan
Bruxelles	Chartres	Lyon	Grenoble (4)	Paris	Compiègne	Paris	Namur	Paris	Strasbourg (3)
Bruxelles	Le Mans	Lyon	Lyon (9)	Paris	Coutances (2)	Paris	Nancy	Paris	Thionville
Bruxelles	L'Orient	Lyon	Marseille (4)	Paris	Crefeld	Paris	Nantes (2)	Paris	Toulon (2)
Bruxelles	Orléans	Lyon	Montpellier	Paris	Dijon (4)	Paris	Nemours	Paris	Toulouse (3)
Bruxelles	Paris (7)	Lyon	Paris (18)	Paris	Dourdan	Paris	Neufchâtel	Paris	Tours
Bruxelles	St. Denis	Lyon	Perpignan	Paris	Evreux	Paris	Nîmes (2)	Paris	Valence
Lille	Amiens	Lyon	Toulon	Paris	Ferney	Paris	Nogent-le-Rotrou	Paris	Vannes
Lille	Boulogne-sur-mer	Lyon	Turin	Paris	Gand [Ghent]	Paris	L'Orient	Paris	Versailles (2)
Lille	Le Hâvre	Paris	Amiens (3)	Paris	Genève	Paris	Paris (86)	Paris	Vitry
Lille	Nantes	Paris	Andelys (2)	Paris	Grenoble (5)	Paris	Pau	Paris	Weissembourg
Lille	Orléans (2)	Paris	Angers	Paris	Le Hâvre	Paris	Perigueux	Strasbourg	Hagenau (2)
Lille	Paris (5)	Paris	Anvers	Paris	Laon	Paris	Perpignan (2)	Strasbourg	Landau
Lille	St. Lô	Paris	Bayonne (3)	Paris	Lille	Paris	Provins	Strasbourg	Mayence [Mainz]
Lille	St. Omer	Paris	Beauvais	Paris	L'Orient	Paris	Quimper	Strasbourg	Metz (4)
Lille	Versailles	Paris	Besançon	Paris	Lyon (9)	Paris	Rennes	Strasbourg	Paris (17)
				Paris	Mantes			Strasbourg	Strasbourg (8)

331 large winners (327 of the quaterne, 1 quine, and 3 large ternes) 1798-1833, by the location of the draw (Loterie) and the office where the bet was placed (Bureau). Multiple winners in parentheses. (From Menut)



EXTRAIT de la première carte de la Pierre philosophale *des Loteries impériales de France.*

TABLEAU NATUREL
Perpétuel et Unique, 3.e édit.

Ne point avoir d impatience

Jouer à la loter e est un amusement agréable, quand on a un Jeu avec lequel en le su vant avec principe on est sur de gagner — On peut prouver que toute persônne peut parvenir a gagner de bons lots en su vant et s a tachant a son Jeu particulier c est a dire sur son nom de famille.

NOM DEMANDÉ

NAPOLEON

La Loterie est une institut on bien faite et très intéressante car s'il n y en avait point, une immense quant é de personnes n aurait pas l espoir d augmenter leur for une et d avoir un meilleur avenir, au lieu que par cette ressource il n est pas de tirage où beaucoup de fortunes ne se faisent.

Composition des Numeros produits par les lettres du Nom ci dessus, et comme il est composé de huit lettres, il doit nécessairement produire huit Numeros invariables Voyez verticalement au dessous de chacune desdites lettres, leur Numéro positif y est placé, mais la colonne de l etoile ne sera jamais composée de moins de six Numéros, attendu qu'il faut ce nombre de Nos pour avoir probabilité Pour que l'actionnaire réussisse il faut qu'il se fixe irrévocablement sur le Nos personnels et leurs remplaçans, ainsi qu il est démontré par l instruction

Lettres du Nom dé achées par division.	N	A	P	O	L	E	O	N
Numéros que produisent chacune des lettres remises à l ordre numérique ci-dessous à é oile	68 =	34 =	23 =	87 =	62 =	15 =	40 =	63

Ne faites pas attention aux Numéros ci dessus, attendu qu'ils sont posés comme les lettres les produisent, fixez-vous seulement sur la colonne ci apres N° 1. les Numéros de votre Nom, ou de celui que vous adoptez, sont à leur ordre numérique et comme vous devez les jouer, ceux qui sont dans la colonne N° 2, sont les remplaçans Les colonnes seront toujours au moins de six Numéros, quand même le Nom ne serait que de quatre lettres Les numéros ci dessus sont bien ceux qui, dans les 90, vous appartiennent et vous produiront fortune, en y étant constant

