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**ON THE STRUCTURE OF LOCALLY FINITE PURE SEMISIMPLE  
 GROTHENDIECK CATEGORIES**

by Daniel SIMSON

**INTRODUCTION**

Let  $\mathcal{U}$  be a locally finitely presented Grothendieck category and denote by  $fp(\mathcal{U})$  the full subcategory of  $\mathcal{U}$  consisting of all finitely presented objects. We recall from [8] that  $\mathcal{U}$  is pure semisimple if each of its objects is a direct sum of finitely presented objects. In [1] it is proved that if  $R$  is an Artin algebra then the category  $R\text{-Mod}$  of all left  $R$ -modules is pure semisimple iff  $R$  is of finite representation type, i. e. the category  $R\text{-mod}$  of all finitely generated left  $R$ -modules has only a finite number of isomorphism classes of indecomposable modules. Furthermore we know from [9] that if  $\mathcal{U}$  is a pure semisimple Grothendieck category with the property:

(EA) *The endomorphism ring of any noetherian injective object in  $\mathcal{U}$  is an Artin algebra*

and  $M$  is an object in  $fp(\mathcal{U})$  then there exist an Artin algebra  $R$  of finite representation type and a pair of additive functors

$$(R\text{-mod})^{op} \xrightarrow{T} fp(\mathcal{U}) \xrightarrow{F} (R\text{-mod})^{op}$$

such that  $FT = id$  and  $M = TF(M)$ . This shows that  $fp(\mathcal{U})$  can be locally approximated by categories of the form  $(R\text{-mod})^{op}$  where  $R$  is an Artin algebra of finite representation type.

In the paper we establish a stronger result on the structure of the category  $fp(\mathcal{U})$  under the assumption that  $\mathcal{U}$  is locally finite Grothendieck and pure semisimple category with the property (EA). We show that there exist an inverse system of ring surjections  $\{R_\alpha, f_{\alpha\beta}\}_{\alpha, \beta \in T}$  and full and exact embeddings  $(R_\alpha\text{-mod})^{op} \hookrightarrow fp(\mathcal{U})$ ,  $\alpha \in T$ , such that:

- (i)  $R_\alpha$  is an Artin algebra of finite representation type for all  $\alpha \in T$

and the pseudocompact ring of the category  $\mathfrak{A}$  (cf. [3]) is the limit of the system  $\{R_\alpha, f_{\alpha\beta}\}_{\alpha, \beta \in T}$ ,

(ii)  $fp(\mathfrak{A})$  is the union of all categories  $(R_\alpha\text{-mod})^{op}$ ,  $\alpha \in T$ ,

(iii) For each  $\alpha \in T$  there exists a  $\beta \in T$ ,  $\beta \geq \alpha$ , such that for any  $\gamma \geq \beta$  we have a factorization  $f_{\alpha\beta} = f_{\alpha\gamma}g_{\gamma\beta}$  where  $g_{\gamma\beta}: R_\beta \rightarrow R_\gamma$  is a homomorphism of left  $R_\gamma$ -modules.

The result was announced in [11].

**1. NOTATION AND PRELIMINARIES.**

If  $R$  is a ring we denote by  $R\text{-Mod}$  and  $\text{Mod}\cdot R$  the categories of all left and all right  $R$ -modules, respectively.  $R\text{-mod}$  and  $\text{mod}\cdot R$  will denote categories of finitely presented left  $R$ -modules and right  $R$ -modules.

We recall that a ring  $R$  is an Artin algebra if the center  $C$  of  $R$  is an Artinian ring and  $R$  is a finitely generated  $C$ -module.

Pure semisimple Grothendieck categories are investigated in [7-10, 2 and 4], where the reader is referred for details. The following result is taken from [9] (see also [1], Theorem A).

**THEOREM 1.1.** *A Grothendieck category  $\mathfrak{A}$  is pure semisimple iff  $\mathfrak{A}$  is locally Noetherian and for any sequence*

$$M_1 \xrightarrow{f_1} M_2 \longrightarrow \dots \longrightarrow M_n \xrightarrow{f_n} M_{n+1} \longrightarrow \dots$$

*of monomorphisms between indecomposable Noetherian objects in  $\mathfrak{A}$  there exists an integer  $n$  such that  $f_i$  is an isomorphism for all  $i > n$ .*

In the next section we will need the following theorem proved in [8 and 9].

