CAHIERS DE
TOPOLOGIE ET GÉOMÉTRIE DIFFÉRENTIELLE
CATÉGORIQUES

AFEWORK SOLOMON

A category model proof of the cogluing theorem

Cahiers de topologie et géométrie différentielle catégoriques, tome 48, nº 3 (2007), p. 200-219

<http://www.numdam.org/item?id=CTGDC_2007__48_3_200_0>

CAHIERS DE TOPOLOGIE ET GEOMETRIE DIFFERENTIELLE CATEGORIQUES
Volume XLVIII-3 (2007)

A CATEGORY MODEL PROOF OF THE COGLUING THEOREM

by Afework SOLOMON

Résumé. Cet article présente une preuve, basée sur les catégories modèles, d'un théorème de recollement qui généralise la preuve donnée par Brown et Heath pour la catégorie des espaces topologiques et des applications continues. Le but de l'article est de donner des conditions pour qu'une application entre produits fibrés soit une équivalence faible dans une catégorie modèle.

1 Introduction

An important problem in homotopy theory is to determine when two spaces are of the same homotopy type. This is often a difficult problem, and sometimes it may be easier to prove that two spaces X and Y are not of the same homotopy type by distinguishing their homotopy type invariants. This is accomplished, for example, by using the fundamental group or, using other invariants such as homology groups. However, to prove that X and Y are of the same homotopy type, we must construct a homotopy equivalence from X into Y. To do this, we must know something about the spaces X and Y. If the spaces X and Y happen to be expressed as pullbacks, then the cogluing theorem gives conditions under which the maps on the respective components imply that the induced map between the pullbacks is a weak equivalence. This result was originally proved by *Brown* and *Heath* in [3, *Theorem* 1.2] in the category of topological spaces and continuous maps.

The objective of this paper is to give a proof of the cogluing theorem in a model category $\mathcal{M}$ in the sense of Quillen [7, page 1-1]. Consider the following diagram in a model category $\mathcal{M}$

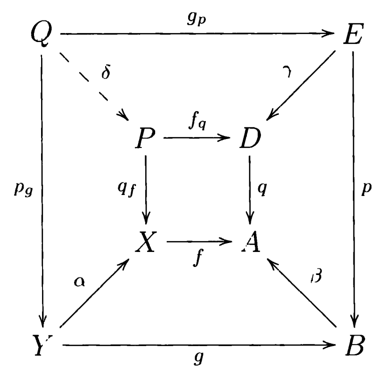

fig. 1.1

We think of the pullbacks $QEYB$ and $PDXA$ as the front and back faces of the diagram respectively. The cogluing theorem is the following: (refer to $fig.$ 1.1)

Theorem 1.1 *Suppose that p and q are fibrations and α, β, γ are weak equivalences, where A,X,B, and Y are fibrant, then δ is a weak equivalence.*

Our model category proof is preceded by the following lemma which says, roughly, that a pullback of a weak equivalence along a fibration is a weak equivalence. This is also referred to as right proper in the literature.

Lemma 1.1 *Let $AXBY$ be a pullback square as shown below :*

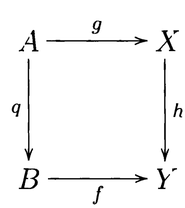

fig. 1.2

Suppose h is a fibration, f is a weak equivalence. B and Y are fibrant. Then, g is a weak equivalence.

In the ordinary category of topological spaces where the weak equivalences are homotopy equivalences, the proof by *Brown* and *Heath* in [3, Corollary 1.4] in fact gives Lemma 1.1 as a consequence of Theorem 1.1. The proof in *Brown* and *Heath* [3,Corollary 1.4] easily generalizes to the situation here. Thus, in fact we show that Theorem 1.1 and Lemma 1.1 are equivalent.
Other versions of the cogluing theorem exist in the literature. Baues[1, Lemma 1.2(b)] proves a gluing theorem(i.e. dual to Theorem 1.1 of this paper) in a category of cofibrant objects by incorporating left properness(which is dual to Lemma 1.1 of this paper) into his axioms.Thus, the approach taken in this paper is in sharp contrast to that of Baues. Kamps and Porter discuss the gluing theorem[6,Proposition 2.29, Theorem 2.27] in the category of cofibrant objects and their method of proof depends on the well known "Factorization Lemma" of Kenneth Brown[2,Factorization Lemma, p.421] originally proved for a category of fibrant objects. A similar approach is taken by Paul Goeress and John F.Jardine[5,Lemma 8.4, Lemma 8.8]. Hence the proof presented in this paper is direct in the sense that it only utilizes Quillen's axioms of a model category and some elementary consequences of the axioms along with the universal property of pullbacks to factor maps. In addition, the proof presented in this paper doesn't depend on K.Brown's Factorization Lemma as is the case with the other versions existing in the literature. Finally, we remark that the requirement of fibrancy on the spaces B and Y of Lemma 1.1 of this paper cannot be omitted. Examples of the failure of properness are discussed in[8, 2.10]. The same holds in Theorem 1.1 where it is required that the objects A, X, B, Y are fibrant. Nonetheless, the presentation of this paper requires the fibrancy condition on some of the spaces involved in the statements of the Lemma and the Theorem. That is, this paper doesn't require that all spaces be fibrant or dually cofibrant.
The paper is divided into three main parts. The first part, section 2, sets forth the model category definitions and theorems pertinent to the proofs of the Lemma and the Cogluing Theorem. Section 3 constitutes the second part. Here we prove Lemma 1.1. Finally, in the third part,

