JOURNÉES ÉQUATIONS AUX DÉRIVÉES PARTIELLES

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*Journées Équations aux dérivées partielles* (1999), p. 1-13
<http://www.numdam.org/item?id=JEDP_1999____A10_0>

Journées *Équations aux dérivées partielles*
Saint-Jean-de-Monts, 31 mai–4 juin 1999
GDR 1151 (CNRS)

# Resonances for transparent obstacles

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**Abstract**

This paper is concerned with the distribution of the resonances near the real axis for the transmission problem for a strictly convex bounded obstacle $\mathcal{O}$ in $\mathbf{R}^n$, $n \geq 2$, with a smooth boundary. We consider two distinct cases. If the speed of propagation in the interior of the body is strictly less than that in the exterior, we obtain an infinite sequence of resonances tending rapidly to the real axis. These resonances are associated with a quasimode for the transmission problem the frequency support of which coincides with the corresponding gliding manifold $\mathcal{K}$. To construct the quasimode we first find a global symplectic normal form for pairs of glancing hypersurfaces in a neighborhood of $\mathcal{K}$ and then we separate the variables microlocally near the whole glancing manifold $\mathcal{K}$. If the speed of propagation inside $\mathcal{O}$ is bigger than that outside $\mathcal{O}$, than there exists a strip in the upper half plane containing the real axis, which is free of resonances. We also obtain an uniform decay of the local energy for the corresponding mixed problem with an exponential rate of decay when the dimension is odd, and polynomial otherwise. It is well known that such a decay of the local energy holds for the wave equation with Dirichlet (Neumann) boundary conditions for any nontrapping obstacle. In our case, however, $\mathcal{O}$ is a trapping obstacle for the corresponding classical system.

## 1. Introduction.

Let $\mathcal{O} \subset \mathbf{R}^n$ be a bounded domain with a connected $C^\infty$ boundary $\Gamma$ and $\Omega = \mathbf{R}^n \setminus \overline{\mathcal{O}}$. Consider the operator

$$\Delta_g := c(x)^2 \sum_{i,j=1}^{n} \partial_{x_i}(g_{ij}(x)\partial_{x_j}) \ , \ x \in \mathcal{O} \ ,$$

where $c$, $g_{ij} \in C^\infty(\overline{\mathcal{O}})$ and $c(x) \geq c_0 > 0$. We suppose that the principal symbol, $g(x,\xi)$, of $-\Delta_g$ satisfies

$$g(x,\xi) := c(x)^2 \sum_{i,j=1}^{n} g_{ij}(x)\xi_i\xi_j \geq C|\xi|^2, \qquad \forall (x,\xi) \in T^*\mathcal{O}\,,$$

with some positive constant $C$. Our main example will be $g(x,\xi) = c^2 g^0(x,\xi)$, where $g^0(x,\xi) = |\xi|^2$ and $c \neq 1$ is a positive number. Denote by $\mathcal{G}$ the Riemannian metric $\sum_{i,j=1}^n G_{ij}(x)dx_i dx_j$ in $\overline{\mathcal{O}}$ associated with the Hamiltonian $g$, where $(G_{ij}(x))_{i,j=1}^n$ is the inverse matrix to $(c(x)^2 g_{ij}(x))_{i,j=1}^n$. Given $x' \in \Gamma$, we denote by $\nu'(x')$ the interior unit normal to $\Gamma$ at $x'$ with respect to the Riemannian metric $\mathcal{G}$, and by $\nu(x')$ the exterior one with respect to the Euclidean metric. Fix a constant $\alpha > 0$. The complex number $\lambda \in \mathbf{C}$ is said to be a resonance for the transmission problem associated to $\mathcal{O}$ (see [7]), if the following problem has a nontrivial solution

$$\begin{cases} (\Delta_g + \lambda^2)u_1 &= 0 \quad \text{in } \mathcal{O}, \\ (\Delta + \lambda^2)u_2 &= 0 \quad \text{in } \Omega, \\ u_1 - u_2 &= 0 \quad \text{on } \Gamma, \\ \partial_{\nu'} u_1 + \alpha \partial_\nu u_2 &= 0 \quad \text{on } \Gamma, \\ u_2 &- \quad \lambda - \text{outgoing}\,. \end{cases} \tag{1.1}$$

Recall that a function $v$ is said to be $\lambda$-outgoing if for some $\rho_0 \gg 1$ we have

$$v|_{|x|\geq\rho_0} \;=\; R_0(\lambda) g|_{|x|\geq\rho_0}\,,$$

where $g \in L^2_{\text{comp}}(\Omega)$ has a compact support and $R_0(\lambda)$ is the free outgoing resolvent of $\Delta$ in $\mathbf{R}^n$. Here "outgoing" means that

$$R_0(\lambda) \in \mathcal{L}(L^2(\mathbf{R}^n), L^2(\mathbf{R}^n))$$

for $\operatorname{Im}\lambda < 0$. Then all the resonances are in the upper half-plane $\operatorname{Im}\lambda > 0$.

We are going to study the distribution of the resonances for the transmission problem under suitable geometric conditions on $g$ and $\Gamma$. Choose a function $f \in C^\infty(\mathbf{R}^n)$ which defines the boundary $\Gamma$, i.e. $f < 0$ in $\mathcal{O}$, $f > 0$ in $\Omega$ and $df \neq 0$ on $\Gamma$. The boundary $\Gamma$ is said to be $g$-strictly concave with respect to $\mathcal{O}$ (or $g$-strictly convex with respect to $\Omega$) iff for any $(x,\xi)$ satisfying

$$f(x) = 0, \quad g(x,\xi) = 1, \quad \{g,f\}(x,\xi) = 0,$$

we have

$$\{g,\{g,f\}\}(x,\xi) > 0\,, \;\; \{f,\{f,g\}\}(x,\xi) > 0\,, \tag{1.2}$$

where $\{\cdot,\cdot\}$ denotes the Poisson brackets. Notice that the second inequality is automatically fulfilled in our case, since $\{f,\{f,g\}\}(x,\xi) = 2g(x, df(x)) > 0$. Then $F = \{(x,\xi) \in T^*\mathbf{R}^n : f(x) = 0\}$ and $G = \{(x,\xi) \in T^*\mathbf{R}^n : g(x,\xi) - 1 = 0\}$ is a pair of glancing hypersurfaces ([9]). We denote by $\mathcal{K}$ the corresponding glancing manifold

$$\mathcal{K} \;=\; \{(x,\xi) \in T^*(\mathbf{R}^n) : f(x) = g(x,\xi) - 1 = \{g,f\}(x,\xi) = 0\}\,.$$