Numéros du Nom c dessus remis à l o dre numérique et comme on do t les jouer sur un s u billet et cela à perpétui é	N° 1	15 =	23 =	34 =	40 =	62 =	63 =	68 =	87
Numéros dés gnés pour remplacer ceux au-dessous desquels ils sont	N° 2	83 =	41 =	25 =	8 =	78 =	22 =	14 =	56

Il arrive quelquefois que les Numéros sortent avec leurs remplaçans au même tirage en ce cas, que faut-il faire? On prend la dixaine au dessus ou celle au-dessous du Numéro qui devait être joué, et dans le cas ou la dixaine au dessus, ou celle au-dessous, donnerait un Numero qui serait un de ceux du jeu, il faudrait remonter ou redescendre par dixaine jusqu'à ce que l on ait un Numéro dissemblable Le jeu de la Pierre philosophale n'est pas susceptible des éteindre, c'est à dire que par le m ye n des remplaçans on peut toujours jouer, *exemple* en supposant que le N° 15 soit le remplaçant du N° 60 (*Numéro personnel*) le 60 sortant est remplacé par le 15, et le 15 par le 60

FIG 6 – A worked example from Marseille (1807)

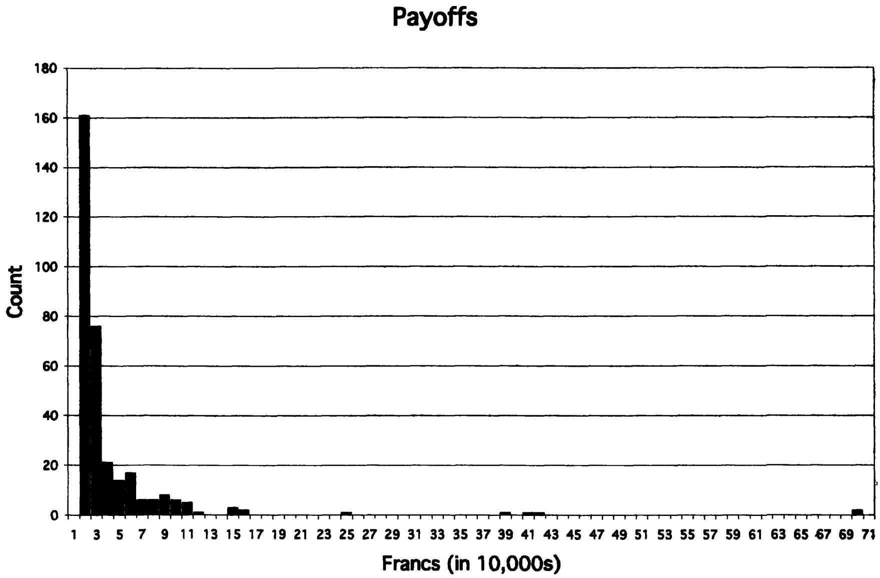


FIG. 7. – The frequency distribution of all 331 large payoffs in the Loterie (including 327 quaternes) from 1798-1833. Based upon data from Menut (1834).

331 winners over this period, by location of the tirage and by location of the bureau where the bet was placed.

Recall that Parisian bettors could purchase tickets for a tirage at any Loterie, while others had a choice of betting on the tirages in Paris or in the Loterie in their section of France.

The overwhelming impression from these data is that Paris was the center of activity: 200 of the 331 winners (or 60%) were drawn in Parisian tirages; 143 of 331 (or 43%) were paid to bets placed in Parisian bureaus. Now, in 1836 the population of France was recorded by census as 33,333,019 (exclusive of Corsica), of whom 909,126 were in Paris, 1,106,891 in the Department of the Seine. By those numbers Paris was greatly over-represented in the Loterie. But at that time France was predominantly agricultural, and (again by the 1836 census) only 2,427,992 of the citizens of France resided in towns of population 25,000 or greater; 37% of these were in Paris. Viewed this way the bets on the Loterie were not that far from being uniformly distributed across the urban population of France at that time. And the list in Table 4 shows that the bets did come from all over, from small towns as well as large. Presumably the descendents of Fermat were betting in Toulouse, and the relatives of Laplace and Quetelet were betting in Caen and in Gand. With small bets spread broadly over the urban population, the appeal of the Loterie seems to have been universal.