section 4, we give a proof of our main theorem the Cogluing Theorem and conclude the paper by demonstrating the equivalence of Theorem 1.1 and Lemma 1.1.
The author wishes to thank Peter Booth and Phil Heath for their useful suggestions in the presentation and organization of this paper as well as continuous encouragement during this work. [fig.3.21 is due to Peter Booth].

2 Preliminaries

The definition below is originally due to Quillen [7, p 1.1].

Definition 2.1 *A Model Category is a category $\mathcal{M}$ with three distinguished classes of maps called weak equivalences, denoted by $\sim$, fibrations, and cofibrations each of which is closed under composition and contains all identity maps.*
A map which is a fibration and and a weak equivalence is called an acyclic fibration. A map which is a cofibration and a weak equivalence is called an acyclic cofibration. We require the following axioms :

MC1 (Limit Axiom) : Finite limits and colimits exist.

MC2 (Two out of three axiom) : If f and g are maps in $\mathcal{M}$ such that gf is defined and two of f, g and gf are weak equivalences, so is the third.

MC3 (Retract Axiom) : If f and g are maps in $\mathcal{M}$ such that f is a retract of g and g is a weak equivalence, a fibration, or a cofibration, then so is f.

MC4 (Lifting Axiom) : Given the commutative solid arrow diagram in $\mathcal{M}$ as shown below :

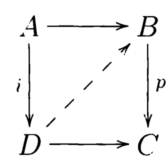

fig. 2.1

the dotted arrow exists if either i is a cofibration and p is an acyclic fibration or, i is an acyclic cofibration and p is a fibration.

MC5 (Factorization Axiom) : Any map f can be factored in two ways : $f = pi$, i is a cofibration and p is an acyclic fibration, and $f = pi$, i is an acyclic cofibration and p is a fibration.

Consider the following commutative diagram in a Category $\mathcal{M}$:

$$\begin{array}{ccccc} Q & \xrightarrow{g} & P & \xrightarrow{\bar{f}} & E \\ {\scriptstyle \bar{\bar{p}}}\downarrow & & \downarrow{\scriptstyle \bar{p}} & & \downarrow{\scriptstyle p} \\ Y & \xrightarrow[g]{} & X & \xrightarrow[f]{} & B \end{array}$$

fig. 2.2

The following properties are well known. We list them in order to make the paper self contained.

Lemma 2.1 *Let $\mathcal{M}$ be a (Model) Category .*

(i) If $PEXB$ and $QPYX$ are pullbacks in $\mathcal{M}$, then so is $QEYB$.

(ii) If $QEYB$ and $PEXB$ are pullbacks in $\mathcal{M}$, then so is $QPYX$.

Definition 2.2 *An object A of a category $\mathcal{M}$ is said to be cofibrant if $\iota \rightarrow A$ is a cofibration; and fibrant if $A \rightarrow *$ is a fibration where ι is an initial object of M and $*$ is a terminal object of M.*

Proposition 2.1 (4, proposition 3.14) *Let $\mathcal{M}$ be a Model Category.*

(i) The class of cofibrations in $\mathcal{M}$ is stable under cobase change (i.e., closed under pushouts).

(ii) The class of acyclic cofibrations in $\mathcal{M}$ is stable under cobase change (i.e., closed under pushouts).

(iii) The class of fibrations in $\mathcal{M}$ is stable under base change (i.e., closed under pullbacks).