In particular, the domain $\mathcal{O}$ is strictly convex in the usual sense iff $\Gamma$ is $g^0$-strictly convex with respect to $\Omega$, where $g^0(x,\xi) = |\xi|^2$. The Hamiltonian $g$ induces a Hamiltonian $r$ on $T^*\Gamma$ as follows. Identifying any $\xi' \in T^*_{x'}\Gamma$ with the covector $\xi = j(\xi') \in T^*_{x'}\mathbf{R}^n$, such that $\xi_{|T_{x'}\Gamma} = \xi'$ and $\xi(\nu'(x')) = 0$, we set $r(x',\xi') = g(x', j(\xi'))$. Similarly, we define $r_0(y',\eta')$ for the free Laplacian $\Delta = \sum_{j=1}^n \partial^2_{x_j}$.

First we suppose that

$$r(x',\xi') \ < \ r_0(x',\xi') \ , \ \forall\,(x',\xi') \in T^*\Gamma\setminus 0\,. \tag{1.3}$$

If $g(x,\xi) = c^2 g^0(x,\xi)$, $c > 0$, this condition is equivalent to $0 < c < 1$. Consider the classical system corresponding to the transmission problem in this case. Because of (1.3), any ray coming from infinity (a null-bicharacteristic of $g^0(x,\xi) - 1$) splits into two when interacting with the boundary. One of them reflects by the usual law of the geometric optics and goes again to infinity but the other one enters the obstacle. The latter is a refracted ray. This is true even for the diffractive rays, that is null-bicharacteristics of $g^0 - 1$ which are tangent to the boundary. Consider now a ray $\gamma$ traveling in the interior of the obstacle, i.e. a null-bicharacteristic of $g(x,\xi) - 1$. If $\gamma$ hits the boundary far enough from the glancing manifold $\mathcal{K}$ the same phenomenon occurs. It splits into two rays, one of them remaining in the obstacle while the other one leaves the obstacle and goes to infinity. On the other hand, because of (1.3), if $\gamma$ hits the boundary sufficiently close to $\mathcal{K}$ then it remains in the interior after the reflection without giving rise to a refracted ray in the exterior. In other words, there is a total reflection near the gliding region. The gliding rays do not give rise to refracted rays neither. Moreover, there is a kind of "effective" stability of the billiard flow in $\mathcal{O}$ near the glancing manifold. Namely, for each $N > 0$ there exists $C_N > 0$ such that if a broken ray $\gamma(t)$ is $\varepsilon$ close to $\mathcal{K}$ for $t = 0$ (in a suitable metric) then it remains $2\varepsilon$ close to $\mathcal{K}$ for $|t| \le C_N\varepsilon^{-N}$. In particular, those rays conserve a considerable amount of energy in the obstacle for a long period of time. Then it is natural to expect that the Lax - Phillips conjecture is true in this case, i.e. that there exist infinitely many resonances approaching the real axis. If the boundary is strictly convex and analytic, the "effective" stability is valid in an exponentially large time interval $|t| \le Ce^{b/\varepsilon}$ (see [3]) with some $C$, $b > 0$ and we believe that there exists a sequence of resonances which tends exponentially fast to the real axis in this case.

**Theorem 1.1** *Let $\Gamma$ be $g$-strictly concave with respect to $\mathcal{O}$. Suppose that (1.3) holds. Then there exists an infinite sequence $\{\lambda_j\}$ of different resonances of (1.1) such that*

$$0 < \operatorname{Im}\lambda_j \le C_N|\lambda_j|^{-N}, \quad \forall N \gg 1.$$

Moreover, we are going to localize $\{\lambda_j\}$ near the eigenvalues $\{k_j\}$ of a suitable elliptic pseudodifferential operator $Q$ on the boundary $\Gamma$ with a nonhomogeneous principal symbol, given by (2.10).

Consider now the case opposite to (1.3). We are going to show that there are no resonances in a strip in the upper half-plane containing the real axis. We consider a little bit more general case putting inside $\mathcal{O}$ an unpenetrable body $\mathcal{O}_1$. More precisely, let $\mathcal{O}_1 \subset \mathcal{O}_2$ be two bounded obstacles with smooth connected boundaries $\Gamma_1$ and $\Gamma$ respectively such that $\Gamma_1 \cap \Gamma = \emptyset$. Set $\mathcal{O} = \mathcal{O}_2 \setminus \overline{\mathcal{O}}_1$ and $\Omega = \mathbf{R}^n \setminus \overline{\mathcal{O}}_2$, and add a Dirichlet or Neumann boundary condition $Bu_1|_{\Gamma_1} = 0$ to (1.1). Theorem 1.1 holds also in this more general case.