Table 5 gives a slightly different view of a portion of these data dealing with bets made in the cities with regional Loteries. In particular it displays the trade between cities: The bets predominantly originated in Paris; three-quarters of these were made for Parisian tirages, but a quarter were directed to the regions. On a per-capita basis the distribution of originating bets among the cities was fairly even, with the population of Lyon being slightly less interested in betting, although this difference does not pass the screen of a significance test. And the size of the bets – as measured by the average sizes of the winnings – are fairly uniform as well, with one exception. Bets originating in Paris and placed on the Lyon Loterie yielded an average payoff of 65,258 francs, more than twice that for any other combination. This is due to the 683,000 francs won on the earlier mentioned terne, bet in Paris and won on October 9, 1824; without that prize the average winnings drops to 28,920 francs. But coupled with the earlier noted phenomenon of a cluster of four quaternes in Lyon won by Parisian bettors in seven months, there is a suspicion that something was afoot. After all, is it not surprising that a bettor would wager 125 francs (on the order of 2,500 Euros today) on a 11,748 to 1 bet, to be drawn in another city? Of course people have always done surprising things.

TABLE 5. – Large wins in cities with regional Loteries, 1798-1833 Calculated on the basis of data from Menut (1834).

TABLE 5.a – Number of large wins 1798-1833, cross-classified by location of tirage and location of bureau where the bet was placed [8].

Loterie	Bureaux: Paris	Lyon	Strasb.	Bord.	Brux.	Lille	Total	Pop 1836	Pop/wins
Paris	86	9	3	5	0	1	104	909	8.7
Lyon	18	9	0	0	0	0	27	151	5.6
Strasbourg	17	0	8	0	0	0	25	58	2.3
Bordeaux	10	0	0	11	0	0	21	99	4.7
Bruxelles	7	0	0	0	0	0	7	103	-
Lille	5	0	0	0	0	0	5	72	-
Total	143	18	11	16	0	1	189		
Pop 1836	909	151	58	99	103	72			
Pop/wins	6.4	8.4	5.3	6.2	-	-			

TABLE 5.b – Total amount won 1798-1833, on the same basis as table 5.a.

Loterie	Bureaux: Paris	Lyon	Strasbourg	Bordeaux	Bruxelles	Lille	Total
Paris	2,426,945	200,880	69,254	107,598	0	5,012	2,809,689
Lyon	1,174,645	156,306	0	0	0	0	1,330,951
Strasbourg	450,970	0	214,582	0	0	0	665,552
Bordeaux	164,790	0	0	217,882	0	0	382,672
Bruxelles	65,740	0	0	0	0	0	65,740
Lille	154,231	0	0	0	0	0	154,231
Total	4,437,321	357,186	283,836	325,480	0	5,012	5,408,835

8. For each city the population (in 1,000s) is from the 1836 census (except for Bruxelles, which uses 1835 data). The regional Loterie moved from Bruxelles to Lille in 1814.

TABLE 5.c – The average size of a large win per bet won, 1798-1833, on the same basis as table 5.a.

Loterie	Bureaux						
	Paris	Lyon	Strasbourg	Bordeaux	Bruxelles	Lille	Overall
Paris	28,220	22,320	23,085	21,520	0	5,012	27,016
Lyon	65,258	17,367	0	0	0	0	49,294
Strasbourg	26,528	0	26,823	0	0	0	26,622
Bordeaux	16,479	0	0	19,807	0	0	18,222
Bruxelles	9,391	0	0	0	0	0	9,391
Lille	30,846	0	0	0	0	0	30,846
Overall	31,030	19,844	25,803	20,343	0	5,012	28,618

10. Suppression

The Loterie was a very large and (with one exception, to be noted) a quite profitable operation. Before its first suppression in 1793 it had grown to some 700 bureaus covering the kingdom, and with a budget of over 36 million francs in 1788 it had contributed over 10 million to the state treasury. With the coming of the revolution, the revenues had dwindled, from a high of 13 million in 1790 to 8 million in 1791, and that in an inflated currency. The suppression in 1793 lasted until a budget deficit of 20 million in 1796 led to a call for new taxes. The reinstatement in 1797 rejuvenated the Loterie; initially 650 bureaus were created, and that figure grew to a peak of over 1000 in 1811. The highest receipts were in 1812, when the state profited by 24 million, although the average was closer to 12 million per year, about 2% of the national budget. (Leonnet, 1963). The Loterie had lost money only in the year 1814, when extraordinary political events had led to its closure for various periods of time in Strasbourg, Lyon, and Bordeaux, and the Bruxelles regional Loterie had been closed and then moved to Lille. The Loterie was finally and permanently suppressed by a law enacted on May 21, 1836.