**THEOREM 1.2.** *Let  $\mathfrak{A}$  be a pure semisimple Grothendieck category and suppose that  $\mathfrak{A}$  has only a finite number of isomorphism classes of simple objects. If the endomorphism ring  $S$  of the minimal injective cogenerator in  $\mathfrak{A}$  is an Artin algebra then  $\mathfrak{A}$  is equivalent with the category  $\text{Mod}\cdot S$  and  $S$  is of finite representation type.*

We also will need the following simple lemma.

LEMMA 1.3. *Let  $\mathcal{C}$  be an abelian category such that each of its objects has finite length. Given a set  $\mathcal{F}$  of simple objects in  $\mathcal{C}$  we denote by  $\mathcal{C}(\mathcal{F})$  the full subcategory of  $\mathcal{C}$  consisting of all objects having composition series with factors from  $\mathcal{F}$ . Then:*

(a)  *$\mathcal{C}(\mathcal{F})$  is abelian and  $\mathcal{F}$  is the set of representatives of isomorphism classes of simple objects in  $\mathcal{C}(\mathcal{F})$ .*

(b) *The embedding  $\mathcal{C}(\mathcal{F}) \hookrightarrow \mathcal{C}$  is exact.*

(c) *Every object of  $\mathcal{C}$  has a unique maximal subobject which belongs to  $\mathcal{C}(\mathcal{F})$ .*

PROOF. [6], Theorem 1.2.

Throughout this paper we follow the terminology and notation of [5, 7 and 8]. In particular given an abelian category  $\mathcal{C}$  we denote by  $Lex\mathcal{C}$  the category of all left exact additive functors from  $\mathcal{C}$  to the category of abelian groups. If  $X$  and  $Y$  are objects of  $\mathcal{C}$  we denote by  $(X, Y)$  the abelian group of all morphisms from  $X$  into  $Y$ .  $\mathcal{C}^{op}$  denotes the category opposite to  $\mathcal{C}$ .

## 2. THE MAIN RESULT.

We recall that a Grothendieck category  $\mathcal{U}$  is locally finite if  $\mathcal{U}$  has a set of generators of finite length.

Let  $\mathcal{U}$  be a locally finite Grothendieck category and denote by  $Q$  the direct sum of representatives of a fixed family of isomorphism classes of indecomposable injective objects in  $\mathcal{U}$ . Fix a directed family of subobjects  $L_t \subset Q$  of finite length such that  $Q = \bigcup_t L_t$ . The pseudocompact ring of the category  $\mathcal{U}$  is the ring  $R = End Q$  equipped with the linear topology defined by the ideals  $(Q/L_t, Q)$  of  $R$  (see [3]). We also know from [3] that the functor

$$D: \mathcal{U} \rightarrow R\text{-Mod} \text{ given by } D(-) = (-, Q)$$

defines a duality of  $\mathcal{U}$  and the full subcategory  $R\text{-PC}$  of  $R\text{-Mod}$  consisting of all pseudocompact modules and continuous  $R$ -homomorphisms. The category of all left discrete  $R$ -modules will be denoted by  $R\text{-Dis}$ .

We are now able to prove the main result of the paper.

**THEOREM 2.1.** *Let  $\mathcal{A}$  be a locally finite pure semisimple Grothendieck category and suppose that the endomorphism ring of any injective object of finite length in  $\mathcal{A}$  is an Artin algebra. Let  $R$  be the pseudocompact ring of the category  $\mathcal{A}$  and  $R'$  the pseudocompact ring of the category  $R\text{-Dis}$ . Then there exist a directed set  $T$ , a directed family of full and exact abelian subcategories  $\mathcal{C}_\alpha$  of  $\text{fp}(\mathcal{A})$ ,  $\alpha \in T$ , and inverse systems of rings  $\{R_\alpha, f_{\alpha\beta}\}_{\alpha, \beta \in T}$  and  $\{R'_\alpha, f'_{\alpha\beta}\}_{\alpha, \beta \in T}$  such that the following assertions hold:*

