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Local existence theory for the generalized Schrödinger equation

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This note is based on a forthcoming joint work with Carlos E. Kenig and Luis Vega.

We are interested in the local well posedness of the initial value problem (IVP) for nonlinear Schrödinger equations of the form

$$\begin{cases} \partial_t u = i\mathcal{L}u + P(u, \nabla_x u, \overline{u}, \nabla_x \overline{u}), & t \in \mathbb{R},\ x \in \mathbb{R}^n, \\ u(x,0) = u_0(x), \end{cases} \tag{1.1}$$

where $u = u(x,t)$ is a complex valued function, $\mathcal{L}$ is a non-degenerate constant coefficient, second order operator

$$\mathcal{L} = \sum_{j\leq k} \partial_{x_j}^2 - \sum_{j>k} \partial_{x_j}^2, \qquad \text{for some} \quad k \in \{1,..,n\}, \tag{1.2}$$

and

$$P : \mathbb{C}^{2n+2} \longrightarrow \mathbb{C}.$$

is a polynomial of the form

$$P(z) = P(z_1, .., z_{2n+2}) = \sum_{l_0 \leq |\alpha| \leq d} a_\alpha z^\alpha, \qquad l_0 \geq 2. \tag{1.3}$$

When a special form of the nonlinear term P is assumed, for example,

$$D_{\partial_{x_j} u} P, \qquad \text{are real for } j = 1, .., n,$$

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standard energy estimates provide the desired result, see [Kt]. In these particular cases, the dispersive part of the equation, modeled by the operator $\mathcal{L}$, does not play any role in the proof, i.e. the same local result applies to the IVP

$$\begin{cases} \partial_t u = P(u, \nabla_x u, \overline{u}, \nabla_x \overline{u}), & t \in \mathbb{R},\ x \in \mathbb{R}^n, \\ u(x,0) = u_0(x). \end{cases}$$

Another approach used to overcome the "loss of derivatives" introduced by the nonlinear term, is restricted to work in suitable analytic function spaces, see [Hy1], [SiTa]. For the semi-linear case $P = P(u, \overline{u})$ see [Cz] and references therein.

In [KePoVe1], we proved that (1.1) is locally well posed for "small" data, in the Sobolev space $H^s(\mathbb{R}^n)$, for s large enough, when $l_0 \geq 3$ in (1.3), and in its weighted version, if $l_0 = 2$ in (1.3). This result applies to the general form of $\mathcal{L}$ in (1.2).

To explain the method of proof in [KePoVe1] as well as the restriction on the size of the data, we consider integral equation version of the IVP (1.1)

$$u(t) = e^{it\mathcal{L}} u_0 + \int_0^t e^{i(t-t')\mathcal{L}} P(u, \nabla_x u, \overline{u}, \nabla_x \overline{u})(t')dt', \tag{1.4}$$

and use the following estimates,

$$\begin{cases} (i) \quad \sup_{[0,T]} \|e^{it\mathcal{L}} u_0\|_2 = \|u_0\|_2, \\ \\ (ii) \quad |||D^{1/2} e^{it\mathcal{L}} u_0|||_T = \sup_{\mu \in \mathbb{Z}^n} \left(\int_0^T \int_{Q_\mu} |D^{1/2} e^{it\mathcal{L}} u_0|^2 dx dt \right)^{1/2} \leq c\|u_0\|_2, \\ \\ (iii) \quad |||\nabla_x \int_0^t e^{i(t-t')\mathcal{L}} F(t')dt'|||_T \leq c|||F|||'_T, \end{cases} \tag{1.5}$$

where $\{Q_\mu\}_{\mu \in \mathbb{Z}^n}$ is a family of unit cubes of side one with disjoint interiors covering $\mathbb{R}^n$, and $D = (-\Delta)^{1/2}$.

The local smoothing effect described in (ii) was proven by Constantin-Saut [CoSa], Sjölin [Sj], and Vega [Ve].

The inhomogeneous version (iii) of this local effect was established in [KePoVe1].