In a sense the cause of death was old age. With time and familiarity came boredom, ennui. The enemies of the Loterie, whose voices had never been stilled, came to rule the day. In 1819 when the finances were in excellent health and a tax cut was being considered, Pierre Simon Laplace had argued to the Chambre des Pairs (séance du 16 juillet) that instead of decreasing taxes the Loterie should be closed. He argued on moral grounds, pointing to the mathematical disadvantage it gave to those who played, and arguing that this was visited to large degree upon the poor :

> "The poor, excited by the desire for a better life and seduced by hopes whose unlikelihood it is beyond their capacity to appreciate, take to this game as if it were a necessity. They are attracted to the combinations that permit the greatest benefit, the same that we see are the least favorable. ... We applaud the orator who would turn his audience away from the lottery, forcefully recounting the thefts, the misery, the bankruptcies, and the suicides that are its children. ...

> We are told that the tickets of foreign lotteries will be introduced among us. But the surveillance of the government can prevent that, or at least make them so rare that they will not reach the interior of the kingdom, and with a bit of vigilance the bets on these lotteries would amount to less than a fiftieth of the current bets of the Loterie de France. We are told further that this tax is voluntary. No doubt it is voluntary for each individual, but for the set of all individuals it is a necessity, just as their marriages, births, and all sorts of variable effects are necessary, and nearly the same each year when their number is large, just as the revenues from the lottery are as constant as is agricultural production (Laplace, 1819 [1912])."

Laplace argued that the state's annual profit of 10 to 12 million was offset by a hidden tax upon the poor of 40 to 50 million per year in lost investment; he was unsuccessful. But over time the public's interest diminished. The data on large winning bets can be used to estimate the number of bets. Figure 8 shows there was no marked trend in the size of bets, but Figure 9 indicates that the number of bets was in fairly steady decline from 1800.

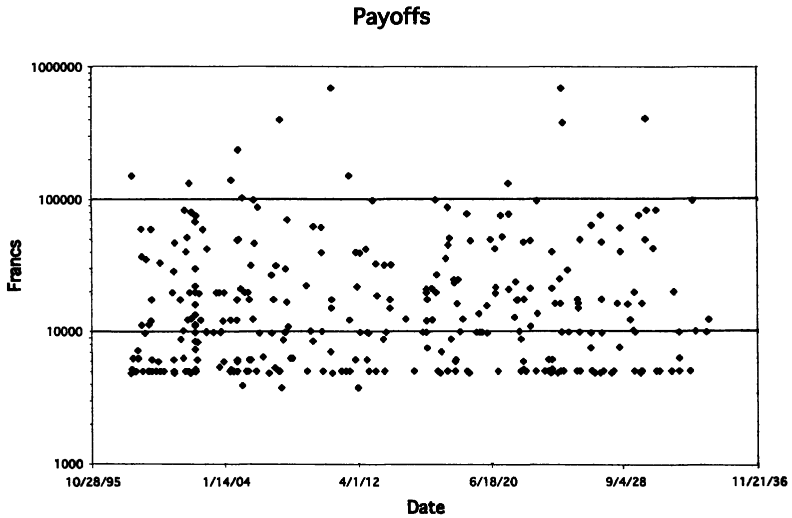


FIG. 8. – The same 331 payoffs as given in Figure 7, displayed by date of payoff.

We can crudely estimate that over 1800-1833 the number of quaternes bet per bureau for each tirage averaged about 100 to 200, but that this average decreased from around 200 in 1800 to around 30 in 1833. During the six years surrounding 1810 the Parisians bet on about 63,000 quaternes; during the six years surrounding 1830 they bet on about 48,000 quaternes, a decrease of 25%. In the regional Loteries the decrease was greater: In Lyon there was an over 40% decline. By 1836 the political balance had shifted. The ban on lotteries in France lasted nearly half a century this time, although it may have been incompletely successful. Descotils and Guilbert's illustrated history of the lottery included a picture of a ticket from Lyon dated 1839.