(iv) The class of acyclic fibrations in $\mathcal{M}$ is stable under base change (i.e., closed under pullbacks).

Notation : For X and Y objects of a Model Category $\mathcal{M}$; $pr_0 : X \times Y \to X$ and $pr_1 : X \times Y \to Y$ will henceforth denote the canonical projections onto the first and second factors.

Definition 2.3 *A path object for X is an object X^I of a Model Category $\mathcal{M}$ together with a diagram $X \xrightarrow{\sigma} X^I \xrightarrow{p} X \times X$ (σ a weak equivalence) which factors the diagonal map $\Delta : X \to X \times X$. We denote the two maps $X^I \to X$ by $p_0 = pr_0 p$ and $p_1 = pr_1 p$.*

Definition 2.4 *A path object X^I is called a good path object, if $X^I \to X \times X$ is a fibration. By MC5(ii) at least one good path object exists for X.*

Proposition 2.2 (4, *Lemma* 4.14) *If X is fibrant and X^I is a good path object for X, then the maps $i_0, i_1 : X^I \to X$ are acyclic fibrations.*

Proposition 2.3 *If B and Y are fibrant, then the canonical projections $B \times Y \to B$ and $B \times Y \to Y$ are fibrations.*

Proof : Follows from Proposition 2.1 (iii) and the pullback diagram shown below.

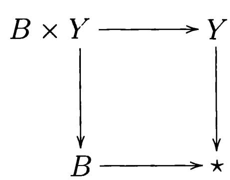

fig. 2.3

3 Proof of Lemma 1.1

Proof of Lemma 1.1 : Refer to fig. 1.2

Let Y^I be a good path object for Y. See *Definitions* 2.3 and 2.4

Since Y is fibrant and Y^I is a good path object for Y, then $p_0, p_1 : Y^I \to Y$ are acyclic fibrations by Proposition 2.2.

We construct P as the pullback of the following diagram:

$$\begin{array}{ccc} P & \xrightarrow{\bar{p}} & B \times Y \\ \downarrow{\scriptstyle \xi} & & \downarrow{\scriptstyle f \times 1_Y} \\ Y^I & \xrightarrow[p=(p_0,p_1)]{} & Y \times Y \end{array}$$

fig. 3.1

The above pullback is motivated by the double mapping track as defined in *Brown* and *Heath*. [3, *pp* 322]

Consider the following diagram :

$$\begin{array}{ccc} B \times Y & \longrightarrow & B \\ \downarrow & & \downarrow{\scriptstyle f} \\ Y \times Y & \longrightarrow & Y \\ \downarrow & & \downarrow \\ Y & \longrightarrow & \star \end{array}$$

fig. 3.2

Since the outer rectangle is a pullback and the lower rectangle is a pullback, it follows that the upper rectangle is a pullback by Lemma 2.1(ii).

Next define $\alpha : P \to B$ and $\beta : P \to Y$ to be the respective composites $P \xrightarrow{\bar{p}} B \times Y \to B$ and $P \xrightarrow{\bar{p}} B \times Y \to Y$.

It follows readily from Proposition 2.3 that α and β being composites of fibrations are fibrations.

Hence, by the universal property of products and commutativity of $fig.3.1$ we see that:

$$p_0\xi = f\alpha \quad and \quad p_1\xi = \beta \tag{1}$$

Hence we have the following composed pullback rectangle.

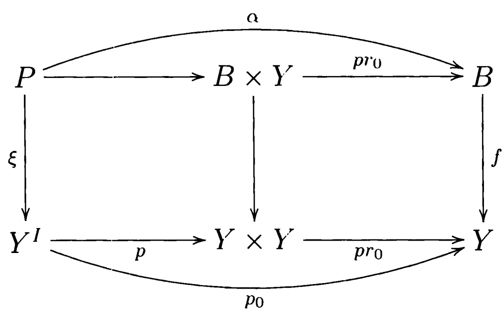

fig. 3.3

Since p_0 is an acyclic fibration, it follows that α is an acyclic fibration by Proposition 2.1(iv).
Define S to be the pullback of $P \xrightarrow{\alpha} B \xleftarrow{q} A$ and construct the following diagram.