Introduce the Hilbert space $H = L^2(\mathcal{O}; \alpha^{-1}c(x)^{-2}dx) \oplus L^2(\Omega; dx)$. Consider the operator

$$Pu := (\Delta_g u_1, \Delta u_2), \qquad u = (u_1, u_2) \in D(P),$$

with domain of definition $D(P)$ consisting of all $(u_1, u_2) \in H$ such that $u_1 \in H^2(\mathcal{O})$, $u_2 \in H^2(\Omega)$, and

$$Bu_1|_{\Gamma_1} = 0\,,\ u_1|_\Gamma = u_2|_\Gamma\,,\ \partial_{\nu'} u_1|_\Gamma + \alpha \partial_\nu u_2|_\Gamma = 0\,,$$

where $B$ denotes either the Dirichlet or Neumann boundary condition on $\Gamma_1$ if $\mathcal{O}_1 \neq \emptyset$. One can see that $P$ is a selfadjoint elliptic operator, $P \leq 0$, and the spectrum of $P$ is absolutely continuous with no embedded eigenvalues. Moreover, the resonances of the transmission problem given by (1.1) coincide with the poles of the meromorphic continuation of the cutoff resolvent

$$R_\chi(\lambda) := \chi(P + \lambda^2)^{-1}\chi : H \to H$$

from $\operatorname{Im}\lambda < 0$ to the whole complex plane $\mathbf{C}$ if $n$ is odd, and to the logarithmic Riemann surface if $n$ is even. Here $\chi \in C_0^\infty(\mathbf{R}^n)$ and $\chi = 1$ in a neighborhood of $\mathcal{O}_2$.

We make the following assumption

(H) There exists $T > 0$ such that for any generalized $g$-geodesic $\gamma(t)$ with $\gamma(0) \in \overline{\mathcal{O}}$ there is $t = t_\gamma$, $0 \leq t \leq T$, such that $\gamma(t) \in \Gamma$.

Recall that any generalized $g$-geodesic in $\overline{\mathcal{O}}$ is a projection of a generalized null-bicharacteristic of the Hamiltonian $g(x,\xi) - 1$ as defined by Melrose and Sjöstrand [10]. Clearly, (H) is fulfilled when $\mathcal{O}_1 = \emptyset$ and $g(x,\xi) = c^2|\xi|^2, c = Const > 0$.

We suppose also that

$$r(x',\xi') \ > \ r_0(x',\xi')\ ,\ \forall\,(x',\xi') \in T^*\Gamma \setminus 0. \tag{1.4}$$

If $g(x,\xi) = c^2 g^0(x,\xi)$, $c = Const > 0$, this condition is equivalent to $c > 1$.

Consider now the classical system corresponding to the transmission problem in this case. Because of (H) and (1.4), any ray issuing from $\overline{\mathcal{O}}$ reaches $\Gamma$ in a finite time and it splits into two at $\Gamma$. One of them reflects by the usual law of the geometric optics and keeps moving in $\mathcal{O}$, and the other one leaves the obstacle. Moving only on the internal rays, we stay in the obstacle, hence, there is a lot of rays trapped by the obstacle. On the other hand, any time when the ray hits $\Gamma$, a portion of its energy goes out of the obstacle. Hence, one can expect that there is a strip in the upper half-plain containing the real axis which is free of poles. Indeed we have:

**Theorem 1.2** *Let $\Gamma$ be both $g$ and $g^0$-strictly concave with respect to $\mathcal{O}$ and let* (H) *and (1.4) be fulfilled. Then there exist positive constants $\gamma$, $C_1$ and $C_2$ such that the cut-off resolvent $R_\chi(\lambda)$ is holomorphic in the strip*

$$\Lambda \ = \ \{z \in \mathbf{C}\ :\ |\operatorname{Re} z| \geq C_2\,,\ 0 \leq \operatorname{Im} z \leq \gamma\}$$

*and*

$$\|\lambda R_\chi(\lambda)\|_{\mathcal{L}(H)} \leq C_1\ ,\ \lambda \in \Lambda\,. \tag{1.5}$$

Note that for nontrapping perturbations there is a larger free of resonances region of the form $\operatorname{Im}\lambda \leq N\log|\lambda| - C_N$, $\forall N > 0$. Moreover, in the case of scattering by strictly convex obstacles (with Dirichlet boundary conditions) there is a free region of the form $\operatorname{Im}\lambda \leq C_1|\lambda|^{1/3} - C_2$, with some $C_1, C_2 > 0$ (see [4]). It is easy to check, however, that when $\mathcal{O}_1 = \emptyset$, $\mathcal{O}_2$ is a ball, and $g(x,\xi) = c^2|\xi|^2, c = Const > 1$, there exist infinitely many resonances $\{\lambda_j\}$ of $P$ such that $\operatorname{Im}\lambda_j \to \gamma_0 > 0$. This example shows that one can not expect a free of resonances region near the real axis larger than a strip.

Denote by $u(t)$ the solution of the equation

$$\left\{\begin{array}{c}(\partial_t^2 - P)u(t) = 0,\\ u(0) = f_1, \partial_t u(0) = f_2.\end{array}\right.$$

Given a compact $K \subset \mathbf{R}^n \setminus \mathcal{O}_1$, we denote by $p_0(t)$ the function

$$\sup\left\{\frac{\|\nabla_x u\|_{L^2(K)} + \|\partial_t u\|_{L^2(K)}}{\|\nabla_x f_1\|_{L^2(K)} + \|f_2\|_{L^2(K)}}, (0,0) \neq (f_1, f_2) \in [C^\infty(\mathcal{O}) \oplus C^\infty(\Omega)]^2, \operatorname{supp} f_j \subset K\right\}.$$

**Corollary 1.3** *Under the assumptions of Theorem 1.2, there exist constants $C, \beta > 0$ such that*

$$p_0(t) \leq \left\{\begin{array}{ll} Ce^{-\beta t}, & n \quad odd,\\ Ct^{-n}, & n \quad even.\end{array}\right.$$

In other words, we have the same uniform decay of the local energy as in the case of nontrapping perturbations.

We are going to sketch the proof of the main results. Detailed proofs are given in [1], [12] and [13].

## 2. Construction of quasimodes.

The main ingredient in the proof of Theorem 1.1 is the construction of a quasimode, the frequency support of which is concentrated at the glancing manifold $\mathcal{K}$.