For the problem considered here, it is essential that in the inhomogeneous case one gains one derivative. This allows to use the contraction principle in (1.4). However, the use of the $|||\cdot|||$ norm forces us to use the following norm

$$\|G\|_{l^1_\mu(L^\infty(Q_\mu\times[0,T]))}=\sum_\mu \sup_{t\in[0,T]}\sup_{Q_\mu}|G(x,t)|.$$

This appears as factor which cannot be made small by taking T small, except if $G(t)$ is small at $t=0$. It is here where the restriction on the size of the data appears.

In the one dimensional case, $n=1$, the smallness assumption on the size of the data in was removed by N. Hayashi and T. Ozawa [HyOz]. They used a change of variable to obtain an equivalent system with a nonlinear term independent of $\partial_x u$, which can be treated by the standard energy method.

In [Ch1] H. Chihara, for the elliptic case $\mathcal{L}=\Delta$, removed the size restriction on the data in any dimension. Using an invertible classical pseudo-differential operator of order zero, K, he changes variables, and rewrites the equation as a system in terms of $(Ku, K\overline{u})$, where the commutator $[K;i\Delta]$ basically allows to control the terms in $\nabla_x u$. A main step in his proof is a diagonalization method in which the assumption on the ellipticity of $\mathcal{L}$ is essential.

Equations of the form described in (1.1) with $\mathcal{L}$ non-elliptic arise in several situations, for example, in the study of water wave problems, the Davey-Stewartson [DaSe], and Zakharov-Shulman [ZaSc] systems, in ferromagnetism, the Ishimori system [Ic], and as higher dimension completely integrable models, see [AbHa].

For example, consider the Davey-Stewartson system

$$\text{(1.6)}\qquad \begin{cases} i\partial_t u + c_0\partial_{x_1}^2 u + \partial_{x_2}^2 u = c_1|u|^2u + c_2 u\partial_{x_1}\varphi, \\ \partial_{x_1}^2\varphi + c_3\partial_{x_2}^2\varphi = \partial_x|u|^2, \\ u(x,y,0)=u_0(x,y), \end{cases}$$

where $u=u(x,y,t)$ is a complex-valued function, $\varphi=\varphi(x,y,t)$ is a real-valued function, (when $(c_0,c_1,c_2,c_3)=(-1,1,-2,1)$ or $(1,-1,2,-1)$ the system in (1.1) is known in inverse scattering as the DSI and DSII respectively).

In the case, $c_3<0$ where the equation in (1.6) is essentially not semi-linear, several results has been established, see [GhSa], [Ch2], [Hy-Sa], [Hy2], [LiPo].

However, in the particular case $c_3 < 0$, $c_0 < 0$, i.e. nonsemilinear and nonelliptic dispersion, the only existence result available are for analytic data, see [Hy-Sa], or "small" data, see [LiPo] .

In the IVP for the Ishimori system can be written as

$$\begin{cases} i\partial_t u + \partial_{x_1}^2 u \mp \partial_{x_2}^2 u = \frac{2\overline{u}}{1+|u|^2}(u_{x_1}^2 - u_{x_2}^2) + ib(\varphi_{x_1}u_{x_2} - \varphi_{x_2}u_{x_1}), \quad b \in \mathbb{R}, \\ \partial_{x_1}^2\varphi \pm \partial_{x_2}^2\varphi = 4i\frac{u_{x_1}\overline{u}_{x_2} - \overline{u}_{x_1}u_{x_2}}{(1+|u|^2)^2}, \\ u(x,y,0) = u_0(x,y). \end{cases}$$

For the $(-,+)$ case, see [So]. For the case $(+,-)$ the only existence results available are restricted to the class of analytic data, see [Hy-Sa].

Our main results are as follows:

Theorem A.

Let $l_0 \geq 3$, see (1.3). Then there exists $s = s(n;P) > 0$ such that, for any $u_0 \in H^s(\mathbb{R}^n)$, the IVP (1.1) has a unique solution $u(\cdot)$, defined in the time interval $[0,T]$, $T = T(\|u_0\|_{H^s}) > 0$, satisfying

$$u \in C([0,T] : H^s(\mathbb{R}^n)), \tag{1.7}$$

and

$$|||J^{s+1/2}u|||_T \equiv \sup_{\mu\in\mathbb{Z}^n} \left(\int_0^T \int_{Q_\mu} |J^{s+1/2}u(x,t)|^2 dxdt\right)^{1/2} < \infty. \tag{1.8}$$

We remark that this result is new even in the elliptic since we do not need weighted Sobolev spaces as in Chihara's work.

