RAIRO. INFORMATIQUE THÉORIQUE

GHEORGHE PĂUN

Some further remarks on the family of finite index matrix languages

RAIRO. Informatique théorique, tome 13, nº 3 (1979), p. 289-297
<http://www.numdam.org/item?id=ITA_1979__13_3_289_0>

R.A.I.R.O. Informatique théorique/Theoretical Informatics
(vol. 13, n° 3, 1979, p. 289 à 297)

SOME FURTHER REMARKS ON THE FAMILY OF FINITE INDEX MATRIX LANGUAGES (*)

by Gheorghe PĂUN (1)

Communicated by Wilfried BRAUER

Abstract. — *It is proved that the family of finite index matrix languages coincides with the families of finite index random context languages generated with forbidding sets and with or without λ-rules and is included in the family of finite index conditional languages. Finally, the Szilard languages associated to finite index matrix grammars are briefly investigated.*

Résumé. — *On montre que la famille de langages matriciels avec l'index fini est identique à la famille de langages engendrés par « random context » grammaires avec l'index fini et l'ensemble de symboles interdits et avec ou sans λ-règles. Alors on montre que ces familles sont contenues dans la famille de langages engendrés par les grammaires indépendantes de contexte avec restrictions régulières. Dans la dernière partie de l'article les langages de Szilard associés à une grammaire matricielle avec l'index fini sont aussi brièvement discutés.*

1. INTRODUCTION

In this paper we investigate the relation between the families of languages generated by (context-free) matrix grammars, by random context grammars, and by conditional grammars, each of finite index. Similar to other cases ([6, 7, 8]), the finiteness of the index is a very strong condition. For example, there is no difference between the families of finite index matrix languages generated with or without λ-rules, appearance checking, or by leftmost derivations only [7], although the corresponding families of languages with arbitrary index are very different [11].

For the families investigated here the situation is similar: the use of λ-rules does not change the generative capacity of finite index random context grammars with forbidding sets and the obtained family coincides with the family of finite index matrix languages. We then show that the family of 2-conditional

(*) Received May 1978, revision received September 1978.
(1) Institute of Mathematics, Bucuresti, Romania.

languages [5] of finite index includes the family of finite index matrix languages and is included in the family of 1-conditional ([2, 9]) of finite index.

Finally, the Szilard language associated to a finite index matrix grammar is briefly investigated.

2. THE INDEX OF GRAMMARS AND LANGUAGES

The formal language terminology used in what follows is that of [11]. We specify only some notations.

A context-free (shortly, c. f.) matrix grammar is a quadruple $G=(V_N, V_T, S, M)$, where V_N is the nonterminal vocabulary, V_T is the terminal vocabulary, $S \in V_N$ is the start symbol of the grammar and M is the set of matrix rules. We write a matrix rule as $m: (r_1, \ldots, r_n)$, $n \geqq 1$, where $r_i: A_i \to x_i$, $x_i \in V_G^*$, $A_i \in V_N$. ($V_G = V_N \cup V_T$ and V^* denotes the free monoid generated by V under the operation of concatenation with the null element λ). For $x, y \in V_G^*$ and $r: A \to z$ in some matrix of M we write $x \overset{r}{\Rightarrow} y$ iff $x = x_1 A x_2$, $y = x_1 z x_2$. For $x, y \in V_G^*$ we write $x \Rightarrow y$ iff $x = x_1 \overset{r_1}{\Rightarrow} x_2 \overset{r_2}{\Rightarrow} \ldots \overset{r_n}{\Rightarrow} x_{n+1} = y$ for some $m: (r_1, \ldots, r_n) \in M$ and $x_i \in V_G^*$. If $\overset{*}{\Rightarrow}$ is the reflexive transitive closure of $\Rightarrow$ then we define

$$L(G) = \{ x \in V_T^* \mid S \overset{*}{\Rightarrow} x \}.$$

Following [1] we define the index of a matrix grammar G (and, in a similar manner, of any type of regulated c. f. grammars) in the following way.