A quasimode for the transmission problem is defined as an infinite sequence

$$\mathcal{Q} = \{(k_j, (u_1^{(j)}, u_2^{(j)})) : j \in \mathbf{N}\},$$

where $k_j \in \mathbf{C}$, $|k_j| \to \infty$, $\operatorname{Re} k_j \geq 1$, and $u_1^{(j)} \in C^\infty(\overline{\mathcal{O}})$, $u_2^{(j)} \in C^\infty(\overline{\Omega})$ have supports in a small compact neighbourhood $\mathcal{U}$ of $\Gamma$, $\|u_1^{(j)}|_\Gamma\|_{L^2(\Gamma)} = 1$, and

$$\left\{\begin{array}{rcl} \|(c^2\Delta + k_j^2)u_1^{(j)}\|_{L^2(\mathcal{O})} &=& O(|k_j|^{-\infty}),\\ \|(\Delta + k_j^2)u_2^{(j)}\|_{L^2(\Omega)} &=& O(|k_j|^{-\infty}),\\ \|u_1^{(j)}|_\Gamma - u_2^{(j)}|_\Gamma\|_{H^2(\Gamma)} &=& O(|k_j|^{-\infty}),\\ \|(\partial_{\nu'} u_1^{(j)} + \alpha\partial_\nu u_2^{(j)})|_\Gamma\|_{H^2(\Gamma)} &=& O(|k_j|^{-\infty}).\end{array}\right. \tag{2.1}$$

If $\mathcal{O}_1 \neq \emptyset$ we suppose that $\mathcal{U} \cap \Gamma_1 = \emptyset$, hence, the boundary condition on $\Gamma_1$ is automatically fulfilled. Hereafter, given an infinite sequence $\{z_j\}$ of (complex) numbers, we say that $z_j = O(|k_j|^{-\infty})$, if for each $N > 0$ there exists $C_N > 0$ such that $|z_j| \leq C_N|k_j|^{-N}$ for all $j$.

Using a result of Tang and Zworski [17] (see also [15]), we can localize resonances of the transmission problem near the sequence of "quasi-resonances" $\{k_j\}$. More precisely, we have:

**Proposition 2.1** *Let $Q$ be a quasimode for the transmission problem. Then*

$$0 < \operatorname{Im} k_j = O(|k_j|^{-\infty}),$$

*and there exists an infinite sequence $\Lambda = \{\lambda_j\}$ of resonances of $P$ such that*

$$\operatorname{dist}(k_j, \Lambda) = O(|k_j|^{-\infty}).$$

*In particular,* $0 < \operatorname{Im} \lambda_j = O(|\lambda_j|^{-\infty})$.

In what follows, we are going to associate to each zero of the Airy function $\operatorname{Ai}(s)$ a suitable elliptic pseudodifferential operator $Q$ of order one on $\Gamma$ with a real-valued principal symbol given by (2.10). The quasi-resonances $k_j$ will be the eigenvalues of $Q$.

First we microlocalize near the glancing manifold $\mathcal{K}$, using a class $\mathrm{OP}^{0,0}$ of pseudo-differential operators with a large parameter $\lambda$ ($\lambda - \Psi$DOs) [2]. We shall also allow complex values for the parameter $\lambda$ in

$$\mathcal{L} = \{\lambda \in \mathbf{C} : |\operatorname{Im} \lambda| \le C_1, \ \operatorname{Re} \lambda \ge 1\}, \ C_1 > 0. \tag{2.2}$$

First we reduce (2.1) to a suitable interior problem. Set $\mathcal{D}^2 = \mathcal{D}_1^2 + \cdots + \mathcal{D}_n^2$, where $\mathcal{D}_j = (i\lambda)^{-1}\partial/\partial x_j$. Denote $\Sigma = \{(y, \eta) \in T^*\Gamma : r(y', \eta') \le 1\}$. Because of (1.3), the boundary $\partial\Sigma$ of $\Sigma$ belongs to the elliptic region for the exterior Dirichlet problem for the $\lambda$-differential operator $\mathcal{D}^2 - 1$. Then for any family $f(\cdot, \lambda) \in C^\infty(\Gamma)$, $\lambda \in \mathcal{L}$, with $\|f\|_{L^2} = 1$ and frequency support $\widetilde{\mathrm{WF}}(f)$ contained in $\partial\Sigma$, and hence in the elliptic region, there exist $u_2(\cdot, \lambda) \in C^\infty(\overline{\Omega})$, $\lambda \in \mathcal{L}$, supported in a compact neighborhood of $\Gamma$ such that

$$\begin{cases} (\mathcal{D}^2 - 1)u_2 &= O(|\lambda|^{-\infty})f \quad \text{in } \Omega, \\ u_2 &= f + O(|\lambda|^{-\infty})f \quad \text{on } \Gamma, \end{cases}$$

where $u_2(\cdot, \lambda) = R(\lambda)f(\cdot, \lambda)$ and $R(\lambda)$ is a $\lambda$-Fourier integral operator with a complex phase function (see the appendix in [2]). Moreover, the restriction of $\partial_\nu R(\lambda)$ to $\Gamma$ is a $\lambda - \Psi$DO on $\Gamma$ and we obtain

$$\lambda^{-1} N(\partial_\nu u_2)|_\Gamma = u_2|_\Gamma + O(|\lambda|^{-\infty}) u_2|_\Gamma,$$

where $N$ is a $\lambda - \Psi$DO in $\mathrm{OP}^{0,0}(\Gamma)$ with a real-valued principal symbol $\sigma(N)$ which is elliptic on $\partial\Sigma$. Hereafter, $O(|\lambda|^{-\infty}) : H^{-s} \to H^s$, $s \ge 0$. stands for operators which are bounded in the corresponding Sobolev spaces (depending on $\lambda$) with norms $< C_{s,N}|\lambda|^{-N}$ for each $N > 1$. Moreover,

$$\sigma(N)(y, \eta) = (r_0(y, \eta) - 1)^{-1/2} \tag{2.3}$$

in a neighborhood of $\partial\Sigma$. In this way we reduce (2.1) to the following interior problem

$$\begin{cases} (g(x, \mathcal{D}) - 1)u &= O(|\lambda|^{-\infty})f \quad \text{in } \mathcal{O}, \\ u &= f \quad \text{on } \Gamma, \\ \alpha^{-1}\lambda^{-1}N(\partial_{\nu'} u)|_\Gamma + u|_\Gamma &= O(|\lambda|^{-\infty})u|_\Gamma. \end{cases} \tag{2.4}$$

We are going to show that this problem has a solution for infinitely many discrete values $\{k_j\}_{j=1}^{\infty} \subset \mathcal{L}$ of $\lambda$ and a sequence of functions $\{f_j\}_{j=1}^{\infty}$ such that $|k_j| \longrightarrow +\infty$, $\|f_j\|_{L^2} = 1$ and $\widetilde{\mathrm{WF}}(\{f_j\}) \subset \partial\Sigma$. To obtain $k_j$ and $f_j$ we construct a "normal form" of the boundary value problem (2.4) near the glancing manifold $\mathcal{K}$. First we are going to find a symplectic normal form for the corresponding classical problem.