(a) *For each  $\alpha \in T$  the rings  $R_\alpha$  and  $R'_\alpha$  are Artin algebras of finite representation type and for each  $\beta \geq \alpha$  the ring homomorphisms*

$$f_{\alpha\beta}: R_\beta \rightarrow R_\alpha \quad \text{and} \quad f'_{\alpha\beta}: R'_\beta \rightarrow R'_\alpha$$

*are surjective.*

(b)  *$R = \varprojlim_{\alpha \in T} R_\alpha$  and  $R' = \varprojlim_{\alpha \in T} R'_\alpha$ . The canonical maps  $f_\alpha: R \rightarrow R_\alpha$ ,  $f'_\alpha: R' \rightarrow R'_\alpha$  are surjective and the families  $\text{Ker} f_\alpha$ ,  $\alpha \in T$ , and  $\text{Ker} f'_\alpha$ ,  $\alpha \in T$ , form bases of neighborhoods of zero in  $R$  and  $R'$  respectively.*

(c) *For each  $\alpha \in T$  there exist natural equivalences*

$$\mathcal{C}_\alpha \xrightarrow{D_\alpha} (R_\alpha\text{-mod})^{\text{op}} \xrightarrow{D'_\alpha} R'_\alpha\text{-mod}$$

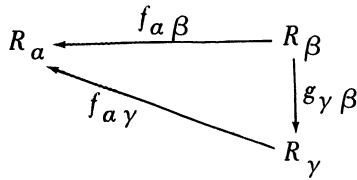
*such that for any pair  $\beta \geq \alpha$  the diagram*

$$\begin{array}{ccccc} \mathcal{C}_\alpha & \xrightarrow{D_\alpha} & (R_\alpha\text{-mod})^{\text{op}} & \xrightarrow{D'_\alpha} & R'_\alpha\text{-mod} \\ \downarrow & & \downarrow & & \downarrow \\ \mathcal{C}_\beta & \xrightarrow{D_\beta} & (R_\beta\text{-mod})^{\text{op}} & \xrightarrow{D'_\beta} & R'_\beta\text{-mod} \end{array}$$

*is commutative where the right hand inclusions are induced by the surjections  $f_{\alpha\beta}$  and  $f'_{\alpha\beta}$  respectively.*

(d) *The category  $\text{fp}(\mathcal{A})$  is the directed union of categories  $\mathcal{C}_\alpha$ ,  $\alpha \in T$ .*

(e) *For each  $\alpha \in T$  there exists a  $\beta \in T$ ,  $\beta \geq \alpha$ , such that for any  $\gamma \geq \beta$  we have a factorization as indicated below where  $g_{\gamma\beta}$  is a homomorphism of left  $R$ -modules:*



PROOF. Let  $S_i, i \in I$ , be a complete list of representatives of isomorphism classes of simple objects in  $\mathcal{A}$  and let  $T$  be the directed set of all finite subsets of  $I$ . Let

$$Q = \bigoplus_{i \in I} \hat{S}_i \quad \text{and} \quad Q_\alpha = \bigoplus_{i \in \alpha} \hat{S}_i, \quad \alpha \in T,$$

where  $\hat{S}_i$  is the injective envelope of  $S_i$  in  $\mathcal{A}$ . Note that each  $\hat{S}_i$  has finite length since  $\mathcal{A}$  is pure semisimple. Now applying the construction in the Lemma 1.3 to

$$\mathcal{C} = fp(\mathcal{A}) \quad \text{and} \quad \mathcal{F} = \mathcal{F}_\alpha = \{S_i, i \in \alpha\}, \quad \alpha \in T,$$

we define an abelian exact subcategory  $\mathcal{C}_\alpha$  of  $fp(\mathcal{A})$  putting

$$\mathcal{C}_\alpha = fp(\mathcal{A})(\mathcal{F}_\alpha), \quad \alpha \in T.$$