Theorem B.

Let $l_0 = 2$, see (1.3). Then there exist $s = s(n;P) > 0$, and $m = m(n;P) > 0$, such that, for any $u_0 \in H^s(\mathbb{R}^n) \cap L^2(\mathbb{R}^n : |x|^{2m}dx)$, the results in Theorem A hold with

$$u \in C([0,T] : H^s(\mathbb{R}^n) \cap L^2(\mathbb{R}^n : |x|^{2m}dx)). \tag{1.9}$$

To explain our method, let us consider a particular case of the I.V.P. (1.1)

$$\begin{cases} \partial_t u = i\mathcal{L}u + u^2 \sum_{j=1}^n (\partial_{x_j}u + \partial_{x_j}\overline{u}), \\ u(x,0) = u_0(x). \end{cases} \tag{1.10}$$

Differentiating the equation above, and using the notation

$$u_\alpha = \partial_x^\alpha u, \qquad |\alpha| < l, \qquad v_\alpha = \partial_x^\alpha u, \qquad |\alpha| = l,$$

with $l \in \mathbb{Z}^+$ to be fixed later, we obtain the system

$$(1.11) \qquad \begin{cases} \partial_t u_\alpha = i\mathcal{L}u_\alpha + P_\alpha((u_\beta, \overline{u}_\beta)_{|\beta|<l}, (v_\beta, \overline{v}_\beta)_{|\beta|=l}), & |\alpha| < l, \\ \\ \partial_t v_\alpha = i\mathcal{L}v_\alpha + u^2 \sum_{j=1}^{n} (\partial_{x_j} v + \partial_{x_j} \overline{v}) + \\ \qquad + P_\alpha((u_\beta, \overline{u}_\beta)_{|\beta|<l}, (v_\beta, \overline{v}_\beta)_{|\beta|=l}), & |\alpha| = l, \end{cases}$$

where the P's are polynomials in their variables.

Since the equations for $(u_\beta)_{|\beta|<l}$ are semi-linear, they can be easily handled if the equations for the v_α's can be solved in $H^s(\mathbb{R}^n)$. Thus, the problem reduces to considering the equations for v_α's, which can be rewritten as

$$(1.12) \qquad \begin{aligned} \partial_t v_\alpha = i\mathcal{L}v_\alpha + u_0^2 \sum_{j=1}^{n} (\partial_{x_j} v_\alpha + \partial_{x_j} \overline{v}_\alpha) + (u^2 - u_0^2) \sum_{j=1}^{n} (\partial_{x_j} v_\alpha + \partial_{x_j} \overline{v}_\alpha) \\ + P_\alpha((u_\beta, \overline{u}_\beta)_{|\beta|<l}, (v_\beta, \overline{v}_\beta)_{|\beta|=l}), \quad |\alpha| = l. \end{aligned}$$

The nonlinear terms in (1.12) are either semilinear ones, whose bound depends on the L^2-well posedness of the associated linear problem, or those involving $(\nabla_x v_\beta, \nabla_x \overline{v}_\beta)_{|\beta|=l}$. These appear in a form such that if $u(\cdot)$ is a solution of (1.12) then they vanish at $t = 0$. Thus, in a small time interval $[0, T]$, they must remain "small". Hence, if the solution of the associated linear problem in (1.12), satisfies local estimates similar to those for the generalized free Schrödinger group $\{e^{it\mathcal{L}}\}_{t=-\infty}^{\infty}$ described in (1.5), then our previous argument provides the desired result.