We can suppose that the Hamiltonian $g$ is defined in a neighborhood of $T^*\mathcal{O}$. Given $0 < \delta \ll 1$, we set $\mathbf{D} = \Gamma \times (-\delta, \delta)$ and consider the so called "normal" to the boundary geodesic coordinates $y = (y', y_n) \in \mathbf{D}$, where $y'$ are local coordinates in $\Gamma$ and

$$x = y'(x) + y_n(x)\nu'(y'(x)).$$

In these coordinates, $f(y) = y_n + O(y_n^2)$ and the principal symbol of $-\Delta_g$ becomes

$$g(y, \eta) = \eta_n^2 + r(y', \eta') + y_n r_1(y, \eta'),$$

where $r(y', \eta')$ is the induced Hamiltonian. Moreover, $r$ is just the principal symbol of the Laplace-Beltrami operator on $\Gamma$ equipped with the Riemannian metric on $\Gamma$ induced by the metric $\mathcal{G}$, while

$$k(y', \eta') := 2^{-1} r_1(y', 0, \eta', 0) = -4^{-1}\{g, \{g, y_n\}\}(y', 0, \eta', 0) \geq C|\eta'|^2, \ C > 0, \quad (2.5)$$

could be identified with the second fundamental form of $\Gamma$ (associated with $\mathcal{G}$ and $\nu'$) on $\partial\Sigma$.

First we obtain a global symplectic normal form for the pair of glancing hypersurfaces $F$ and $G$ near the gliding manifold $\mathcal{K}$. To do this we make use of the approximate interpolating Hamiltonian $\zeta$ of the corresponding billiard ball map $B$ (see [8], [3], [6], and the references there). Recall that the $\zeta$ defines $\partial\Sigma$ ($\zeta = 0$ and $d\zeta \neq 0$) on $\partial\Sigma$ and $\zeta > 0$ inside $\Sigma$, $\zeta < 0$ outside $\Sigma$. Hence, there exists a positive function $b \in C^\infty(T^*\Gamma)$ such that

$$\zeta(y', \eta') \ = \ b(y', \eta')\,(1 \ - \ r(y', \eta')) \,. \quad (2.6)$$

**Theorem 2.2** *There exists an exact symplectic transformation $\chi : T^*\mathbf{D} \ \to \ T^*\mathbf{D}$ such that $\tilde{f} = f \circ \chi$ and $\tilde{g} = g \circ \chi - 1$ have the form*

$$\tilde{f}(x, \xi) \ = \ b(x', \xi')^{1/2}(x_n + O(x_n^2)) \ ,$$

$$\tilde{g}(x, \xi) \ = \ h(x, \xi')(\xi_n^2 + x_n - \zeta(x', \xi')) + O(x_n^\infty) + O(\xi_n^\infty) + O(\zeta^\infty)$$

*in a neighborhood of $\mathcal{K} = \{x_n = \xi_n = \zeta = 0\}$ in $T^*\mathbf{D}$, where $h \in C^\infty(T^*\Gamma \times [-\delta, \delta])$, $h > 0$ in a neighborhood of $\partial\Sigma \times \{0\}$. Moreover, $\chi(x, \xi) = (y(x, \xi), \eta(x, \xi))$ has the form*

$$y_n \ = \ x_n b(x', \xi')^{1/2} + O(x_n^2) \ , \ (y', \eta', \eta_n) \ = \ (x', \xi', \xi_n b(x', \xi')^{-1/2}) + O(x_n) \,,$$

*and*

$$\begin{aligned} b(x', \xi') \ &= \ (2\,k(x', \xi'))^{-2/3} + O(\zeta(x', \xi')) \,, \quad (2.7) \\ h(x, \xi') \ &= \ b(x', \xi')^{-1}(1 + O(x_n)) \,. \end{aligned}$$

Denote by $L(x', \mathcal{D}', \lambda)$ a $\lambda - \Psi$DO with a principal symbol $\zeta(x', \xi')$ and subprincipal symbol 0 and set

$$P_0(x, \mathcal{D}, \lambda) \;=\; \mathcal{D}_n^2 \,+\, x_n \,-\, L(x', \mathcal{D}', \lambda)\,.$$

Denote by $\mathcal{R}$ the ideal of all $\lambda - \Psi$DOs $R \in \mathrm{OP}^{0,0}(\mathbf{D})$ having in any local chart complete symbols of the form $\sum_{j=0}^{\infty} \lambda^{-j} R_j(x, \xi)$, where

$$R_j(x, \xi) \;=\; O(x_n^{\infty}) \,+\, O(\xi_n^{\infty}) \,+\, O(\zeta^{\infty})\,,\; j \geq 0\,,$$

Next using Fourier integral operators with a large parameter $\lambda$, we transform (microlocally near $\mathcal{K}$) the boundary value problem (2.4) to a boundary value problem

$$\begin{cases} (P_0(x, \mathcal{D}, \lambda) + R(x, \mathcal{D}, \lambda))v \;=\; O(|\lambda|^{-\infty})v\,,\; \widetilde{\mathrm{WF}}\, v \subset \mathcal{K}\,, \\ \qquad\qquad (i\, W\mathcal{D}_n v \,+\, v + \tilde{C}v)|_{x_n=0} \;=\; O(|\lambda|^{-\infty})v\,, \end{cases} \tag{2.8}$$

where $C$, $\tilde{C} \in \mathcal{R}$ and $W \in \mathrm{OP}^{0,0}(\Gamma)$. The principal symbol $\sigma(W)$ of $W$ is real valued and taking into account (2.3) we get

$$\sigma(W)(x', \xi') \;=\; \alpha^{-1} b(x', \xi')^{-1/2} (r_0(x', \xi') - 1)^{-1/2} \tag{2.9}$$

in a neighborhood of $\partial\Sigma$.