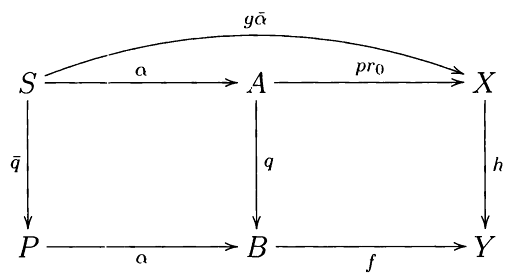

fig. 3.4

The gist of the proof is to factorize $g\overline{\alpha} = \epsilon\, \overline{p}_0$ where both ϵ and $\overline{p}_0$ are weak equivalences. We conclude that g is a weak equivalence by $MC2$.

To this end, we begin by defining P_0 and P_1 respectively as the pullbacks of $Y^I \overset{p_0}{\rightarrow} Y \overset{h}{\leftarrow} X$ and $Y^I \overset{p_1}{\rightarrow} Y \overset{h}{\leftarrow} X$ (see diagram below)

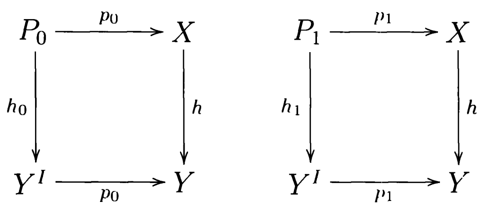

fig. 3.5

It follows readily from the diagram that since h is a fibration, h_0 and h_1 are fibrations and from the fact that p_0 and p_1 are acyclic fibrations that $\overline{p}_0$ and $\overline{p}_1$ are acyclic fibrations.
By universality , there exists a unique δ_0: $X \longrightarrow P_0$ such that $\overline{p}_0\delta_0 = 1_X$ and $h_0\ \delta_0 = \sigma\ h$. Now $\overline{p}_0\ \delta_0 = 1_X \Rightarrow \delta_0$ is a weak equivalence. Similarly, there exists a unique $\delta_1 : X \longrightarrow P_1$ such that δ_1 is also a weak equivalence. We now construct the pullback of $h_1 : P_1 \longrightarrow Y^I$ and $h_0 : P_0 \longrightarrow Y^I$ and call it Z.

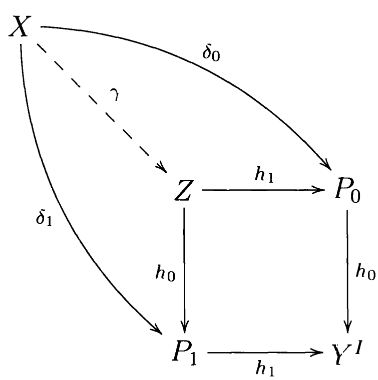

fig. 3.6

Again, by universality, there exists a unique $\gamma : X \rightarrow Z$ such that $\bar{h}_1\gamma = \delta_0$ and $\bar{h}_0\gamma = \delta_1$
By MC5, the map $\gamma : X \rightarrow Z$ can be factored as

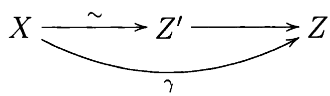

fig. 3.7

Hence,

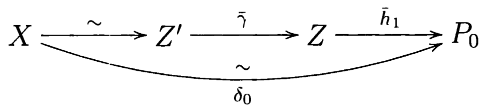

fig. 3.8

implies that $\bar{h}_1\bar{\gamma} : Z' \to P_0$ is an acyclic fibration. Similarly, $\bar{h}_0\bar{\gamma} : Z' \to P_1$ is an acyclic fibration. That is,

$$\overline{h}_1\overline{\gamma} \quad and \quad \overline{h}_0\overline{\gamma} \tag{2}$$

are acyclic fibrations.

We factorize f as the composite $B\xrightarrow{\varphi}P \xrightarrow{\beta} Y$ where φ is the unique map defined from the universal property of P by the maps $B \xrightarrow{f} Y\xrightarrow{\sigma} Y^I$ and the unique map $(1_B, f) : B \to B \times Y$ induced by $1_B : B \to B$ and $f : B\to Y$. It then follows from the universal property of pullbacks that

$$\overline{p}\varphi = (1_B, f) = (\alpha\varphi, \beta\varphi) \tag{3}$$

$$\Rightarrow 1_B = \alpha\varphi \quad and \quad f = \beta\varphi \tag{4}$$

Consequently, φ and β are both weak equivalences by $MC2$. In fact β is an acyclic fibration as we have earlier on demonstrated that it is a fibration.

We now construct R as the pullback of $X\xrightarrow{h} P \xleftarrow{\beta} Y$.

