INFORMATIQUE THÉORIQUE ET APPLICATIONS

D. ARUN KUMAR
C. PANDU RANGAN

Approximation algorithms for the traveling salesman problem with range condition

Informatique théorique et applications, tome 34, nº 3 (2000), p. 173-181
<http://www.numdam.org/item?id=ITA_2000__34_3_173_0>

Theoretical Informatics and Applications
Theoret. Informatics Appl. **34** (2000) 173–181

APPROXIMATION ALGORITHMS FOR THE TRAVELING SALESMAN PROBLEM WITH RANGE CONDITION*

D. Arun Kumar[1,2] and C. Pandu Rangan[2]

Abstract. We prove that the Christofides algorithm gives a $\frac{4}{3}$ approximation ratio for the special case of traveling salesman problem (TSP) in which the maximum weight in the given graph is at most twice the minimum weight for the *odd degree restricted* graphs. A graph is *odd degree restricted* if the number of odd degree vertices in any minimum spanning tree of the given graph is less than $\frac{1}{4}$ times the number of vertices in the graph. We prove that the Christofides algorithm is more efficient (in terms of runtime) than the previous existing algorithms for this special case of the traveling salesman problem. Secondly, we apply the concept of stability of approximation to this special case of traveling salesman problem in order to partition the set of all instances of TSP into an infinite spectrum of classes according to their approximability.

AMS Subject Classification. 68W25, 05C85, 68W40.

1. Introduction

In the *Traveling Salesman Problem* ("TSP"), we are given n vertices and for each pair $\{i, j\}$ of distinct vertices a weight $w(i, j)$. We desire a closed path that visits each vertex exactly once and incurs a least weight (which is the sum of the weights along the path).

In the *metric TSP* the vertices lie in a metric space (*i.e.* the distances satisfy the triangle inequality). In the *Euclidean TSP* the vertices lie in $\mathbb{R}^2$ (or more generally in $\mathbb{R}^d$ for some d). Note that the Euclidean TSP is a subclass of the metric TSP. Unfortunately, even the Euclidean TSP is NP-hard [7, 11] and the metric TSP

* *The work on this paper has been supported by DFG grant Hr 14/5-1 and Deutscher Akademischer Austauschdienst (DAAD).*

[1] Lehrstuhl für Informatik I, RWTH Aachen, 52056 Aachen, Germany.

[2] Department of Computer-Science, Indian Institute of Technology, Madras 600036, India; e-mail: `rangan@iitm.ernet.in`

is APX-hard [12]. Christofides [5] designed an *approximation algorithm* that on every instance of the metric TSP computes a tour of cost at most 1.5 times the optimum. Improving this performance has been a major open problem for more than two decades.

Many special cases of the traveling salesman problem appear in the literature. In [12], the traveling salesman problem in which the distances are either one or two was considered. There, a polynomial-time approximation algorithm with worst-case approximation ratio of $\frac{7}{6}$ was presented. In [3], the authors considered the TSP with sharpened triangle inequality. The sharpened triangle inequality is defined as the following condition on the weights of the edges,

$$w(\{u,v\}) \leq \beta \cdot (w(\{u,x\}) + w(\{x,v\}))$$

for all vertices u, v, x and $\frac{1}{2} \leq \beta < 1$.

The authors presented different algorithms, where the approximation ratio lies between 1 and $\frac{3}{2}$ depending on β. If $\beta > 1$ then the resulting problem is called the TSP with relaxed triangle inequality which has been considered in [1, 2, 4]. It has been proved that the relaxed triangle inequality can be approximated in polynomial time with approximation ratio $\min\{4\beta, \frac{3}{2}\beta^2\}$.

In this paper we study an interesting case of the traveling salesman problem, in which the distances satisfy the following *range* condition. Throughout this paper let $w_{\max}$ and $w_{\min}$ denote the maximum and minimum weight respectively of a given complete graph G. Then the *range* condition is defined as

$$w_{\max} \leq 2 \cdot w_{\min}.$$

We denote by *range*-TSP, the TSP whose weights satisfy the *range* condition. The following observation shows that the *range*-TSP is a special case of metric TSP.