It is clear that  $\mathcal{C}_\alpha \subset \mathcal{C}_\beta$  whenever  $\alpha \leq \beta$ , and each  $Q_\alpha$  is an object of a certain  $\mathcal{C}_\beta$ . We know from Lemma 1.3 that for every pair  $\alpha, \beta \in T$  there exists a unique maximal subobject  $Q_\alpha^\beta$  of  $Q_\alpha$  which is an object in  $\mathcal{C}_\beta$ . It is easy to check that  $Q_\alpha^\alpha$  is an injective cogenerator in  $\mathcal{C}_\alpha$  for any  $\alpha \in T$ . Now given  $\alpha \in T$  we put  $R_\alpha = End Q_\alpha^\alpha$ . It is easy to see that the natural inclusions  $Q_\alpha^\alpha \subset Q_\alpha^\beta \subset Q_\alpha, \alpha \geq \beta$ , induce isomorphisms

$$t_\alpha: (Q_\alpha^\alpha, Q_\alpha^\alpha) \rightarrow (Q_\alpha^\alpha, Q) \quad \text{and} \quad t_{\alpha\beta}: (Q_\alpha^\alpha, Q_\alpha^\alpha) \rightarrow (Q_\alpha^\alpha, Q_\beta^\beta).$$

We denote by  $f_\alpha: R \rightarrow R_\alpha$  and  $f_{\alpha\beta}: R_\beta \rightarrow R_\alpha, \beta \geq \alpha$ , the composed maps

$$\begin{aligned}
 (Q, Q) &\longrightarrow (Q_\alpha^\alpha, Q) \xrightarrow{t_\alpha^{-1}} (Q_\alpha^\alpha, Q_\alpha^\alpha), \\
 (Q_\beta^\beta, Q_\beta^\beta) &\longrightarrow (Q_\alpha^\alpha, Q_\beta^\beta) \xrightarrow{t_{\alpha\beta}^{-1}} (Q_\alpha^\alpha, Q_\alpha^\alpha).
 \end{aligned}$$

It is obvious that  $f_\alpha$  and  $f_{\alpha\beta}$  are surjective ring homomorphisms such that

$$f_\alpha f_{\alpha\beta} = f_\beta \quad \text{and} \quad f_{\alpha\beta} f_{\beta\gamma} = f_{\alpha\gamma} \quad \text{provided} \quad \gamma \geq \beta \geq \alpha.$$

We observe that for each  $\alpha \in T$  there exists  $\beta \geq \alpha, \beta \in T$ , such that

$Q_\alpha \subset Q_\beta^\beta \subset Q_\beta$ . Then we have

$$Q = \bigcup_{\alpha \in T} Q_\alpha = \bigcup_{\beta \in T} Q_\beta^\beta$$

and therefore we get

$$R = (Q, Q) = \varprojlim_{\beta \in T} (Q_\beta^\beta, Q) = \varprojlim_{\beta \in T} (Q_\beta^\beta, Q_\beta^\beta) = \varprojlim_{\beta \in T} R_\beta.$$

Then we have proved the part of (b) related to the ring  $R$ .

Now we are going to prove that each  $R_\alpha$ ,  $\alpha \in T$ , is an Artin algebra of finite representation type. For this purpose we fix  $\alpha \in T$  and consider the category  $\mathcal{A}_\alpha = \text{Lex } \mathcal{C}_\alpha^{\text{op}}$ . We know from [3] that  $\mathcal{A}_\alpha$  is a locally finite Grothendieck category.  $\text{fp}(\mathcal{A}_\alpha) \approx \mathcal{C}_\alpha$  and the natural embedding  $\mathcal{C}_\alpha \rightarrow \mathcal{A}_\alpha$  is exact. It follows that the finite set  $S_i$ ,  $i \in \alpha$ , is a set of representatives of isomorphism classes of simple objects in  $\mathcal{A}_\alpha$ . Since  $\mathcal{A}$  is pure semi-simple then by Theorem 1.1 the category  $\mathcal{A}_\alpha$  is pure semisimple, too. Hence the injective cogenerator  $Q_\alpha^\alpha$  in  $\mathcal{C}_\alpha$  is an injective cogenerator in  $\mathcal{A}_\alpha$ . Furthermore the inclusion  $Q_\alpha^\alpha \subset Q_\alpha$  induces the ring surjection

$$(Q_\alpha, Q_\alpha) \longrightarrow (Q_\alpha^\alpha, Q_\alpha^\alpha) = R_\alpha.$$

Since by our assumption  $(Q_\alpha, Q_\alpha)$  is an Artin algebra then so is  $R_\alpha$ . Then by Theorem 1.2 the category  $\mathcal{A}_\alpha$  is equivalent with  $\text{Mod-}R_\alpha$  and  $R_\alpha$  is of finite representation type, as required.