Since $AXBY$ is a pullback by hypothesis, there exists a unique $\overline{\varphi} : A \to R$ such that $\overline{\beta}\,\overline{\varphi} = g$ and the following diagram commutes : (*follows* *from equation 4 and fig. 1.2)*

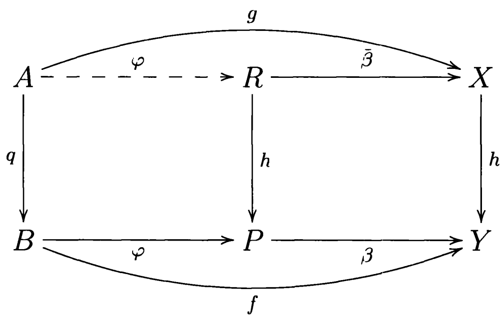

fig. 3.9

Since, $AXBY$ and $RXDY$ are pullbacks, the square $ARBP$ is a pullback.

Since $RXPY$ is a pullback square, h is a fibration $\Rightarrow \overline{h}$ is a fibration ; and β is an acyclic fibration $\Rightarrow \overline{\beta}$ is an acyclic fibration.
Consider the following pullback :

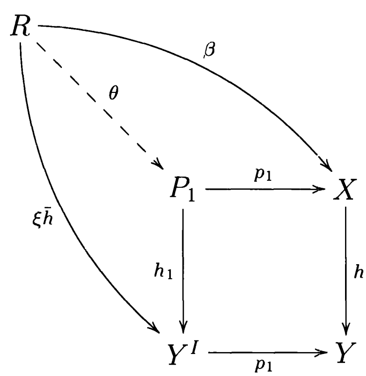

fig. 3.10

By universality, there exists a unique $\theta : R \to P_1$ such that

$$\overline{p}_1\theta = \overline{\beta} \quad and \quad h_1\theta = \xi\overline{h} \tag{5}$$

Hence, we have the following factorization of $\overline{\beta}$:

$$\begin{array}{ccccc} R & \xrightarrow{\theta} & P_1 & \xrightarrow[\sim]{\bar{p}_1} & X \end{array}, \qquad \bar{\beta} : R \xrightarrow{\sim} X$$

fig. 3.11

Therefore, $\theta : R \to P_1$ is a weak equivalence by MC2.
Consider the following composite square :

$$\begin{array}{ccccc} R & \xrightarrow{\theta} & P_1 & \xrightarrow{\bar{p}_1} & X \\ \downarrow{\scriptstyle \bar{h}} & & \downarrow{\scriptstyle h_1} & & \downarrow{\scriptstyle h} \\ P & \xrightarrow[\xi]{} & Y^I & \xrightarrow[p_1]{} & Y \end{array} \qquad \bar{\beta} : R \to X, \quad \beta : P \to Y$$

fig. 3.12

The outer rectangle is a pullback and right rectangle is a pullback implies that the left-hand rectangle is a pullback. Therefore, RP_1PY^I is a pullback.
Let Q be the pullback square defined below :

$$\begin{array}{ccc} Q & \xrightarrow{\tilde{\theta}} & Z' \\ {\scriptstyle \tilde{t}}\downarrow{\scriptstyle \sim} & & {\scriptstyle \sim}\downarrow{\scriptstyle \bar{h}\circ\bar{\gamma}} \\ R & \xrightarrow[\theta]{\sim} & P_1 \end{array}$$

fig. 3.13

Now, $Z' \to P_1$ is an acyclic fibration (equation 2) $\Rightarrow Q \to R$ is an acyclic fibration. Therefore, $\tilde{\theta} : Q \to Z'$ is a weak equivalence.

Consider the following diagram:

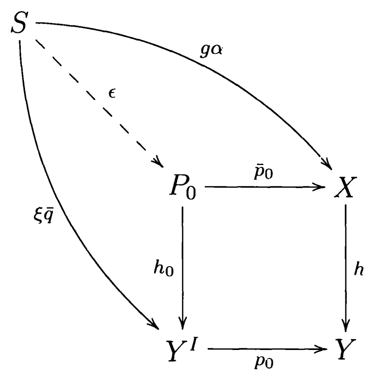

fig. 3.14

Commutativity of diagram follows from the following equalities:
$p_0\ \xi\ \overline{q} = f\ \alpha\overline{q} = f\ q\ \overline{\alpha} = h\ g\overline{\alpha}$ (follows from *equation*1, *fig.* 1.2 and *fig.* 3.4).
By universality, there exists a unique ϵ: $S \rightarrow P_0$ such that:

$$\overline{p}_0\epsilon = g\overline{\alpha} \quad and \quad h_0\epsilon = \xi\overline{q} \tag{6}$$