Observation 1.1. *Let G be a weighted complete graph. If G satisfies the range condition, then G satisfies the triangle inequality.*

Proof. Let $w_{\max}$, $w_{\min}$ denote the maximum and minimum weight of the edges in G. G satisfies the *range* condition, so,

$$w_{\max} \leq 2 \cdot w_{\min}.$$

Let x, y, z denote the weights of the sides of a triangle. Then,

$$x \leq w_{\max} \leq 2 \cdot w_{\min} \leq (y + z).$$

Thus the *range* condition implies the triangle inequality condition. □

We say that a graph is *odd degree restricted*, if the number of odd degree vertices in any minimum spanning tree of the graph is less than or equal to $\frac{1}{4}$ times the number of vertices in the given graph.

In this paper, we prove that the classical Christofides algorithm gives $\frac{4}{3}$-approximation for the *range*-TSP for *odd degree restricted* class of graphs. Note that Papadimitriou and Yannakakis [12] derived also a polynomial-time $\frac{4}{3}$-approximation algorithm for input instances of TSP taking values 1 and 2 and this algorithm works for *range* condition, too. But the complexity of this algorithm has not been exactly analyzed up till now and we know only that it is in $O(n^c)$ for some large c. On the other hand Christofides algorithm is efficient and so very practical.

We also study the $\varepsilon-$*extended range* condition where the weights satisfy the following condition

$$w_{\max} \leq (2+\varepsilon) \cdot w_{\min},$$

where $\varepsilon \geq 0$.

We denote by *range*-TSP$_\varepsilon$, the TSP whose weights satisfy the $\varepsilon-$*extended range* condition for any $\varepsilon \geq 0$.

We prove that the cycle cover algorithm presented in [3] gives a $\frac{4+\varepsilon}{3}$ approximation for the *range*-TSP$_\varepsilon$, *i.e.* the algorithm is stable according to the *range* condition (see [9] for the definition of stability of approximation).

This paper is organized as follows. In Section 2 we prove the upper bound on the approximation ratio of the Christofides algorithm for the *range*-TSP and in Section 3 the cycle cover approach for *range*-TSP$_\varepsilon$ and its stability is presented.

2. Christofides algorithm for the *range*-TSP

In this section we analyze the approximation ratio of the Christofides algorithm for the *range*-TSP. We prove that we can improve the $\frac{3}{2}$-approximation ratio to $\frac{4}{3}$ for the *odd degree restricted* class of graphs which satisfy the *range* condition.

First we present the classical Christofides algorithm.

Input: A complete graph $G = (V, E)$ with a *weight* function *weight*: $E \longrightarrow \mathbb{R}^{>0}$ satisfying the *range* condition and G is a *odd degree restricted* graph.

1. Construct a minimal spanning tree T of G and find a matching M with the minimal weight on the vertices of T with odd degree.
2. Construct a Eulerian tour D on $G' = T \cup M$.
3. Construct a Hamiltonian tour H from D by avoiding the repetition of vertices in the Eulerian tour.
4. Output H.

Theorem 2.1. *The Christofides algorithm is a $\frac{4}{3}$-approximation algorithm for the range-TSP for odd degree restricted graphs.*

Proof. Let H be the Hamiltonian tour computed by Christofides algorithm for an input $(G, weight)$. Let H_{OPT} be the optimal Hamiltonian tour for $(G, weight)$. Let for any set of edges $E' \subseteq E, w(E') = \sum_{e \in E'} w(e)$. Let $w_{\max}$, $w_{\min}$ denote the maximum and minimum weights in the given graph G. First we will prove that

$$w(H) \quad \leq \quad \frac{3}{2} \cdot w(H_{\mathrm{OPT}}) - \frac{n}{8} \cdot w_{\max} \tag{1}$$

where n is the number of vertices of G.

Let T be the minimum spanning tree of G produced in the first step of the Christofides algorithm. Let $v_1, v_2, v_3, \dots, v_k$ be the vertices of odd degree in T in the order as they appear in H_{OPT}.

Consider the sets $M_1 = \{\{v_1, v_2\}, \{v_3, v_4\}, \dots, \{v_{k-1}, v_k\}\}$ and $M_2 = \{\{v_2, v_3\}, \{v_4, v_5\}, \dots, \{v_k, v_1\}\}$. Obviously, M_1 and M_2 are matchings on the vertices $v_1, v_2, v_3, \dots, v_k$ of T.