Let $D: S = x_1 \Rightarrow x_2 \Rightarrow \ldots \Rightarrow x_k = y \in V_T^*$ be a derivation in G and let $N(x_i)$ be the string which is obtained from x_i by deletion of the terminal symbols. We put

$$\operatorname{ind}(D, G) = \max_{1 \leqq i \leqq k} |N(x_i)|,$$

$$\operatorname{ind}(y, G) = \min_{D} \operatorname{ind}(D, G),$$

where D exhausts the set of derivations of y in the grammar G. ($|x|$ denotes the length of x). Then we define

$$\operatorname{ind}(G) = \sup_{y \in L(G)} \operatorname{ind}(y, G),$$

and, for a language L,

$$\operatorname{ind}(L) = \inf \{ \operatorname{ind}(G) \mid L = L(G) \}.$$

Generally, the family of finite index languages in a given family $\mathscr{L}$ is denoted by $\mathscr{L}_f$.

Let $\mathscr{L}_i$, $i=0, 1, 2, 3$, be the four families of languages in the Chomsky hierarchy, and $\mathscr{M}$, $\mathscr{M}^\lambda$, $\mathscr{M}_{ac}$, $\mathscr{M}^\lambda_{ac}$ be the families of languages generated by λ-free c. f. matrix grammars, by arbitrary c. f. matrix grammars, by λ-free c. f. matrix grammars operating in the appearance checking mode, and, respectively, by arbitrary c. f. matrix grammars operating in the appearance checking mode [11].

3. RANDOM CONTEXT GRAMMARS OF FINITE INDEX

DEFINITION: A random context c. f. grammar is a triple (G, p, f), where $G=(V_N, V_T, S, P)$ is a c. f. grammar and p, f are mappings of P into $\mathscr{P}(V_N)$. [$p(r)$ is called the permitting set and $f(r)$ is the forbidding set associated to r.] For x, $y \in V_G^*$ we write $x \Rightarrow y$ iff $x=x_1 A x_2$, $y=x_1 z x_2$, $A \to z$ is in P and every symbol in $p(A \to z)$ is in $x_1 x_2$, but no symbol in $f(A \to z)$ is in $x_1 x_2$. The language $L(G, p, f)$ is defined in the usual way.

We denote by $\mathscr{R}$, $\mathscr{R}^\lambda$, $\mathscr{R}_{ac}$, $\mathscr{R}^\lambda_{ac}$ the sets of languages generated by random context grammars with: 1) λ-free rules and $f(r)=\emptyset$ for all r; 2) arbitrary c. f. rules and $f(r)=\emptyset$ for all r; 3) λ-free rules and arbitrary $f(r)$, and, respectively, with 4) arbitrary c. f. rules and arbitrary $f(r)$.

The following relations are known [4]:

(i) $\mathscr{R} \subset \mathscr{R}^\lambda \subset \mathscr{R}^\lambda_{ac}$, $\mathscr{R} \subset \mathscr{R}_{ac} \subset \mathscr{R}^\lambda_{ac}$;

(ii) $\mathscr{R} \subset \mathscr{M}$, $\mathscr{R}^\lambda \subset \mathscr{M}^\lambda$, $\mathscr{R}_{ac} = \mathscr{M}_{ac}$, $\mathscr{R}^\lambda_{ac} = \mathscr{M}^\lambda_{ac}$.

It is an open problem whether or not the inclusions in (ii) are proper [4].

The proofs of lemmas 1, 2, 5 and 6 in [4] do not modify the finiteness of the index of the involved grammars hence we obtain $\mathscr{R}_f \subset \mathscr{M}_f$, $\mathscr{R}^\lambda_f \subset \mathscr{M}^\lambda_f$, $\mathscr{R}_{acf} \subset \mathscr{M}_{acf}$, $\mathscr{R}^\lambda_{acf} \subset \mathscr{M}^\lambda_{acf}$ and $\mathscr{M}^\lambda_{acf} \subset \mathscr{R}^\lambda_{acf}$. In [8] it was proved that $\mathscr{M}^\lambda_f = \mathscr{M}_{acf} = \mathscr{M}^\lambda_{acf}$. In the following lemma 1 we shall prove that $\mathscr{M}_f \subset \mathscr{R}_{ac,f}$, therefore we have:

THEOREM 1: $\mathscr{R}_f \subset \mathscr{R}^\lambda_f \subset \mathscr{R}_{acf} = \mathscr{R}^\lambda_{acf} = \mathscr{M}_f$.