Now, $SXPY$ being the composite of the pullbacks $SAPB$ and $AXBY$, is itself a pullback.
Hence, the outer rectangle shown below is a pullback :(for commutativity see *equation 1, fig. 3.4 and fig. 1.2*)

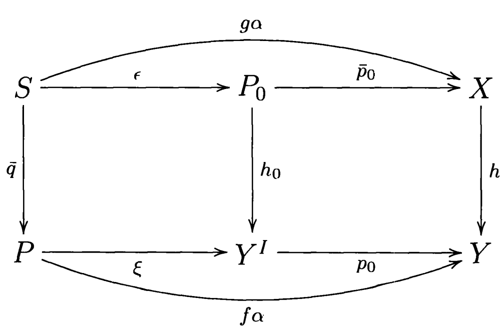

fig. 3.15

It follows then that SP_0PY^I is a pullback.
It is easy to see that the following composed diagrams commute and ,

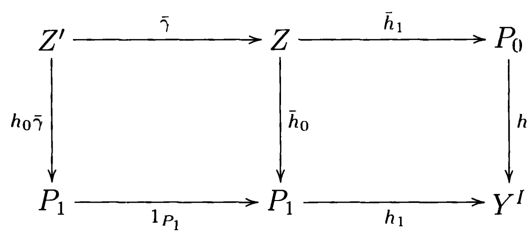

fig. 3.16

give rise to the following diagram:

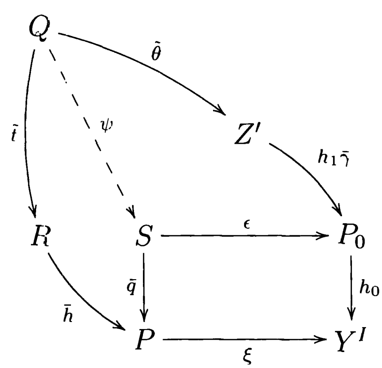

fig. 3.17

which is commutative since $h_0\ \bar{h}_1\ \bar{\gamma}\ \tilde{\theta} = h_1\ \bar{h}_0\ \bar{\gamma}\ \tilde{\theta} = h_1\ \theta\ \tilde{t} = \xi\ \bar{h}\ \tilde{t}$ (*follows from fig. 3.6, fig 3.13, and equation 5*)
Therefore, there exists a unique $\psi : Q \to S$ such that:

$$\epsilon\psi = (\overline{h}_1\overline{\gamma})\tilde{\theta} \quad and \quad \bar{q}\psi = \overline{h}\tilde{t} \tag{7}$$

Therefore, $QZ'SP_0$ is commutative.
Claim : $QZ'SP_0$ is pullback.

Consider the following composite squares :

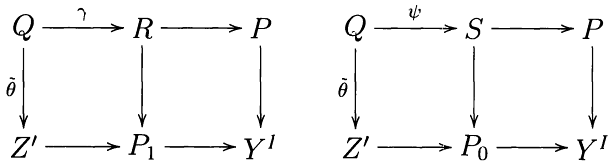

fig. 3.18

The composed square on the left hand, that is, $QPZ'Y^I$is a pullback being a composite of pullbacks. Observe the following, $Q \to R \to P$ is the same as $Q \to S \to P$; and$Z' \to P_0 \to Y^I$ is the same as $Z' \to P_1$ $\to Y^I$. (*see fig. 3.16 and fig. 3.17*)

Therefore, the righthand composed diagram $QPZ'Y^I$ is a pullback. Since SPP_0Y^I is a pullback, it follows that $QZ'SP_0$ is a pullback. (*for commutativity see equation 7*)
See diagram below :

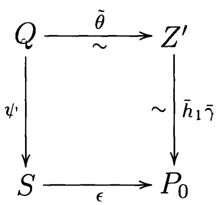

fig. 3.19

Now, $Z' \to P_0$ is an acyclic fibration (*equation2*) $\Rightarrow \psi$ is an acyclic fibration $\Rightarrow \epsilon$ is a weak equivalence by MC2. Hence $g\bar{\alpha} = \bar{p}_0\epsilon$ is a weak equivalence .