Let $x_1, x_2, x_3, \dots, x_m$ denote the vertices of even degree in T in the order as they appear in H_{OPT}. Clearly, $k + m = n$. Denote, $V_{\mathrm{odd}} = \{v_1, v_2, \dots, v_k\}$ and $V_{\mathrm{even}} = \{x_1, x_2, \dots, x_m\}$.

Let A be the set of all edges in H_{OPT} with at least one end vertex an odd degree vertex in T, *i.e.*,

$$A = \{\{w_p, w_q\} \mid w_p \in V_{\mathrm{odd}} \text{ or } \mathrm{w_q} \in \mathrm{V_{odd}} \text{ and } \{w_p, w_q\} \in H_{\mathrm{OPT}}\}.$$

Obviously, A will have at most $2k$ edges.

We will show that

$$w(A) \quad \geq \quad w(M_1) + w(M_2). \tag{2}$$

Set, $A' = \{\{z_1, z_2\} \in A \mid z_1, z_2 \in V_{\mathrm{odd}}\}$ and $A'' = \{\{z_1, z_2\} \in A \mid z_1 \in V_{\mathrm{odd}}$ and $z_2 \in V_{\mathrm{even}}\}$.

We see that, $A' \cap A'' = \emptyset$ and $A = A' \cup A''$, *i.e.* (A', A'') is a partition of A.

Set $M' = \{\{z_1, z_2\} \mid \{z_1, z_2\} \in H_{\mathrm{OPT}}$ and $\{\mathrm{z_1}, \mathrm{z_2}\} \in \mathrm{M_1} \cup \mathrm{M_2}\}$ and $M'' = \{\{z_1, z_2\} \mid \{z_1, z_2\} \notin H_{\mathrm{OPT}}$ and $\{\mathrm{z_1}, \mathrm{z_2}\} \in \mathrm{M_1} \cup \mathrm{M_2}\}$.

Obviously, $M' \cap M'' = \emptyset$. and $M_1 \cup M_2 = M' \cup M''$.

It is clear that $A' = M'$. So, $w(A') = w(M')$.

It remains to prove $w(A'') \geq w(M'')$.

Let $J = \{i \in \{1, \dots, k\} \mid \{v_i, v_{i+1}\} \in M''\}$, where $v_{k+1} = v_1$. For all $i \in J$ there exist $\{x_{j_1}, \dots, x_{j_i + l_i}\} \in V_{\mathrm{even}}$, such that $v_i, x_{j_i}, \dots, x_{j_i + l_i}, v_{i+1}$ is a part of H_{OPT} and $\{v_i, x_{j_i}\}, \{x_{j_i + l_i}, v_{i+1}\} \in A''$.

Due to the *range* condition,

$$w(\{v_i, x_{j_i}\}) + w(\{x_{j_i + l_i}, v_{i+1}\}) \quad \geq \quad w(\{v_i, v_{i+1}\}).$$

Furthermore, we know, $\bigcup_{i \in J} \{\{v_i, v_{i+1}\}\} = M''$ and $\bigcup_{i \in J} \{\{v_i, x_{j_i}\}, \{x_{j_i + l_i}, v_{i+1}\}\} = A''$ and $\{\{v_p, x_{j_p}\}, \{x_{j_p + l_p}, v_{p+1}\}\} \cap \{\{v_q, x_{j_q}\}, \{x_{j_q + l_q}, v_{p+1}\}\} = \emptyset$ for $p, q \in J$, $p \neq q$.

It follows that, $w(A'') \geq w(M'')$ and thus we have,

$$w(A) = w(A') + w(A'') \geq w(M') + w(M'') \geq w(M_1) + w(M_2).$$

Thus we have proved the relation (2).

Let B be the set of edges which are complementary to the edges of A in H_{OPT}, *i.e.* $B = H_{\mathrm{OPT}} - A$. A has at most $2k$ edges. So B will have at least $n-2k$ edges. Therefore we can write,

$$w(B) \quad \geq \quad (n-2k)\cdot w_{\min}. \tag{3}$$

We know that $w(H_{\mathrm{OPT}}) = w(A) + w(B)$. Substituting for $w(A)$ and $w(B)$ from (2) and (3) we can write,

$$w(H_{\mathrm{OPT}}) \quad \geq \quad w(M_1) + w(M_2) + (n-2k)\cdot w_{\min}. \tag{4}$$