This enables us to "separate" the variables microlocally near $\mathcal{K}$. To illustrate it, we consider the equation $P_0 v = O(|\lambda|^{-\infty})v$ with Dirichlet boundary conditions $v|_{x_n=0} = O(|\lambda|^{-\infty})v$ instead of those in (2.8). Then we can take $v(x, \lambda) = \mathrm{Ai}(\tau + \lambda^{2/3} x_n) w(x', \lambda)$, where $\tau < 0$ is a zero of $\mathrm{Ai}(s)$ and $w(\cdot, \lambda) \in C^{\infty}(\Gamma)$ satisfies the equation

$$L(x', \mathcal{D}', \lambda)w \,+\, \tau\lambda^{-2/3} w \;=\; O(|\lambda|^{-\infty})w\,.$$

The boundary condition in (2.8), however, makes the construction more complicated. We are looking for an asymptotic solution of $(\mathcal{D}_n^2 + x_n - L)v = O(|\lambda|^{-\infty})v$ of the form:

$$v \;=\; v(x, \lambda) \;\sim\; \sum_{k=0}^{\infty} \lambda^{-k/3}\, \mathrm{Ai}^{(k)} \left(\tau + \lambda^{2/3} x_n\right) X_k w(x', \lambda)\,, \quad |\lambda| \to \infty\,,$$

where $\mathrm{Ai}^{(k)}$ is the derivative of order $k \geq 0$ of the Airy function $\mathrm{Ai}(s)$, $X_k$ are suitable $\lambda$-pseudodifferential operators on $\Gamma$, $X_0$ is the identity, and $w(\cdot, \lambda) \in C^{\infty}(\Gamma)$ has a frequency set in $\partial\Sigma = \{\zeta = 0\}$. Then we obtain $R(x, \mathcal{D}, \lambda)v = O(|\lambda|^{-\infty})v$, hence, $v$ turns out to be also an asymptotic solution of the first equation in (2.8). Using the boundary condition in (2.8), we determine the operators $X_k$, $k \geq 1$, and we obtain a pseudodifferential equation for $w$ of the form

$$\lambda L(x', \mathcal{D}', \lambda)w \,+\, \tau\lambda^{1/3} w \,+\, Zw \;=\; O(|\lambda|^{-\infty})w\,,$$

where $Z \sim \sum_{j=0}^{\infty} \lambda^{-j/3} Z_j$, $Z_j \in \mathrm{OP}^{0,0}(\mathbf{D})$, and $Z_0 = -W$. Finally, using (2.6) and (2.7) we transform this equation to a spectral problem $Q(x', D')w = \lambda w$, where $Q \in \Psi_{\mathrm{phg}}^{1,1/3}(\Gamma)$ is a classical elliptic pseudodifferential operator on $\Gamma$ with a polyhomogeneous symbol of order one and step $1/3$ (see [5]). The principal symbol of $Q$ is real valued and it has the form

$$\sigma(Q)(x', \xi') \;=\; r(x', \xi')^{1/2} \,-\, \tau 2^{-1} \left(2k(x', \xi')\right)^{2/3} r(x', \xi')^{-1/2}\,, \tag{2.10}$$

where $k(x',\xi')$ is defined by (2.5). Then $\lambda = k_j$ and $w = w_j$ are just the eigenvalues and the corresponding eigenfunction of $Q(x',D')$.

## 3. Proof of Theorem 1.2.

We are going to sketch the proof of Theorem 1.2. It is enough to prove (1.5) for real $\lambda$, then Theorem 1.2 and Corollary 1.3 follow from the results in [19]. To do this we obtain a priori estimates for the interior and the exterior Dirichlet problem.

*1. A priori estimates for interior boundary value problems.* Let $\mathcal{O} \subset \mathbf{R}^n$ be as in the introduction and let $u \in H^2(\mathcal{O})$ satisfy the equation

$$\begin{cases} (\Delta_g + \lambda^2)u = \lambda v \quad \text{in} \quad \mathcal{O}, \\ Bu|_{\Gamma_1} = 0, \\ u|_\Gamma = f, \quad \partial_{\nu'} u|_\Gamma = \lambda h. \end{cases} \tag{3.1}$$

Then we have

**Theorem 3.1** *Let $\Gamma$ be both $g$ and $g_0$-strictly concave with respect to $\mathcal{O}$, and let the assumption (H) be fulfilled. Then there exist constants $C, \lambda_0 > 0$ so that for real $\lambda \geq \lambda_0$ we have*

$$\|u\|_{H^1(\mathcal{O})} \leq C\left(\|v\|_{L^2(\mathcal{O})} + \|f\|_{L^2(\Gamma)} + \|h\|_{L^2(\Gamma)}\right). \tag{3.2}$$

Consider now the problem

$$\begin{cases} (\Delta_g + \lambda^2)u = \lambda v \quad \text{in} \quad \mathcal{O}, \\ u|_\Gamma = f, \\ \lambda^{-1}\partial_{\nu'} u|_\Gamma + A(\lambda) f = h, \end{cases} \tag{3.3}$$

where

$$A(\lambda) = O(1) : H^1(\Gamma) \to L^2(\Gamma)\,. \tag{3.4}$$

Suppose that

$$\operatorname{Re}\langle A(\lambda)f, f\rangle_{L^2(\Gamma)} \leq o(1)\|f\|^2_{L^2(\Gamma)}\,, \qquad \forall f \in H^1(\Gamma). \tag{3.5}$$