We do not known whether the above inclusions are proper.

LEMMA 1: $\mathscr{M}_f \subset \mathscr{R}_{acf}$.

Proof: Let $L \in \mathscr{M}_f$, $L \subset V^*$. For $a \in V$ we define

$$\partial_a(L) = \{ x \in V^* \mid ax \in L \}.$$

According to [6], the family $\mathscr{M}_f$ is a full AFL, therefore $\partial_a(L) \in \mathscr{M}_f$ for any $a \in V$. On the other hand,

$$L = \bigcup_{a \in V} \{a\} \partial_a(L)$$

and $\mathscr{R}_{acf}$ is clearly closed under union. Thus it is sufficient to prove that for any $L' \in \mathscr{M}_f$, $L' \subset V^*$ and $a \in V$, the language $\{a\} L' \in \mathscr{R}_{acf}$.

Let $L' \in \mathscr{M}_f$ be generated by a λ-free c. f. matrix grammar $G = (V_N, V, S, M)$ of finite index. In view of theorem 6.8, p. 183 [11] and of its proof, we may assume the matrices in M to be of one of the following forms:

(a) $$(S \to XA),$$

(b) $$(X \to Y, A \to w),$$

(c) $$(X \to \lambda, A \to w), X, Y, A \in V_N, w \in V_G^* - \{\lambda\}.$$

Replacing each rule $X \to \lambda$ by $X \to a$ we obtain a λ-free c. f. matrix grammar for the language $\{a\} L'$ with matrices of the forms (a), (b) above and (c) $(X \to a, A \to w)$.

Let $G' = (V_N, V, S, M')$ be this grammar. We denote by $M_{(\alpha)}$ the set of matrices of the form (α), $\alpha \in \{a, b, c\}$. We construct the random context grammar (G'', p, f) with

$$G'' = (V'_N, V, S, P),$$

where $V'_N = V_N \cup \{R_i \mid i = 1, 2, \ldots, n\}$ (we assume $M_{(b)} \cup M_{(c)} = \{m_1, \ldots, m_n\}$) and P contains the rules in the matrices in $M_{(a)}$ with $p(r) = f(r) = \emptyset$ and the following groups of rules [the sets $p(r)$ and $f(r)$ are fiven too]:

1) for every matrix m_i: $(X \to Y, A \to w)$ in $M_{(b)}$ with $w = xz$, $x \in V_G^*$, $z \in V_G$, we introduce in P the rules

$$\begin{array}{lll} A \to x R_i, & p(r) = \{X\}, & f(r) = \{R_1, \ldots, R_n\}, \\ X \to Y, & p(r) = \{R_i\}, & f(r) = \emptyset, \\ R_i \to z, & p(r) = \{Y\}, & f(r) = \emptyset. \end{array}$$

2) for every matrix m_i: $(X \to a, A \to w)$ in $M_{(c)}$ with $w = xz$, $x \in V_G^*$, $z \in V_G$, we introduce the rules

$$\begin{array}{lll} A \to x R_i, & p(r) = \{X\}, & f(r) = \{R_1, \ldots, R_n\}, \\ X \to a, & p(r) = \{R_i\}, & f(r) = \emptyset, \\ R_i \to z, & p(r) = \emptyset, & f(r) = V_N. \end{array}$$

Let $r(m_i)$ be the set of rules associated to $m_i \in M_{(b)} \cup M_{(c)}$ as above.

Clearly, if $x \Rightarrow y$ in G' by using a matrix $m \in M_{(b)} \cup M_{(c)}$, then $x \Rightarrow y$ in the grammar G'' by using the three rules in $r(m)$. Therefore, $L(G') \subset L(G'')$.