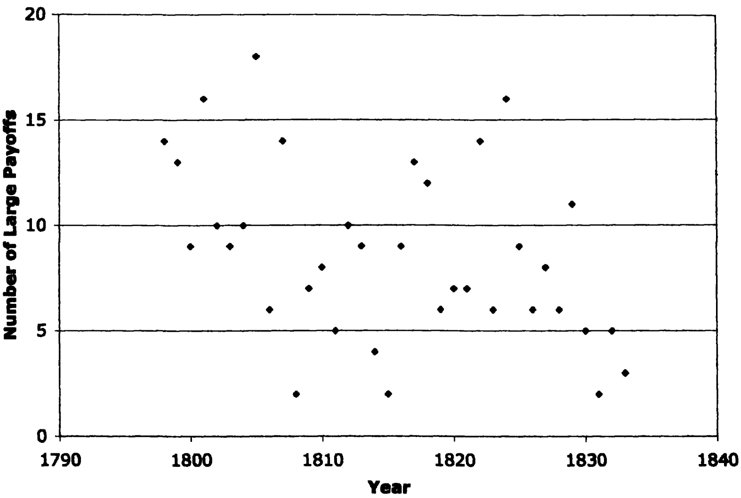


FIG. 9. – The count per year of large payoffs, indicating an estimated 50% drop off in the average volume of business in the Loterie over the years 1798-1833.

11. Risk adversity

Laplace was hardly alone in condemning lotteries, or in pleading that the bettors took part in them because they were deluded as to their true chances. With a different set of lotteries in mind, Adam Smith wrote (in *The Wealth of Nations*) "*That the chance of gain is naturally overvalued, we may learn from the universal success of lotteries*". (Smith, 1776, Vol. I, Book I, Chap. X, p. 132). Augustus De Morgan in 1839 gave this definition, "*Lotteries are schemes by which some modern governments have raised a revenue from their subjects, by taking advantage of that feeling of confidence in their own good fortune which is entertained by a large proportion of mankind*". (De Morgan, 1839). In France there was a long tradition of deriding the Loterie as a scourge and illusion (*e.g.* Mercier, 1782, 1783, and Dusaulx, 1775, 1779). But it seems hard to maintain that only delusion – or lack of access to the calculations of mathematical probability – was responsible for the success of the Loterie. The Loterie's attraction was consistent over a very long time, despite both theoretical and empirical reasons why it might not have been. There was in fact a widespread knowledge of the correct odds, and even a casual observer would have seen there was no steady parade of his fellows from rags to riches.

Almost from the beginning there was easy access to the results of an analysis of the game; the odds were not hidden. In modern lotto it is extremely difficult to evaluate the expected value of a lottery ticket accurately, because with parimutuel pools, with roll-over of unclaimed prize money, and with available prizes split among winners, your chances hinge upon the action of other players. The Loterie de France had none of those complications; an evaluation of the chances of winning any one of the bets was a simple problem in combinatorial probability, and those without the mathematical education to complete the calculation themselves would have no trouble finding help in the form of books and articles that presented these results to all levels of mathematical literacy. The clear inference from this is the existence of a large segment of the French population who were, as far as the Loterie went, risk seeking. The available data do not permit some questions to be answered, such as the relative amounts wagered on the different available bets, questions that could help understand the attractions of risk in a way that modern lottery data cannot.

The menu of bets available showed a wide variety of expected values. The extrait simple was offered at unusually good odds, while the quaterne and the quine (when it was available) were priced at much less attractive rates. No doubt this was a reflection of the risk adversity of the state, which was revealed so clearly in Casanova's conversation with the Councilors (and even more starkly in today's lottos, where the state's parimutuel pool is a signal of an unwillingness to accept any risk at all).

Interestingly there was a device available to the French bettors that could have improved the odds on the longshots considerably. From 1800 on, the instructions the Loterie administration issued to the bureaus placed a limit on the maximum size of a bet on the quaterne (12 francs) and the quine (3 francs). Parisot (1810, p. 263) states that before the revolution a decree of the Council had in addition put a maximum of 10,000 francs on the extrait, 600 francs on the ambe, and 150 francs on the terne, but these were not part of the instructions issued to bureaus after 1797. From 1797, a bettor who wagered one franc on an extrait simple with the intention of reinvesting all winnings on another extrait, and repeating as long as he won, up to a total of six tirages, would gain a return of 11,390,625 with probability 1/34,012,224, a strategy that dominated the quine. A bettor attracted by a payoff of the order of magnitude of a quaterne could simply try the same strategy for only four tirages (gaining 50,625 with probability 1/104,976, as opposed to betting on a quaterne and gaining 75,000 with probability 1/511, 038). It is impossible to tell if anyone availed themselves of this opportunity, but the large number of quaternes bet suggests that few if any did. [9]

