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(Y. AMICE, G. CHRISTOL, P. ROBBA)
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ON THE CLASS GROUP OF AFFINOID SPACES

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The field k is supposed to be complete with respect to a non-archimedean valuation and to be stable (i. e. $|k^*| = \sqrt{|k^*|}$, and for every finite field extension ℓ of k one has $[\ell : k] = [\bar{\ell} : \bar{k}]$). By $\overset{\circ}{k}$ and $\bar{k}$, we denote the valuation ring and the residue field of k .

The affinoid k-algebra A is supposed to be reduced and to be provided with its spectral norm $\|.\|$. Furthermore we assume that $\|.\|$ takes its value in $|k^*|$. By $\bar{A}$, we denote the affine $\bar{k}$-algebra $\overset{\circ}{A} \otimes \bar{k}$, where $\overset{\circ}{A} := \{x \in A ; \|x\| \leq 1\}$. The group of isomorphism classes of projective, rank 1 , A-modules is called the class group $Cl(A)$ of A .

THEOREM.

(i) <u>There is a natural injective map</u>

$$\alpha : Cl(\bar{A}) \hookrightarrow Cl(A) .$$

(ii) <u>If</u> $\bar{A}$ <u>is regular,</u> α <u>is also surjective.</u>

<u>Definition of</u> α .

(a) $\beta : Cl(\overset{\circ}{A}) \longrightarrow Cl(\bar{A})$

$$\beta([M]) := [M \otimes_{\overset{\circ}{A}} \bar{A}]$$

It is easy to show that β is a bijective homomorphism.

(b) $\gamma : Cl(\overset{\circ}{A}) \longrightarrow Cl(A)$

$$\gamma([M]) := [M \otimes_{\overset{\circ}{A}} A] .$$

The homomorphism γ is injective, and we define

$$\alpha := \gamma \circ \beta^{-1} .$$

Several examples show that in general α is not surjective.

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These examples are constructed geometrically : every 1-dimensional, normal, connected affinoid space $Y = Sp(A)$ is an affinoid subspace of a non-singular complete curve. We consider special affinoid subspaces of Tak-curves, of an elliptic curve with good reduction, of a Mumford-curve of genus 2 with stable reduction

We also consider the product of two of these subspaces with the A-dimensional unit disk.

A detailed paper "Über die Picardgruppen affinoider Algebren", by M. van der PUT and E. HEINRICH is to appear.