Now we consider the duality

$$D: \mathcal{A} \rightarrow R\text{-PC} \quad \text{given by } D(-) = (-, Q).$$

Since  $Q_\alpha^\alpha$  is an injective cogenerator in  $\mathcal{C}_\alpha$ ,  $\alpha \in T$ , and for any object  $N$  in  $\mathcal{C}_\alpha$  we have  $(N, Q_\alpha^\alpha) = (N, Q)$  then the restriction of the functor  $D$  to the category  $\mathcal{C}_\alpha$  defines an equivalence  $D_\alpha: \mathcal{C}_\alpha \rightarrow (R_\alpha\text{-mod})^{\text{op}}$  (see [9], Proposition 2.3).

In order to define the inverse system  $\{R'_\alpha, f'_{\alpha\beta}\}_{\alpha, \beta \in T}$  we consider the subcategory  $R\text{-Dis}$  of  $R\text{-PC}$ . We know from [3] that  $R\text{-Dis}$  is a locally finite Grothendieck category, the objects  $S_i^i = D(S_i)$ ,  $i \in I$ , form a complete list of representatives of isomorphism classes of simple objects in  $R\text{-Dis}$  and

$$D(\text{fp}(\mathcal{A})) = \text{fp}(R\text{-Dis}) = \bigcup_{\alpha \in T} R_\alpha\text{-mod}.$$

We consider the following injective objects in  $R\text{-Dis}$  :

$$E = \bigoplus_{i \in I} \hat{S}_i^! \quad \text{and} \quad E_\alpha = \bigoplus_{i \in \alpha} \hat{S}_i^!, \quad \alpha \in T,$$

and for each pair  $\alpha, \beta \in T$  we define a left module  $E_\alpha^\beta$  over the ring  $R_\beta = R/\text{Ker } f_\beta$  by formula

$$E_\alpha^\beta = \{ x \in E_\alpha, (\text{Ker } f_\beta) x = 0 \}.$$

Since  $R_\beta$  is of finite representation type then  $E_\alpha^\beta$  is a direct sum of modules of finite length. Then  $E_\alpha^\beta$  has finite length because the socle of  $E_\alpha^\beta$  is finite. Furthermore, we observe that for each  $\alpha \in T$  the module  $E_\alpha^\alpha$  is an injective cogenerator in  $R_\alpha\text{-mod}$ . Given  $\alpha \in T$  we put  $R'_\alpha = \text{End } E_\alpha^\alpha$ . Similarly as in the first part of the proof we define ring surjections

$$f'_\alpha: R' \rightarrow R'_\alpha \quad \text{and} \quad f'_{\alpha\beta}: R'_\beta \rightarrow R'_\alpha, \quad \beta \geq \alpha, \quad \text{where } R' = \text{End } E.$$

Furthermore the duality

$$D': R\text{-Dis} \rightarrow R'\text{-PC} \quad \text{given by} \quad D'(-) = (-, E)$$

yields an equivalence  $D'_\alpha: (R_\alpha\text{-mod})^{op} \rightarrow R'_\alpha\text{-mod}$  for each  $\alpha \in T$ . Now it is easy to verify the statements (a)-(d).

In order to prove the last statement we consider the inverse system  $\{R_\alpha, f_{\alpha\beta}\}_{\alpha, \beta \in T}$  in the category  $R\text{-PC}$  and observe that under  $D$  it is the image of the direct system  $Q_\alpha^a, \alpha \in T$ , in the category  $fp(\mathfrak{U})$ . Since  $\mathfrak{U}$  is pure semisimple then by [7], Theorems 6.3, 5.4, 3.16, the system  $Q_\alpha^a, \alpha \in T$ , is factorizable in the sense of [7], Definition 3.1. Hence (e) follows and the theorem is proved.

REMARK. We don't know if the statements (a)-(e) in Theorem 2.1 are sufficient for the pure semisimplicity of the category  $\mathfrak{U}$ .

Now we give a simple example to illustrate our main theorem.