Conversely, let us observe that any derivation according to the grammar G'' begins by an application of a rule which occurs in a matrix in $M_{(a)}$, continues by using some groups of rules $r(m)$ for $m \in M_{(b)}$ and ends by using a group of rules $r(m)$ associated to a matrix $m \in M_{(c)}$. Indeed, a string $y \in (V_N \cup V)^* - \{S\}$ can be derived in G'' only using rules of the form $A \to x R_i$. After introducing a symbol R_i the only possibility to go further is to eliminate this symbol *via* the second and the third rule in the corresponding group $r(m)$. But, if $r(m) = \{r_1, r_2, r_3\}$ and $x \overset{r_1}{\Rightarrow} x_1 \overset{r_2}{\Rightarrow} x_2 \overset{r_3}{\Rightarrow} y$ in the grammar G'', then clearly, $x \Rightarrow y$ in G' by using the matrix m. Consequently, $L(G'') \subset L(G')$ and the proof is complete.

4. CONDITIONAL GRAMMARS OF FINITE INDEX

DEFINITION [2, 9]: An 1-conditional grammar is a pair (G, ρ) where $G = (V_N, V_T, S, P)$ is a c. f. grammar and ρ is a mapping of P into the family of regular languages over V_G. For $x, y \in V_G^*$ we write $x \Rightarrow y$ iff $x = x_1 A x_2$, $y = x_1 z x_2$ for some rule $A \to z \in P$ such that $x \in \rho\ (A \to z)$.

Let $(\mathscr{H}_1)$ $\mathscr{H}_1^\lambda$ be the family of languages generated by (λ-free) 1-conditional grammars.

DEFINITION [5]: A 2-conditional grammar is a triple (G, ρ_1, ρ_2) where G is a c. f. grammar and ρ_1, ρ_2 are mappings of P into the family of regular languages over V_G. For $x, y \in V_G^*$ we write $x \Rightarrow y$ iff $x = x_1\ A\ x_2$, $y = x_1\ z x_2$ for some $A \to z \in P$ such that $x_1 \in \rho_1 (A \to z)$ and $x_2 \in \rho_2\ (A \to z)$.

Let $(\mathscr{H}_2)$ $\mathscr{H}_2^\lambda$ be the family of languages generated by (λ-free) 2-conditional grammars.

In [9] it is proved that

$$\mathscr{H}_1^\lambda = \mathscr{H}_2^\lambda = \mathscr{L}_0, \mathscr{H}_1 = \mathscr{H}_2 = \mathscr{L}_1.$$

For finite index families the situation is different.

THEOREM 2: $\mathscr{M}_f \subset \mathscr{H}_{2f}$.

Proof: As $\mathscr{M}_f = \mathscr{R}_{ac\,f}$, it follows that it is sufficient to prove that $\mathscr{R}_{ac\,f} \subset \mathscr{H}_{2f}$. Let (G, p, f) be a random context grammar with $G = (V_N, V_T, S, P)$, ind $(G, p, f) = k$. We construct the 2-conditional grammar (G', ρ_1, ρ_2) with $G' = (V'_N, V_T, S, P')$ in the following way.

For any rule r: $A \to w$ in P we define

$$M(r) = \{(x_1, x_2) \mid x_1, x_2 \in V_N^*, |x_1 x_2| \leqq k,$$
$$\text{all the symbols in } p(r) \text{ occur in } x_1\ x_2$$
$$\text{but no symbol in } f(r) \text{ occurs in } x_1\ x_2\}.$$

The set $M(r)$ is finite for any r. Let us label the pairs in $M(r)$ by q_i^r, $i=1, 2, \ldots$, card $M(r)$. For each pair q_i^r we consider a new symbol Y_i^r. Let V_N'' be the set of all these symbols. Then we take $V_N' = V_N \cup V_N''$.

For each rule r: $A \to w$ in P we introduce in P' the following rules

$$A \to Y_i^r, \qquad i=1, 2, \ldots, \text{card } M(r),$$
$$Y_i^r \to w, \qquad i=1, 2, \ldots, \text{card } M(r).$$

For an arbitrary vocabulary V and for $x, y \in V^*$ we define [3]:

$\text{Shuff}(x, y) = \{ x_1 y_1 \ldots x_p y_p \mid p \geqq 1, x_i, y_i \in V^*, \ x = x_1 \ldots x_p, y = y_1 \ldots y_p \}$, and for $L_1, L_2 \subset V^*$ we put

$$\text{Shuff}(L_1, L_2) = \bigcup_{\substack{x \in L_1 \\ y \in L_2}} \text{Shuff}(x, y).$$