Hence, the problem has been reduced to show that, under appropriate assumptions on the smoothness and decay of the coefficients ($b_{k,j} = (b_{k,1}, .., b_{k,n})$, $k = 1, 2$, $j = 1, ..., n$, which depend only on the initial data), the IVP for the linear Schrödinger equation with variable coefficient lower order terms

$$(1.13) \qquad \begin{cases} \partial_t v = i\mathcal{L}v + b_1(x) \cdot \nabla_x v + b_2(x) \cdot \nabla_x \overline{v} + F(x, t), \\ v(x, 0) = v_0, \end{cases}$$

satisfies the following results.

To simplify the exposition we shall assume that the coefficients in (1.13) satisfy $b_1,\ b_2 \in C_0^\infty(\mathbb{R}^n)$, with $\|b_1\|_{L^{N,1}} + \|b_2\|_{L^{N,1}} \le K$, N large.

Theorem C.

Under the above hypothesis, there exists $K = K(n)$ *such that the IVP (1.13) has a unique solution* $u \in C((-\infty,\infty) : L^2(\mathbb{R}^n))$ *verifying that for any* $T > 0$

$$(1.14) \qquad \sup_{-T\le t\le T} \|u(t)\|_2 \le ce^{cT}\{\|u_0\|_2 + |||J^{-1/2}F|||'_T\},$$

and

$$(1.15) \qquad |||J^{1/2}u|||_T \le ce^{cT}\{\|u_0\|_2 + |||J^{-1/2}F|||'_T\},$$

where c *depend only on* n, K, *and* $(J^s f)^\wedge(\xi) =< \xi >^s \hat{f}(\xi)$.

Basically Theorem C extends the estimates in (1.5) to solutions of the equation in (1.13) a Schrödinger equation with first order variable coefficients.

The local solvability of the linear IVP (1.13) with $\mathcal{L} = \Delta$ and $b_2(x) \equiv 0$

$$(1.16) \qquad \begin{cases} \partial_t v = i\Delta v + b_1(x)\cdot\nabla_x v + F(x,t), \\ v(x,0) = v_0. \end{cases}$$

have been considered in several works, [Do1], [Mi], and references therein.

In particular, Mizohata [Mi] proved that the following condition is necessary for the L^2-well posedness of (1.16)

$$(1.17) \qquad \sup_{x\in\mathbb{R}^n,\omega\in\mathbb{S}^{n-1},\lambda>0} \left| \Im \int_0^\lambda b_1(x+r\omega)\cdot\omega dr \right| < \infty.$$

We observe that when $b_1(\cdot)$ is real, the standard energy method provides the desired result.

Also since in our case the coefficients b_k, $k = 1,2$, depend on the initial data u_0, condition (1.17) justifies the weighted hypothesis on the data in Theorem B for P having quadratic terms involving $(\nabla_x u, \nabla_x \overline{u})$. In the case $l_0 \ge 3$ in (1.3), the condition (1.17) follows from the Sobolev theorem.

Without decay assumptions on the coefficient b_2, the regularity of the solution of the IVP (1.13) may depend on the ellipticity of $\mathcal{L}$. Thus the IVP

$$(1.18) \qquad \begin{cases} \partial_t v = i\mathcal{L}v + b_2(x)\cdot\nabla_x\overline{v}, \\ v(x,0) = v_0, \end{cases}$$

with $b_2 \in C_b^2(\mathbb{R}^n)$. In [KePoVe1] we showed that in the case $n=2$, $\mathcal{L} = \partial^2_{x_1x_2}$, and $b_2 = -i$ the IVP (1.18) is well posed, and its solution satisfies

$$(1.19) \qquad |||J^s v|||_T = \sup_{\mu\in\mathbb{Z}^n} \Big(\int_0^T \int_{Q_\mu} |J^s v(x,t)|^2 dx dt\Big)^{1/2} \le c_T \|v_0\|_{L^2},$$

for $0 \le s \le 1/4$. Morever, we proved that (1.19) fails for $s > 1/4$. However, we showed that for $\mathcal{L} = \Delta$ and $b_2 \in C_b^2(\mathbb{R}^n)$ (1.19) holds with $s = 1/2$.