EXAMPLE 2.2. Let  $K$  be a field and denote by  $\mathfrak{R}$  the category of  $K$ -representations of the infinite quiver

$$1 \rightarrow 2 \rightarrow \dots \rightarrow n \rightarrow n+1 \rightarrow \dots$$

(see [2]). Let  $\mathfrak{U}$  be the full subcategory of  $\mathfrak{R}$  consisting of directed unions of objects of finite length. We know from [2] that  $\mathfrak{U}$  is pure semisimple



and every indecomposable object in  $\mathcal{U}$  has the form

$$I_{nm}: 0 \rightarrow \dots \rightarrow 0 \rightarrow K \xrightarrow{n \text{ id}} K \xrightarrow{\text{id}} \dots \xrightarrow{\text{id}} K \xrightarrow{m} 0 \rightarrow 0 \rightarrow \dots, \quad n \leq m < \infty.$$

Let  $\mathcal{F}_n$  be the set of simple objects  $I_{11}, \dots, I_{nn}$ . Then  $\mathcal{C}_n = fp(\mathcal{U})(\mathcal{F}_n)$  consists of all representations of the form

$$V_1 \rightarrow V_2 \rightarrow \dots \rightarrow V_n \rightarrow 0 \rightarrow 0 \rightarrow \dots$$

with  $\dim V_j < \infty$  for  $j = 1, 2, \dots, n$ . Observe that  $fp(\mathcal{U})$  is the union of all  $\mathcal{C}_n$ , the object

$$Q_n = I_{11} \oplus I_{12} \oplus \dots \oplus I_{1n}$$

is the minimal injective cogenerator in the category  $\mathcal{C}_n$ , the ring  $R_n = \text{End} Q_n$  is the ring of  $n \times n$  lower triangular matrices with entries in the field  $K$  and we have  $\mathcal{C}_n \approx (R_n\text{-mod})^{op} \approx \text{mod-}R_n$ .

We conclude this paper with some remarks concerning the pure semi simple property of comodule categories.

Let  $C$  be a coalgebra over a field  $K$  and denote by  $C\text{-Comod}$  the category of left  $C$ -comodules (see [12]). Let  $C_j$ ,  $j \in J$ , be a directed set of finite-dimensional subcoalgebras of  $C$  such that  $C = \bigcup_{j \in J} C_j$ . Then the dual  $K$ -algebra  $C^*$  with the linear topology defined by the two-sided ideals  $\text{Hom}_K(C/C_j, K)$  in  $C^*$ ,  $j \in J$ , is pseudocompact and  $C^* = \varprojlim_{j \in J} C_j^*$  (see [13]). It is clear that  $C\text{-Comod}$  is a locally finite Grothendieck category. Furthermore there exist natural equivalences

$$C\text{-Comod} \approx C^*\text{-Rat} \approx C^*\text{-Dis}$$

where  $C^*\text{-Rat}$  is the category of left rational  $C^*$ -modules (cf. [12] and [13]).

If  $R$  is the pseudocompact ring of the category  $C\text{-Comod}$  and  $R'$  is the pseudocompact ring of the category  $R\text{-Dis}$  then we know from [3] that there is an equivalence  $C\text{-Comod} \approx R'\text{-Dis}$ . The continuous  $K$ -dual space  $C'$  to  $R'$  has a natural  $K$ -coalgebra structure such that  $C'^* \approx R'$  and  $C'$  is a minimal injective cogenerator in  $C'\text{-Comod} \approx C\text{-Comod}$ . It is clear that these properties determine  $C'$  uniquely up to a coalgebra isomorphism and we call any such coalgebra  $C'$  basic.

Now suppose  $C$  is a basic coalgebra and  $C\text{-Comod}$  is pure semisimple. Then by Theorem 2.1 there exists a directed set of finite-dimensional subcoalgebras  $L_\alpha$  of  $C$ ,  $\alpha \in T$ , such that  $C = \bigcup_{\alpha \in T} L_\alpha$ , the dual  $K$ -algebra  $L_\alpha^*$  is of finite representation type for all  $\alpha \in T$  and

$$C^* = \varprojlim_{\alpha \in T} L_\alpha^*.$$

It would be interesting to have a characterization of the coalgebras  $C$  with  $C\text{-Comod}$  pure semisimple. In the cocommutative case such a characterization is given in [7], Theorem 7.1.

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