Let M be the minimum weight matching on the odd vertices in T. Obviously,

$$w(M) \quad \leq \quad \frac{1}{2}\cdot(w(M_1)+w(M_2)).$$

Using (4) we can write,

$$\begin{aligned} w(M) &\leq \frac{1}{2}\cdot(w(H_{\mathrm{OPT}}) - (n-2k)\cdot w_{\min}) \\ &\leq \frac{1}{2}\cdot w(H_{\mathrm{OPT}}) - \left(\left\lceil\frac{n}{2}\right\rceil - k\right)\cdot w_{\min}. \end{aligned} \tag{5}$$

T is the minimum weight spanning tree. We know $w(T) \leq w(H_{\mathrm{OPT}}) - w(e)$, where $e \in H_{\mathrm{OPT}}$. This implies,

$$w(T) \quad \leq \quad w(H_{\mathrm{OPT}}) - w_{\min}. \tag{6}$$

Now, by the algorithm $D = T \cup M$. So, $w(D) = w(T) + w(M)$.

Substituting for $w(T)$ and $w(M)$ from (5) and (6) we have,

$$\begin{aligned} w(D) &\leq \frac{3}{2}\cdot w(H_{\mathrm{OPT}}) - \left(\left\lceil\frac{n}{2}\right\rceil - k\right)\cdot w_{\min} - w_{\min}. \\ &\leq \frac{3}{2}\cdot w(H_{\mathrm{OPT}}) - \left\lceil\frac{n}{4}\right\rceil\cdot w_{\min} - \left(\left\lceil\frac{n}{4}\right\rceil - k\right)\cdot w_{\min}. \end{aligned}$$

Note that, $k \leq \lceil\frac{n}{4}\rceil$, therefore,

$$w(D) \quad \leq \quad \frac{3}{2}\cdot w(H_{\mathrm{OPT}}) - \left\lceil\frac{n}{4}\right\rceil\cdot w_{\min}. \tag{7}$$

Since the weights satisfy the triangle inequality condition, we have,

$$\begin{aligned} w(H) &\leq w(D) \leq \frac{3}{2}\cdot w(H_{\mathrm{OPT}}) - \left\lceil\frac{n}{4}\right\rceil\cdot w_{\min} \\ &\leq \frac{3}{2}\cdot w(H_{\mathrm{OPT}}) - \left\lceil\frac{n}{8}\right\rceil\cdot w_{\max}. \end{aligned}$$

Thus we have proved the relation (1).

Let $\Gamma = \{\gamma \geq 1 \mid w(H) \leq \gamma \cdot w(H_{\text{OPT}}) \leq n \cdot w_{\max}\}$.

Then, $w(H) \leq (\frac{3}{2}) \cdot w(H_{\text{OPT}}) - (\frac{\gamma}{8}) \cdot w(H_{\text{OPT}})$.

Thus,

$$w(H) \leq \min_{\gamma \in \Gamma} \min \left\{ \gamma \cdot w(H_{\text{OPT}}),\ \left(\frac{3}{2}\right) - \left(\frac{\gamma}{8}\right) \cdot w(H_{\text{OPT}}) \right\}.$$

The minimum occurs when, $\gamma = (\frac{3}{2}) - (\frac{\gamma}{8})$. This leads to $\gamma = \frac{4}{3}$.

Thus the algorithm has $\frac{4}{3}$-approximation. □

The following assertion claims that the Christofides algorithm is efficient. So, the Christofides algorithm is the most efficient approximation algorithm among known algorithms for the *range*-TSP for the given condition.

Theorem 2.2. *[8] The Christofides algorithm for the traveling salesman problem on n vertices, where weights obey the triangle inequality, can be implemented in* $O(n^{2.5}(\log n)^{1.5})$ *time and* $O(n^2)$ *space.*

3. Using the cycle cover algorithm for *range*-TSP_ε

In [3, 4, 9] the notion of stability of approximation was introduced and investigated. The idea of the concept of stability of approximation is similar to that of stability of numerical algorithms. But instead of observing the size of the change of the output value according to a small change of the input values, one looks for the size of the change of the approximation ratio according to a small change in the specification (some parameters, characteristics) of the input instances of the problem considered. If the change of the approximation ratio of an algorithm A is small for every small change of the considered input characteristic, then A is (approximation) stable with respect to this characteristic.