Let us now define

$$\rho_1(A \to Y_i^r) = \text{Shuff}(\{ x_1 \}, V_T^*) \qquad \text{for} \quad q_i^r = \text{label of } (x_1, x_2),$$

$$\rho_2(A \to Y_i^r) = \text{Shuff}(\{ x_2 \}, V_T^*) \qquad \text{for} \quad q_i^r = \text{label of } (x_1, x_2),$$

$$\rho_1(Y_i^r \to w) = \rho_2(Y_i^r \to w) = (V_T \cup V_N)^* \qquad \text{for all } r \text{ and } i.$$

Let D be a terminal derivation in the grammar (G, p, f) with $\text{ind}(D, G, p, f) \leqq k$ and let $x \Rightarrow y$ be a direct derivation in D which uses a rule r: $A \to w$. Then $x = x_1 A x_2$ and $(N(x_1), N(x_2)) \in M(r)$. Therefore, $x_1 \in \rho_1(A \to Y_i^r)$ and $x_2 \in \rho_2(A \to Y_i^r)$ for some i hence the derivation $x_1 A x_2 \Rightarrow x_1 Y_i^r x_2$ is a correct derivation according to the grammar (G', ρ_1, ρ_2). The symbol Y_i^r will be replaced by w therefore in this way the derivation $x \Rightarrow y$ is simulated in (G', ρ_1, ρ_2). Thus $L(G, p, f) \subset L(G', \rho_1, \rho_2)$.

Conversely, each derivation $x_1 A x_2 \Rightarrow x_1 Y_i^r x_2$ in the grammar (G', ρ_1, ρ_2) implies $x_1 \in \text{Shuff}(N(x_1), V_T^*)$, $x_2 \in \text{Shuff}(N(x_2), V_T^*)$ and $(N(x_1), N(x_2)) \in M(r)$. Therefore $x_1 x_2$ contains all the symbols in $p(r)$ and no symbol in $f(r)$ occurs in $x_1 x_2$. But, each derivation $x_1 A x_2 \Rightarrow x_1 Y_i^r x_2$ in (G', ρ_1, ρ_2) must be followed by a derivation $x_1' Y_i^r x_2' \Rightarrow x_1' w x_2'$ providing that r: $A \to w$. Therefore, the derivations $x_1 A x_2 \Rightarrow x_1 Y_i^r x_2$, $x_1' Y_i^r x_2' \Rightarrow x_1' w x_2'$ correspond to a derivation $x_1 A x_2 \Rightarrow x_1 w x_2$ in the grammar (G, p, f) which uses the rule r: $A \to w$. Thus, the inclusion $L(G', \rho_1, \rho_2) \subset L(G, p, f)$ follows and we have

$$L(G, p, f) = L(G', \rho_1, \rho_2).$$

Clearly, $\text{ind}(G, p, f) = \text{ind}(G', \rho_1, \rho_2)$. As $\mathscr{L}_3$ is closed under the operation Shuff [3], it follows that $L(G', \rho_1, \rho_2) \in \mathscr{H}_{2f}$ and the theorem is proved.

From the proofs in [9] no relation follows between the families $\mathscr{H}_{1f}$ and $\mathscr{H}_{2f}$ with or without the superscript λ. The problem is investigated in what follows.

THEOREM 3: $\mathscr{H}_{2f} \subset \mathscr{H}_{1f}$, $\mathscr{H}^{\lambda}_{2f} \subset \mathscr{H}^{\lambda}_{1f}$.

Proof: Let (G, ρ_1, ρ_2) be a 2-conditional grammar with $G=(V_N, V_T, S, P)$, and let $\operatorname{ind}(G, \rho_1, \rho_2)=k$. Without loss of the generality we can assume that S does not occur in the right-hand side of any rule. For any $A \in V_N - \{S\}$ we consider the new symbols $A, A_1, A_2, \ldots, A_{k-1}$. Let s be the finite substitution defined by $s(A)=\{A, A_1, \ldots, A_{k-1}\}$, $A \in V_N - \{S\}$, $s(a)=\{a\}$ for $a \in V_T \cup \{S\}$. We consider the grammar $G'=(V'_N, V_T, S, P')$ with

$$V'_N=\{A, A_1, \ldots, A_{k-1} \mid A \in V_N - \{S\}\} \cup \{S\},$$

$$P'=\bigcup_{r \in P} s(r),$$

where

$$s(r)=\{B \rightarrow x \mid B \in s(A),\ x \in s(z) \text{ for } r\colon A \rightarrow z\}.$$