As it was mentioned above, once (1.6)(i)-(iii) are available, the proofs of Theorems 1.1-1.2 reduce to the argument given in our previous works, see [KePoVe1]. We look for an operator C such that C is invertible in L^2, the difference between the commutator $i[\mathcal{L}, C]$ and $Cb_1(x)\cdot\nabla$ can be bounded appropriately , and $\overline{Cv} = C\overline{v}$. This operator C is in the exotic symbol class $S^0_{0,0}$ of Calderón-Vaillancourt. The use of C was suggested from a previous work of Takeuchi ([Tk]).

We may remark that the operator A for which the symbol of $i[\mathcal{L}, A] - Ab_1(x)\cdot\nabla$ essentially vanishes is not a classical ψ.d.o. It is in the class recently studied by Craig-Kappeler-Strauss [CrKaST], which in the non-elliptic case has not been shown to provide L^2 bounded operators, see also [Do2], [KePoVe3].

Thus we reduce the problem to obtain the estimate in (1.14)-(1.15) for solutions of the linear IVP (1.13)

The main step in the proof of Theorem C is the construction of the operator $C(x,D)$.

Construction of the operator C .

Let $C = C(x,D)$ be a L^2-bounded ψ.d.o. with symbol $\sigma(C) = c(x,\xi)$ to be determined. Thus,

$$\frac{d}{dt}\langle Cu, Cu\rangle = \frac{d}{dt}\int_{\mathbb{R}^n} Cu\overline{Cu}dx$$

$$(1.20) \qquad \begin{aligned} &= \langle Ci\mathcal{L}u, Cu\rangle + \langle Cb_1\nabla u, Cu\rangle + \langle Cb_2\nabla\overline{u}, Cu\rangle \\ &+ \langle CF, Cu\rangle + \langle Cu, Ci\mathcal{L}u\rangle + \langle Cu, Cb_1\nabla u\rangle \\ &+ \langle Cu, Cb_2\nabla\overline{u}\rangle + \langle Cu, CF\rangle. \end{aligned}$$

Integration by parts gives

$$\langle i\mathcal{L}Cu, Cu\rangle + \langle Cu, i\mathcal{L}Cu\rangle = 0.$$

Thus we can rewrite (1.20) as

$$(1.21) \qquad \begin{aligned} \frac{d}{dt}\langle Cu, Cu\rangle &= 2\Re\langle i[C\mathcal{L} - \mathcal{L}C]u, Cu\rangle + 2\Re\langle Cb_1\nabla u, Cu\rangle \\ &+ 2\Re\langle Cb_2\nabla\overline{u}, Cu\rangle + 2\Re\langle CF, Cu\rangle. \end{aligned}$$

Consider the first two terms in the right hand side of (1.21). Using the notation

$$q(\xi) = -(\xi_1^2 + .. + \xi_k^2) + (\xi_{k+1}^2 + .. + \xi_n^2), \text{ and } \tilde{\xi} = (\xi_1, .., \xi_k, -\xi_{k+1}, .., -\xi_n),$$

it follows that

$$\widehat{\mathcal{L}f}(\xi) = q(\xi)\hat{f}(\xi),$$

so that

$$C\mathcal{L}f(x) = \int_{\mathbb{R}^n} e^{ix\cdot\xi}c(x,\xi)q(\xi)\hat{f}(\xi)d\xi.$$

Moreover,

$$\mathcal{L}Cf(x) = \int_{\mathbb{R}^n} \mathcal{L}_x(e^{ix\cdot\xi}c(x,\xi))\hat{f}(\xi)d\xi,$$

and

$$\mathcal{L}_x(e^{ix\cdot\xi}c(x,\xi)) = q(\xi)e^{ix\cdot\xi}c(x,\xi) + 2i\tilde{\xi}\cdot\nabla_x c(x,\xi)e^{ix\cdot\xi} + e^{ix\cdot\xi}\mathcal{L}_x c(x,\xi).$$