We also suppose that for any $\chi \in C^\infty(T^*\Gamma)$ which is equal either to zero or to $(1+|\xi'|)^s, s = 0,1,$ outside some compact, and any $\lambda$-pseudodifferential operator $\chi(x',\mathcal{D}',\lambda)$ with a principal symbol $\chi$, we have

$$\|[\chi, A(\lambda)]\|_{\mathcal{L}(H^s(\Gamma))} = o(1)\,, \quad s = 0,1\,. \tag{3.6}$$

Choose now a function $\chi \in C^\infty(T^*\Gamma)$ such that $\chi = 1$ for $r(x',\xi') \leq 1+\varepsilon_0$ and $\chi = 0$ for $r(x',\xi') \geq 1+2\varepsilon_0$, where $0 < \varepsilon_0 \ll 1$. Then the support of $1-\chi$ is contained in the elliptic region for the interior problem with Dirichlet boundary conditions, and we obtain

**Proposition 3.2** *Under the assumptions (3.4)-(3.6), we have*

$$\begin{aligned} &\|(1-\chi(x',\mathcal{D}',\lambda))f\|_{H^1(\Gamma)} \\ \leq\ & O(1)\|h\|_{L^2(\Gamma)} + o(1)\|v\|_{L^2(\mathcal{O})} + o(1)\|f\|_{L^2(\Gamma)} + o(1)\|u\|_{L^2(\mathcal{O})}. \end{aligned} \tag{3.7}$$

*2. A priori estimates for exterior boundary value problems.*

Let $\Omega \subset \mathbf{R}^n$ be the exterior of a bounded strictly convex domain with $C^\infty$ boundary $\Gamma$. Let $u \in H^2_{loc}(\Omega)$ satisfy the equation

$$\begin{cases} (\Delta+\lambda^2)u = \lambda v \quad \text{in} \quad \Omega, \\ u|_\Gamma = f,\ \lambda^{-1}\partial_\nu u|_\Gamma = h, \\ u \ - \ \lambda - \text{outgoing}, \end{cases} \tag{3.8}$$

where $\operatorname{supp} v \subset \Omega_a := \{x \in \Omega : |x| \le a\}, a \gg 1$. Then we have

**Theorem 3.3** *There exist constants $C, \lambda_0 > 0$ so that for real $\lambda \ge \lambda_0$ we have*

$$\|u\|_{H^1(\Omega_a)} + \|h\|_{L^2(\Gamma)} \le C(\|v\|_{L^2(\Omega_a)} + \|f\|_{H^1(\Gamma)}). \tag{3.9}$$

Introduce the Neumann operator

$$N(\lambda)f = \lambda^{-1}\partial_\nu K(\lambda) f|_\Gamma\,,$$

where $K(\lambda)f$ solves the problem

$$\begin{cases} (\Delta+\lambda^2)K(\lambda)f & = & 0 \quad \text{in} \quad \Omega, \\ K(\lambda)f|_\Gamma & = & f, \\ K(\lambda)f & - & \lambda - \text{outgoing}. \end{cases}$$

Applying Theorem 3.3 with $v \equiv 0$ leads to the following

**Corollary 3.4** *For real $\lambda \gg 1$, we have*

$$N(\lambda) = O(1) : H^1(\Gamma) \to L^2(\Gamma).$$

Moreover, we have

**Proposition 3.5** *For real $\lambda \gg 1$, we have*

$$\operatorname{Re}\langle N(\lambda)f, f\rangle_{L^2(\Gamma)} \le C\lambda^{-1/3}\|f\|^2_{L^2(\Gamma)}, \qquad \forall f \in H^1(\Gamma),$$

*with some constant $C > 0$ independent of $\lambda$.*

It follows from Corollary 3.4 and Proposition 3.5 that $A(\lambda) = \alpha N(\lambda)$ satisfies (3.4) and (3.5). It is easy to see that it satisfies also (3.6).

*3. Estimates for the cutoff resolvent on the real axis.* Consider the problem

$$\begin{cases} (\Delta_g+\lambda^2)u_1 & = & \lambda v_1 \quad \text{in} \quad \mathcal{O}, \\ (\Delta+\lambda^2)u_2 & = & \lambda v_2 \quad \text{in} \quad \Omega, \\ u_1|_\Gamma = u_2|_\Gamma & = & f, \\ Bu_1|_{\Gamma_1} & = & 0, \\ \partial_{\nu'}u_1|_\Gamma + \alpha\partial_\nu u_2|_\Gamma & = & 0, \\ u_2 & - & \lambda - \text{outgoing}, \end{cases} \tag{3.10}$$

where $u_1 \in H^2(\mathcal{O}), u_2 \in H^2_{loc}(\Omega)$, $\operatorname{supp} v_2 \subset \Omega_a, a \gg 1$. In what follows $C$ will denote a positive constant independent of $\lambda$. Clearly, the estimate

$$\|\lambda R_\chi(\lambda)\|_{\mathcal{L}(H)} \le C \ , \ \lambda \in \mathbf{R}, \ |\lambda| \ge C_2 \,,$$

is equivalent to

$$\|u_1\|_{L^2(\mathcal{O})} + \|u_2\|_{L^2(\Omega_a)} \le C(\|v_1\|_{L^2(\mathcal{O})} + \|v_2\|_{L^2(\Omega_a)}). \tag{3.11}$$

By Theorems 3.1 and 3.3 we have

$$\|u_2\|_{L^2(\Omega_a)} \le C(\|v_2\|_{L^2(\Omega_a)} + \|f\|_{H^1(\Gamma)}),$$

and

$$\|u_1\|_{L^2(\Omega_a)} \le C(\|v_1\|_{L^2(\mathcal{O})} + \|v_2\|_{L^2(\Omega_a)} + \|f\|_{H^1(\Gamma)}). \tag{3.12}$$