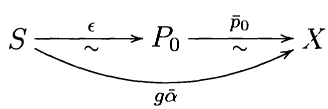

fig. 3.20

Therefore, g is a weak equivalence
Finally, a summary of the proof is contained in the diagram shown below and it proceeds as follows: .

$$
\begin{aligned}
\beta \text{ is a weak equivalence} \quad &\Rightarrow \overline{\beta} \text{ is a weak equivalence} \\
&\Rightarrow \theta \text{ is a weak equivalence} \\
&\Rightarrow \tilde{\theta} \text{ is a weak equivalence} \\
&\Rightarrow \epsilon \text{ is a weak equivalence} \\
&\Rightarrow g\overline{\alpha} : A \rightarrow X \text{ is a weak equivalence} \\
&\Rightarrow g : A \rightarrow X \text{ is a weak equivalence}
\end{aligned}
$$

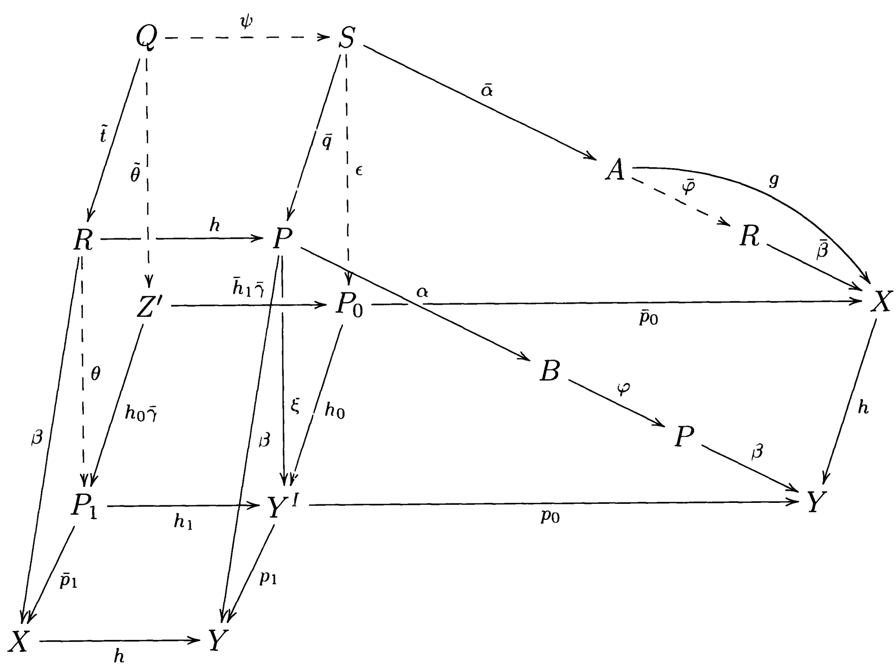

fig. 3.21

4 Proof of Theorem 1.1

Step 1 : Factor p_g and p by constructing R and S in the front face as pullbacks of α and q_f and q and β respectively. δ_1, δ_2, δ_3, exist by universal properties and commutativity of diagram.

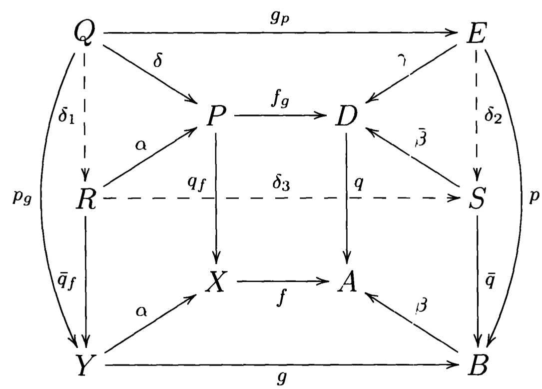

fig. 4.1

Now, $\overline{\alpha}$ and $\overline{\beta}$ are weak equivalences by Lemma 1.1 . Hence it suffices to show that δ_1 is a weak equivalence. $RPYX$ and $PDXA$ are both pullbacks and hence $RDYA$ is a pullback by Lemma 2.1(i) Since f $\alpha = g\ \beta$ and $f_q\ \overline{\alpha} = \overline{\beta}\ \delta_3$ it follows that the composite of $RSYB$ and $SDBA$ is a pullback. Therefore, $RSYB$ is a pullback by Lemma 2.1(ii). Similarly,. $QERS$ is also a pullback. Now. $\delta_2\ \overline{\beta} = \gamma$ and so δ_2 is a weak equivalence by MC2.

Step 2 : Factorize g_p, δ_3 and g and construct pullbacks T and U.

