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*RAIRO. Informatique théorique*, tome 12, n° 3 (1978), p. 201-202

[http://www.numdam.org/item?id=ITA\\_1978\\_\\_12\\_3\\_201\\_0](http://www.numdam.org/item?id=ITA_1978__12_3_201_0)

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## ON LANGUAGES SATISFYING OGDEN'S LEMMA (\*)

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Communicated by M. NIVAT

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Abstract. — *We show that various types of non-context-free languages satisfying Ogden's lemma can very easily be constructed. A simple answer is then given to a question of [6] and [7].*

Define an "Ogden-like" language as a language  $L$  satisfying Ogden's lemma [3, 4]:

DEFINITION: A language  $L$  is *Ogden-like* if there exists an integer  $k_0$  such that if in any word  $f$  of  $L$  any  $k_0$  or more positions (= occurrences of letters) are marked,  $f$  has a factorization  $f = aubvc$  satisfying:

- (1)  $au^nbv^nc \in L, \forall n \geq 0$ ;
- (2) either each of  $a, u$  and  $b$ , or each of  $b, v$  and  $c$  contains a marked position;
- (3)  $ubv$  contains at most  $k_0$  such positions.

Ogden's lemma, which is stronger than the classical Bar-Hillel's lemma [2, 4, 5], ensures that any context-free language is Ogden-like. The aim of this short note is to prove, as an answer to the third question of [6], the following:

PROPOSITION: *There exists properly context-sensitive, properly recursive, properly recursively enumerable and non-recursively enumerable languages which are Ogden-like.*

By properly context-sensitive, we mean non context-free context sensitive, and analogously for properly recursive and properly recursively enumerable.

*Proof:* The proof is very simple. Consider first a subset  $P$  of  $\mathbb{N}$ , and define:

$$A_P = \{ (ab)^n \mid n \in P \},$$
$$B_P = A_P \cup X^* \{ aa, bb \} X^*,$$

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(\*) Received March 1978.

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both being languages over  $X = \{a, b\}$ .

Obviously,  $B_P$  is context-free if  $A_P$  is. Moreover,  $B_P$  is properly context-sensitive (resp. recursive, recursively enumerable) if  $A_P$  is and  $B_P$  is not recursively enumerable if  $A_P$  is not.

It is obvious too that  $B_P$  is Ogden-like with  $k_0 = 4$ .

The proposition is then proved by choosing  $A_P$  properly context-sensitive (resp. recursive, resp. recursively enumerable) or not recursively enumerable. (It is well known that such languages  $A_P$  do exist, see [4] or [5] for instance.)  $\equiv$

REMARK 1: The family of Ogden-like languages is closed under union, product, star and homomorphism. It is not closed under inverse homomorphism, and under intersection with regular sets.

REMARK 2: The family of Ogden-like languages (belonging as usual to some countably infinite alphabet) has cardinality  $C$  (=continuum). The proof is exactly the same as the proof of the corollary of theorem 3 in [6].

REMARK 3: The languages  $B_P$  are built exactly on the same principle as the non-regular language of [1] satisfying the pumping lemma of regular sets. Moreover,  $B_P$  does satisfy this lemma. It can even be observed that  $B_P$  is never properly context-free. However, it is easily shown that the proof of the proposition could be done with

$$A'_P = \{ a^n b^n \mid n \in P \},$$

$$B'_P = \{ a^n b^m \mid n \neq m \} \cup A'_P,$$

which gives other examples of Ogden-like languages in the various classical families.

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