Let $r'\colon X \rightarrow z$ be in P', $r' \in s(r)$ for some rule $r\colon A \rightarrow x$ in P. We define

$$\rho(r')=\{w \in s(\rho_1(r))\, X\, s(\rho_2(r)) \mid w=y_1 X y_2,\ y_1 z y_2$$

$$\text{does not contain two occurrences of any nonterminal symbol}\}.$$

Let $D\colon S \overset{*}{\Rightarrow} x$ be a terminal derivation according to (G, ρ_1, ρ_2). Replacing each symbol $A \in V_N$ by a suitable symbol in $s(A)$ we can obtain a derivation $D'\colon S \overset{*}{\Rightarrow} x$ such that each sentential form in D' contains distinct nonterminal symbols. This derivation is correct according to (G', ρ): the rules used in D' belong to P' and for each direct derivation $x_1 X x_2 \overset{r'}{\Rightarrow} x_1 w x_2$ in D' we have $x_1 X x_2 \in \rho(r')$ since $x_1 \in \rho_1(r)$, $x_2 \in \rho_2(r)$, $r' \in s(r)$. Thus, we obtain $L(G, \rho_1, \rho_2) \subset L(G', \rho)$.

Conversely, any sentential form derivable in (G', ρ) does not contain two occurrences of the same nonterminal. Thus, if $x_1 X x_2 \Rightarrow x_1 z x_2$ by a rule r': $X \rightarrow z$ in P', $r' \in s(r)$ for $r \in P$, then there is only one occurrence of X in the string $x_1 X x_2$. Therefore, we can say that $x_1 \in s(\rho_1(r))$, $x_2 \in s(\rho_2(r))$, hence, replacing each symbol in $s(A)$ by A, $A \in V_N$, we obtain a derivation in the grammar (G, ρ_1, ρ_2) for the same terminal string. Consequently, $L(G', \rho) \subset L(G, \rho_1, \rho_2)$ hence the two grammars are equivalent.

Clearly, $\operatorname{ind}(G', \rho)=k$. As $\rho(r')$ is regular for any $r' \in P'$ and G' is λ-free if G is, it follows that $L(G, \rho_1, \rho_2) \in \mathscr{H}_{1f}$ [respectively, $L(G, \rho_1, \rho_2) \in \mathscr{H}^{\lambda}_{1f}$] and the theorem is proved.

We do not know whether the inclusions in theorem 3 are proper.

REMARK: We have $\mathscr{K}_{1f}^{\lambda} \subset \mathscr{L}_1$. This inclusion can be proved by a natural construction of a type-0 grammar which can simulate the derivation in an 1-conditional grammar (G, ρ) by checking for each rule whether or not the sentential form to be rewritten can be recognized by the finite automaton associated to the rule by ρ. Such a grammar has for any string x a workspace of at most $|x| + \mathrm{ind}(G, \rho) + k$ where k is the number of certain possibly necessary markers used in the derivations. From the workspace theorem [11] it follows that the obtained language is in $\mathscr{L}_1$.

5. FINAL REMARKS

As was shown in [6, 8], the family $\mathscr{M}_f$ has many (closure, decidability, etc.) properties which do not hold or are not known for $\mathscr{M}$. A further property concerning the Szilard language is considered here.

Let $G=(V_N, V_T, S, M)$ be a matrix grammar and let Lab (M) be a finite set of labels for the matrices in M. We denote by $SZ(G)$ the set of all strings in Lab $(M)^*$ describing terminal derivations according to G. We also denote by $\mathscr{C}_3$ the family of languages which contain an infinite regular sublanguage. We have

THEOREM 4: *For any finite index matrix grammar generating an infinite language we have* $SZ(G) \in \mathscr{M}_f \cap \mathscr{C}_3$.