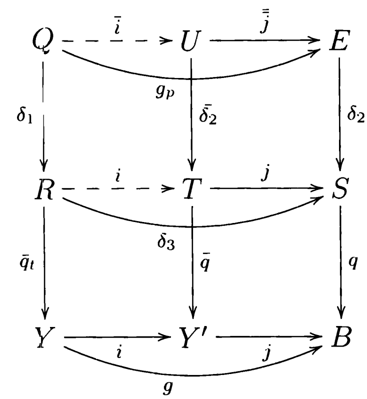

fig. 4.2

$\bar{i}$ and $\bar{\bar{i}}$ exist by the universal property of the pullback and $g = ji$ where i is an acyclic cofibration and j fibration by MC5(ii).
Since $QERS$ is a pullback and $UETS$ is a pullback, it follows that $QURT$ is a pullback. Again, $RSYB$ is a pullback and $TSY'B$ is pullback implies that $RTYY'$ is a pullback.

Since all squares are pullbacks, j is a fibration $\Rightarrow$ $\bar{j}$ and $\bar{\bar{j}}$ are fibrations by Proposition 2.1(iii)

Since B is fibrant and $\bar{q}$ is a fibration, it follows that S is fibrant. Similarly, Y' is fibrant $\Rightarrow$ T is fibrant as $\bar{\bar{q}}$ is a fibration.

Now, δ_2 is a weak equivalence, $\bar{j}$ is fibration and S, T are fibrant implies that $\bar{\delta}_2$ is a weak equivalence by Lemma 1.1., $\bar{q}$ is a fibration implies $\bar{\bar{q}}$ is a fibration and hence $\bar{q}_f$ is a fibration (Proposition 2.1(iii)). So, i is a weak equivalence, $\bar{\bar{q}}$ is a fibration and Y', Y' are fibrant implies $\bar{i}$ is a weak equivalence.

Now, $\bar{q}\ \delta_2 = p \Rightarrow \bar{\bar{q}}\ \bar{\delta}_2$ is a fibration.

Again, by Lemma 1.1, $\bar{\bar{i}}$ is a weak equivalence.

Finally, $\bar{\delta}_2\ \bar{\bar{i}} = \bar{i}\ \delta_1$ is a weak equivalence. $\Rightarrow$ $\bar{i}\ \delta_1$ is a weak equivalence $\Rightarrow$ δ_1 is a weak equivalence since $\bar{i}$ is a weak equivalence.

Corollary 4.1 *Theorem*1.1 $\Rightarrow$ *Lemma*1.1

Proof : The proof is contained in the diagram shown below :

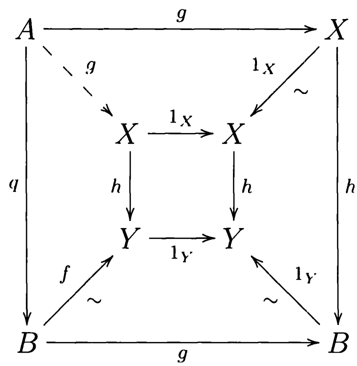

fig. 4.3

References :

1. H . J . Baues , Algebraic Homotopy, Cambridge Studies in Advanced Mathematics **15** , (1989), Cambridge University Press.

2. K . S . Brown , Abstract Homotopy Theory And Generalized Sheaf Cohomology, Trans. Amer. Math. Soc.,**186**,(1973), 419-458.

3. R . Brown ; P .R . Heath, Coglueing Homotopy Equivalences, Math. Z. **113** (1970), 313–315.

4. W . G . Dwyer ; J . Spalinski, Homotopy theories and Model categories, I . M . James(ed.), Handbook of Algebraic Topology, North-Holland, (1995), 73 - 126.

5. P . G . Goerss ; J . F .Jardine, Simplicial Homotopy Theory, Progress in Mathematics, **174** (1999) Birkhauser Verlag

6. H . K . Kamps ; T . Porter, Abstract Homotopy and Simple Homotopy Theory, World Scientific (1996)

7. D . Quillen , Homotopical Algebra, LNM, **43** (1967), Springer–Verlag.

8. C . Rezk , Every Homotopy Theory of Simplicial Algebras admits a proper model, Topology and its Appl. 119 (2002), **1**, 65 - 94.

DEPARTMENT OF MATHEMATICS AND STATISTICS
MEMORIAL UNIVERSITY OF NEWFOUNDLAND
ST.JOHN'S,NEWFOUNDLAND
CANADA, A1C 5S7
afework@math.mun.ca