We omit the formal definitions of the stability of approximations here (one can consult [9] for it) and give a specific definition connected with TSP and the *range* property. Let $(G, weight)$ be an arbitrary input instance of the general TSP. We call the input instances of *range*-TSP **the kernel of the TSP**. We say that the $(G, weight)$ has a distance at most ε from the kernel, $\varepsilon \geq 0$, when the weight satisfies the $\varepsilon-$*extended range* condition,

$$w_{\max} \leq (2 + \varepsilon) \cdot w_{\min}$$

where $\varepsilon > 0$.

Due to this distance measure on the input instances of the TSP we get a partition of the class of all TSP input instances into an infinite spectrum of classes *range*-TSP_ε, where *range*-TSP_ε contains all input instances satisfying the $\varepsilon-$*extended range* condition. Let A be a d-approximation algorithm for the *range*-TSP (the kernel of the general TSP), that is consistent for the general TSP in the sense that it outputs a Hamiltonian tour for any input instance of TSP. We say that A is stable according to the *extended range* condition, if A is a $\delta_{d,\varepsilon}$-approximation algorithm for the *range*-TSP$_\varepsilon$ for every $\varepsilon \geq 0$, where $\delta_{d,\varepsilon}$ is a constant depending on d and ε only (*i.e.*, $\delta_{d,\varepsilon}$ is independent on the size of the input instances).

In this section we show that the **Cycle cover algorithm** presented in [3] is stable according to the *range* condition by showing that it provides a $\frac{4+\varepsilon}{3}$ approximation for the *range*-TSP$_\varepsilon$. First, we present the cycle cover algorithm.

Cycle cover algorithm

Input: A complete graph $G = (V, E)$ with weight function *weight*: $E \to \mathbb{R}^{>0}$ satisfying the *extended range* condition.

1. Construct a minimum cost cycle cover $C = \{C_1, C_2, C_3, \ldots, C_k\}$ of G, *i.e.* a covering of all vertices in G by cycles of length ≥ 3.
2. For $1 \leq i \leq k$, find the cheapest edge $\{a_i, b_i\}$ in every cycle C_i of C.
3. Obtain a Hamiltonian cycle H of G from C by replacing the edges $\{\{a_i, b_i\} \mid 1 \leq i \leq k\}$ by the edges $\{\{b_i, a_{i+1}\} \mid 1 \leq i \leq k-1\}\} \cup \{\{b_k, a_1\}\}$.

A minimum cycle cover can be found in polynomial time [6].

Theorem 3.1. *The* **Cycle Cover algorithm** *is a* $\frac{4+\varepsilon}{3}$*-approximation algorithm for the range-TSP*$_\varepsilon$*, for any* $\varepsilon \geq 0$.

Proof. Let $C = \{C_1, C_2, C_3, \ldots, C_k\}$ be the minimum cycle cover where $C_1, C_2, C_3, \ldots, C_k$ are cycles of length greater than or equal to 3. Let H_{OPT} denote the optimal Hamiltonian tour in G. The minimum cycle cover is a lower bound on the Hamiltonian tour. Hence $w(C) \leq w(H_{\mathrm{OPT}})$.

Let the cycles be,

$$C_i = x_{i,1}, x_{i,2}, \ldots, x_{i,r_i} \qquad \text{for } i \in \{1, \ldots, k\}.$$

Without loss of generality assume $\{x_{i,1}, x_{i,2}\}$, $i \in \{1, \ldots, k\}$ are the cheapest edges in the respective cycles. The algorithm removes these k edges and adds the following k edges, $\{x_{i,2}, x_{i+1,1}\}$ for $i \in \{1, \ldots, k-1\}$.

$\{x_{i,1}, x_{i,2}\}$ is the cheapest edge in the cycle C_i with the length r_i. So, $r_i \cdot w(\{x_{i,1}, x_{i,2}\}) \leq w(C_i)$, and we can write, $w(\{x_{i,1}, x_{i,2}\}) \leq \frac{1}{3} \cdot w(C_i)$ since $r_i \geq 3$.

Now, $w(\{x_{i,2}, x_{i+1,1}\}) \leq w_{\max}$.