Assuming that $(\mathcal{L}_x c(x,\xi))$ is L^2-bounded we have

$$i[C\mathcal{L}-\mathcal{L}C] = -2(\tilde{\xi}\cdot\nabla_x c)(x,D)+\Psi_1,$$

and

$$\sigma(Cb_1(x)\cdot\nabla_x) = ic(x,\xi)b_1(x)\cdot\xi+\Psi_2.$$

Our goal is to make $i[C\mathcal{L}-\mathcal{L}C]+Cb_1(x)\cdot\nabla_x$ as "small" as possible. Thus, we get the equation

$$2\tilde{\xi}\cdot\nabla_x c(x,\xi) = ic(x,\xi)b_1(x)\cdot\xi. \tag{1.22}$$

Defining

$$\phi(x,\xi) = log\ c(x,\xi),$$

it follows that

$$\tilde{\xi}\cdot\nabla_x\phi(x,\xi) = \frac{i}{2}b_1(x)\cdot\xi.$$

Hence

$$\phi(x,\xi) = \frac{i}{2}\int_0^\infty b_1(x+l\frac{\tilde{\xi}}{|\xi|})\cdot\frac{\tilde{\xi}}{|\xi|}dl.$$

We observe that

$$c(x,\xi) = exp((\phi(x,\xi)+\phi(x,-\xi)/2)$$

also solve (1.22).

It is not hard to see that

$$\left|\partial_x^\alpha\partial_\xi^\beta c(x,\xi)\right| \le c_{\alpha,\beta}\left(\frac{\langle x\rangle}{|\xi|}\right)^{|\beta|}, \quad |\alpha|+|\beta|\le N. \tag{1.23}$$

Thus, as it was mentioned above the symbol $c(x,\xi)$ is in the class studied by Craig-Kappeler-Strauss. In the nonelliptic case $\mathcal{L}$ is unknown whether or not it provides a L^2-bounded operator.

We need to truncate the symbol $c(x,\xi)$ in an appropriate manner. Let $\theta(\xi)$ be an even smooth function, $\theta(\xi) = 1$, $|\xi|\ge 2$, and $\theta(\xi)=0$, $|\xi|\le 1$, and $\psi\in C_0^\infty(\mathbb{R})$, $\psi(x)=1$, $|x|<1/2$, and $\psi(x)=0$, $|x|\ge 1$. For $R>1$ we define

$$c_R(x,\xi) = \theta\left(\frac{\xi}{R}\right)\psi\left(\frac{R\langle x\rangle}{\langle \xi\rangle}\right)c(x,\xi).$$

The symbol c_R is in the class $S^0_{0,0}(M)$, i.e. $c_R \in C^M(\mathbb{R}^{2n})$ with

$$|\partial_x^\alpha \partial_\xi^\beta c_R(x,\xi)| \le K,$$

for all multi-indices $\alpha,\beta \le M = M(N)$, depending on our assumptions on the data.

Next we need to verify that $C_R(x,D)$ satisfies the following properties.

(i) Its symbol is even in ξ. This is essential to bound the term $\langle Cb_2\nabla\overline{u}, Cu\rangle$. We use that $C^*C = \Gamma_1 + \Gamma_2$, with $\Gamma_j \in S^0_{0,0}(M)$, $j = 1,2$, Γ_2 "small", for R large, and $\Gamma_1(x,\xi)$ real, and even in ξ, thus $\overline{\Gamma_1 u} = \Gamma_1\overline{u}$.

$$\text{(ii)} \qquad i[C\mathcal{L} - \mathcal{L}C] + Cb_1\cdot\nabla_x = E_{1,R} + E_{2,R},$$

where $E_{1,R} \in S^0_{0,0}(M)$, and $E_{2,R}$ is an operator of "order 1" which satisfies

$$(1.24) \qquad |\int_0^T \langle E_{2,R}u, Cu\rangle dt| \le \frac{K}{R}|||J^{1/2}u|||_T^2 + KT\sup_{|t|\le T}\|u(t)\|_2^2,$$

(iii) for $R \ge R_0 = R_0(A,n)$, C_R is invertible in L^2, with inverse having norm bounded by K.

The fact that $E_{2,R}$ is an operator of "order 1" is due to the use of the class $c_R \in S^0_{0,0}(M)$. However, the bound in (1.24) suffices since the factor K/R can be made as small as we wish. This loss of one derivative is compensated by deducing the inhomogeneous version of the local smoothing effect in solutions of (1.13).

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