Hence, to prove (3.11), it suffices to show that

$$\|f\|_{H^1(\Gamma)} \le C(\|v_1\|_{L^2(\mathcal{O})} + \|v_2\|_{L^2(\Omega_a)}). \tag{3.13}$$

To this end, observe that $u := u_1, v := v_1$ satisfy the equation

$$\left\{\begin{array}{rcl} (\Delta_g + \lambda^2)u & = & \lambda v \quad \text{in} \quad \mathcal{O}, \\ u|_\Gamma & = & f, \\ Bu|_{\Gamma_1} & = & 0, \\ \lambda^{-1}\partial_{\nu'}u|_\Gamma + \alpha N(\lambda) f & = & h, \end{array}\right. \tag{3.14}$$

where $h = -\lambda^{-1}\partial_\nu G(\lambda)v_2|_\Gamma$ and $G(\lambda)v$ solves the problem

$$\left\{\begin{array}{rcl} (\Delta + \lambda^2)G(\lambda)v & = & v \quad \text{in} \quad \Omega, \\ G(\lambda)v|_\Gamma & = & 0, \\ G(\lambda)v & - & \lambda - \text{outgoing}, \end{array}\right.$$

In view of Theorem 3.3, $h$ satisfies the estimate

$$\|h\|_{L^2(\Gamma)} \le C\|v_2\|_{L^2(\Omega_a)}. \tag{3.15}$$

Using (3.14) and Green's formula, we obtain

$$-\alpha\lambda \mathrm{Im}\,\langle N(\lambda)f, f\rangle_{L^2(\Gamma)} = \mathrm{Im}\,\langle \partial_{\nu'}u|_\Gamma, u|_\Gamma\rangle_{L^2(\Gamma)} - \lambda \mathrm{Im}\,\langle h, f\rangle_{L^2(\Gamma)}$$

$$= -\lambda \mathrm{Im}\,\langle c^{-2}u, v\rangle_{L^2(\mathcal{O})} - \lambda \mathrm{Im}\,\langle h, f\rangle_{L^2(\Gamma)}.$$

Then for any $\varepsilon > 0$ we obtain the estimate

$$-\mathrm{Im}\,\langle N(\lambda)f, f\rangle_{L^2(\Gamma)} \le O(\varepsilon^2)\|u\|^2_{L^2(\mathcal{O})} + O_\varepsilon(1)\|v\|^2_{L^2(\mathcal{O})}$$

$$+O(\varepsilon^2)\|f\|^2_{L^2(\Gamma)} + O_\varepsilon(1)\|h\|^2_{L^2(\Gamma)}. \tag{3.16}$$

Choose a cutoff function $\chi \in C^\infty(T^*\Gamma)$ such that $\chi = 1$ in $\{(x', \xi') \in T^*\Gamma : r(x', \xi') \le 1 + \varepsilon_0\}$ and $\chi = 0$ in $\{(x', \xi') \in T^*\Gamma : r(x', \xi') \ge 1 + 2\varepsilon_0\}$, where $0 < \varepsilon_0 \ll 1$. By (3.16),

$$-\mathrm{Im}\,\langle N(\lambda)\chi(x', \mathcal{D}', \lambda)f, \chi(x', \mathcal{D}', \lambda)f\rangle_{L^2(\Gamma)} \le O(\varepsilon^2)\|u\|^2_{L^2(\mathcal{O})} + O_\varepsilon(1)\|v\|^2_{L^2(\mathcal{O})}$$

$$+O(\varepsilon^2)\|f\|^2_{L^2(\Gamma)} + O_\varepsilon(1)\|h\|^2_{L^2(\Gamma)} + O(1)\|(1-\chi(x',\mathcal{D}',\lambda))f\|^2_{H^1(\Gamma)}. \tag{3.17}$$

Because of (1.4), choosing $\varepsilon_0 > 0$ sufficiently small, we can arrange

$$\operatorname{supp}\chi \subset \{(x',\xi') \in T^*\Gamma \,:\, r_0(x',\xi') < 1\},$$

and hence $N(\lambda)\chi(x',\mathcal{D}',\lambda)$ is a $\lambda - \Psi$DO with principal symbol

$$-i\chi(x',\xi')\sqrt{1-r_0(x',\xi')}\,.$$

Therefore

$$-\operatorname{Im}\langle N(\lambda)\chi(x',\mathcal{D}',\lambda)f, \chi(x',\mathcal{D}',\lambda)f\rangle_{L^2(\Gamma)}$$
$$\geq C\|\chi(x',\mathcal{D}',\lambda)f\|^2_{L^2(\Gamma)} - o(1)\|f\|^2_{L^2(\Gamma)}, \quad C > 0. \tag{3.18}$$

By (3.17) and (3.18) we get

$$\|\chi(x',\mathcal{D}',\lambda)f\|_{L^2(\Gamma)} \leq O(\varepsilon)\|u\|_{L^2(\mathcal{O})} + O_\varepsilon(1)\|v\|_{L^2(\mathcal{O})}$$

$$+O(\varepsilon)\|f\|_{L^2(\Gamma)} + O_\varepsilon(1)\|h\|_{L^2(\Gamma)} + O(1)\|(1-\chi(x',\mathcal{D}',\lambda))f\|_{H^1(\Gamma)}.$$

Hence, taking $\varepsilon > 0$ small enough, we obtain

$$\|f\|_{H^1(\Gamma)} \leq C\|\chi(x',\mathcal{D}',\lambda)f\|_{L^2(\Gamma)} + C\|(1-\chi(x',\mathcal{D}',\lambda))f\|_{H^1(\Gamma)}$$

$$\leq O(\varepsilon)\|u\|_{L^2(\mathcal{O})} + O_\varepsilon(1)\|v\|_{L^2(\mathcal{O})}$$

$$+O(\varepsilon)\|f\|_{L^2(\Gamma)} + O_\varepsilon(1)\|h\|_{L^2(\Gamma)} + O(1)\|(1-\chi(x',\mathcal{D}',\lambda))f\|_{H^1(\Gamma)},$$

which combined with Proposition 3.2, (3.12) and (3.15) implies (3.13).

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