By the *extended range* condition,

$$w(\{x_{i,2}, x_{i+1,1}\}) \leq (2+\varepsilon) \cdot w_{\min} \leq (2+\varepsilon) \cdot w(\{x_{i,2}, x_{i+1,1}\}).$$

From this we can conclude,

$$\begin{aligned} w(\{x_{i,2}, x_{i+1,1}\}) + w(\{x_{i,2}, x_{i,3}\}) + \dots, w(\{x_{i,r}, x_{i,i}\}) \\ &\leq ((2+\varepsilon) \cdot w(\{x_{i,2}, x_{i+1,1}\})) + w(\{x_{i,2}, x_{i,3}\}) + \dots w(\{x_{i,r}, x_{i,i}\}) \\ &\leq (1+\varepsilon) w(\{x_{i,2}, x_{i+1,1}\}) + w(C_i) \\ &\leq (1+\varepsilon)\frac{1}{3} \cdot [w(C_i)] + w(C_i) \leq \frac{4+\varepsilon}{3} \cdot w(C_i). \end{aligned}$$

Adding the k equations on the left hand side we have the output of the algorithm, H. So,

$$\begin{aligned} w(H) &\leq \left\lceil \frac{4+\varepsilon}{3} \right\rceil \cdot (w(C_1) + w(C_2) + w(C_3) \dots + w(C_k)) \\ &\leq \left\lceil \frac{4+\varepsilon}{3} \right\rceil \cdot (w(C)) \leq \left\lceil \frac{4+\varepsilon}{3} \right\rceil \cdot (H_{\mathrm{OPT}}). \end{aligned}$$

Thus we have a $[\frac{4+\varepsilon}{3}]$-approximation algorithm for this *range*-TSP_ε, for any $\varepsilon \geq 0$ using the cycle cover algorithm. □

We are very grateful to Juraj Hromkovič for his help. We thank R. Klasing for useful conversations. Thanks to J. Hromkovič and H.-J. Böckenhauer for pointing out various bugs/omissions in earlier drafts of this paper.

References

[1] T. Andreae and H.-J. Bandelt, Performance guarantees for approximation algorithms depending on parametrized triangle inequalities. *SIAM J. Discrete Math.* **8** (1995) 1-16.

[2] M.A. Bender and C. Chekuri, Performance guarantees for the TSP with a parametrized triangle inequality, in *Proc. WADS'99.* Springer, *Lecture Notes in Comput. Sci.* **1663** (1999) 80-85.

[3] H.-J. Böckenhauer, J. Hromkovič, R. Klasing, S. Seibert and W. Unger, An improved lower bound on the approximability of metric TSP and approximation algorithms for the TSP with sharpened triangle Inequality, in *Proc. STACS 2000.* Springer, *Lecture Notes in Comput. Sci.* (to appear).

[4] H.-J. Böckenhauer, J. Hromkovič, R. Klasing, S. Seibert and W. Unger, Towards the Notion of Stability of Approximation Algorithms and the Traveling Salesman Problem, in *Electronic Colloquium on Computational Complexity.* Report No. 31 (1999).

[5] N. Christofides, Worst-case analysis of a new heuristic for the traveling salesman problem. *Technical Report 388, Graduate School of Industrial Administration.* Carnegie-Mellon University, Pittsburgh (1976).

[6] J. Edmonds and E.L. Johnson, Matching: A well-solved class of integer linear programs, in *Proc. Calgary International conference on Combinatorial Structures and Their Applications.* Gordon and Breach (1970) 88-92.

[7] M.R. Garey, R.L. Graham and D.J. Johnson, Some NP-complete geometric problems, in *Proc. ACM Symposium on Theory of Computing* (1976) 10-22.

[8] H.N. Gabow and R.E. Tarjan, Faster scaling algorithms for general graph-matching problems. *J. ACM* **28** (1991) 815-853.

[9] J. Hromkovič, Stability of approximation algorithms for hard optimisation problems, in *Proc. SOFSEM'99.* Springer-Verlag, *Lecture Notes in Comput. Sci.* **1725** (1999) 29-46.

[10] J. Hromkovič, Stability of approximation algorithms and the knapsack problem, in *Jewels are forever*, edited by J. Karhumäki, H. Maurer and G. Rozenberg. Springer-Verlag (1999) 238-249.

[11] C.H. Papadimitriou, Euclidean TSP is NP-complete. *TCS* **4** (1977) 237-244.

[12] C.H. Papadimitriou and M. Yannakakis, The Traveling salesman problem with distances one and two. *Math. Oper. Res.* **18** (1993) 1-11.

Communicated by J. Hromkovič.
Received January 12, 2000. Accepted February 13, 2000.