12. Education

There was one clear side benefit to the Loterie: the encouragement of public interest and education in the theory of probabilities. At least from 1783 and the appearance of Charles Bicquilley's textbook *Du calcul des probabilités*, the chances of ternes, quaternes, and quines became a staple of mathematical education. Others followed: Huyn, Condorcet, Parisot, Lacroix, d'Hauteserve, and Cournot. Even Laplace could not resist the temptation to present the

9. The Loterie was aware of at least some of the possible combinations that could be used that were unfavorable to the bettor – and they acted to protect the bettor in those cases! For example, the bureaus were instructed to not accept a bet by a single person on all 90 extraits, a bet that was sure to lose 22%. Nor were they permitted to accept a bet on a single number as an extrait déterminé for each of the five possible positions, a bet that was inferior to an extrait simple for the same number; similarly for bets of the 10 possible ambes déterminés for a pair of numbers.

simple calculation. While probably too expensive for anyone below the upper-middle classes, there was even a home version: a game called *Loto-Dauphin*, supposedly introduced first to amuse the grandson (and successor) of king Louis XV, it was based upon the rules of the Loterie and both capitalized on the attractions of the Loterie and helped spread knowledge about the chances of winning. Curiously these calculations apparently did not cross the English Channel (at least not into the English language) before William Rouse's 1814 book, even though some quite complicated problems based on English lotteries appeared much earlier (*e.g.* Painter, 1787).

Contributions to mathematical education appeared in other, unexpected ways. The Instructions sent to the bureaus from 1800 on show that the Loterie administrators were sufficiently worried about a rash of mispriced ambes déterminés that they added a section that was essentially a tutorial on computing simple combinations. Clearly some bettors had taken to making wagers such as "*all ambes déterminés consisting of two of the numbers (7, 8, 10, 40) as the third and fourth numbers drawn*". And just as clearly, some agents when faced with this had figured there were six ways of choosing the two numbers and so there were six bets to be charged for, thus missing the other six, with the pairs in the opposite order. They would have then been in effect offering 12 chances to win 5,100 francs for a cost of but 6 francs; since the chance of winning in each bet was 1/8,010 this would give the bettor a positive expected value! The administrators' 10 page tutorial explained the correct approach to this and similar combination bets in detail, and presumably increased mathematical literacy generally.

13. Conclusion

The Loterie de France flourished for three-quarters of a century, with a brief hiatus in the 1790s. The consistency in the demand for the Loterie by a public increasingly well-educated in probability shows an attraction to low cost risk at the individual level that continues today in all societies. The Loterie prospered with the increasingly general knowledge of probability, and without doubt contributed to that knowledge; it was a public laboratory for chance where students could see almost daily the application of the techniques they studied in secondary school and university. The Loterie also serves as an example of the phenomenon of corporate risk aversion by the state. From the resistance Casanova encountered at the founding of the Loterie, to the demise of the quine as an option after about 1803, the state was ever mindful of the fact that it always stood the chance of losing on its bets, millions on the quine and hundreds of thousands on the quaterne. No modern state lottery, whether lotto or sweepstakes, accepts such a risk. The odds were so strongly in favor of the state on these bets that it would seem foolish that they would worry, but against that there was always the specter of undiscovered fraud, and the administrative manuals of the time show they took this possibility very seriously. It is plausible that this risk aversion, coupled with the slow secular decline in public interest as the Loterie became dated, contributed to the Loterie's demise.

This type of lottery (with a menu of bets and guaranteed fixed payoffs even for long odds bets) was never widely adopted internationally – only in parts of Italy and Spain, in some German cities, in Vienna and in France. According to Menut the payoffs in France were superior to the others, for example in Germany they paid only 14 times on an extrait, and 60,000 times on a quaterne. After May 1836 it was never seen again as a sanctioned event. The Loterie survived the revolution, but by 1836 it had run its course. In 1843 Cournot (1843, § 9) could write that "*Everyone knows*" the basic facts of the Loterie, but it soon vanished from textbooks and common knowledge. Nonetheless it left its mark on succeeding generations' understanding of chance.

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