Proof: Theorem 2 in [10] shows that $SZ(G) \in \mathscr{M}$ for any c. f. matrix grammar G. The proof in [10] remains valid for the finite index case hence $SZ(G) \in \mathscr{M}_f$ if G is a finite index matrix grammar.

An infinite regular language in $SZ(G)$, for a finite index matrix grammar G, can be obtained in the following way. We construct

$$V'_N = \{[\alpha] \mid \alpha \in V_N^*, 1 \leqq |\alpha| \leqq k\}, \qquad k = \mathrm{ind}\,(G),$$

$$\begin{aligned} P = \{ [\alpha] \to m[\beta] \mid [\alpha], [\beta] \in V'_N, \alpha \Rightarrow x \\ \text{by the matrix labelled by } m \text{ and } \beta = N(x) \} \\ \cup \{ [\alpha] \to m \mid \alpha \Rightarrow x \text{ by the matrix labelled} \\ \text{by } m \text{ and } N(x) = \lambda \}. \end{aligned}$$

The grammar $G' = (V'_N, \mathrm{Lab}(M), S, P')$ generates an infinite regular sublanguage of $SZ(G)$.

The assertion in theorem 4 does not hold for arbitrary matrix grammars G, For example, the grammar

$$G=(\{A_1, A_2, A_3, S, B, C, D\}, \{a\}, S, \{m_1:(S \to A_1 B),$$
$$m_2: (A_1 \to A_1, B \to CBD), m_3: (A_1 \to A_2),$$
$$m_4: (A_2 \to A_2, C \to a), m_5: (A_2 \to A_3),$$
$$m_6: (A_3 \to A_3, D \to a), m_7: (A_3 \to a, B \to a)\}),$$

has

$$L(G)=\{a^{2n} \mid n \geqq 1\}, SZ(G)=\{m_1 m_2^n m_3 m_4^n m_5 m_6^n m_7 \mid n \geqq 0\}$$

and ind $(G)=\infty$. Obviously, $SZ(G)$ does not contain infinite regular languages (it does not contain even infinite c. f. languages).

Moreover, if we add to G the matrix $m_8: (S \to S)$, then we obtain a grammar G' with ind $(G')=\infty$ but $SZ(G')=\{m_8^p x \mid p \geqq 0, x \in SZ(G)\}$ which contains infinite regular sublanguages. Therefore, the converse of theorem 4 is not true.

ACKNOWLEDGEMENTS

We are very indebted to Professor W. Brauer for many valuable remarks about an earlier version of this paper.

REFERENCES

1. B. Brainerd, *An Analog of a Theorem About Context-Free Languages*, Inform. Control., Vol. 11, 1968, pp. 561-568.
2. J. Friš, *Grammars with Partial Ordering of Rules*, Inform. Control., Vol. 12, 1968, pp. 415-425.
3. S. Ginsburg and S. A. Greibach, *Abstract Families of Languages* in *Studies in Abstract Families of Languages*, Mem. Amer. Math. Soc., Vol. 87, 1969.
4. O. Mayer, *Some Restrictive Devices for Context-Free Grammars*, Inform. Control., Vol. 20, 1972, pp. 69-92.
5. E. Navratil, *Context-Free Grammars with Regular Conditions*, Kybernetika, Vol. 2, 1970, pp. 118-126.
6. Gh. Păun, *On the Index of Grammars and Languages*, Inform. Control., Vol. 35, 1977, pp. 259-266.
7. Gh. Păun, *On the Generative Capacity of Simple Matrix Grammars of Finite Index*, Inform. Processing Letters, Vol. 7, No. 2, 1978, pp. 100-102.
8. Gh. Păun, *On the Family of Matrix Languages of Finite Index*, to appear in J. Comput. System Sc.
9. Gh. Păun, *On the Generative Capacity of Conditional Grammars*, to appear in Inform. Control.
10. Gh. Păun, *On Szilard's Languages Associated to a Matrix Grammar*, to appear in Inform. Processing Letters.
11. A. Salomaa, *Formal Languages*, Academic Press, New York and London, 1973.
12. A. P. J. van der Walt, *Random Context Grammars*, Symp. on Formal Languages, Oberwolfach, Germany, 1970.