COMPOSITIO MATHEMATICA

BEN LICHTIN

Erratum for "Generalized dirichlet series and B-functions"

Compositio Mathematica, tome 72, nº 2 (1989), p. 237-239
<http://www.numdam.org/item?id=CM_1989__72_2_237_0>

Compositio Mathematica **72**: 237–239, **1989**.

Erratum for generalized dirichlet series and B-functions
Vol. 65 no. 1 (1988)

BEN LICHTIN
Department of Mathematics, University of Rochester, Rochester, NY 14627, USA

Received 19 April 1988; accepted in revised form 26 January 1989

The notations and reference numbers of [Li-1] are used below.
(1) The value of $B(N)$ given on page 92 should be $[\![n(N+2)/\alpha]\!] + 1$.
(2) The expression for $\Phi \circ \pi_\alpha(u_1, \ldots, u_n)$ in (5.5) should be

$$\prod_j u_j^{-d \cdot \alpha_j}.$$

(3) Assertion (5.11) (p. 113) and the Remark (p. 82), concerning the validity of (5.14) (p. 116) for arbitrary poles $s = \rho$ with $\rho < \beta_\varphi$, do not follow from the discussion in the proof of Theorem 2. The source of difficulty concerns the statement on p. 114 (line 2). One needs to replace this inclusion by the weaker

$$\mathscr{S}(\sigma, \rho) \subset \Phi_{\theta'}(1/a') \cap [\{x_1 \ldots x_n = 0\} \cup \{Q = 0\}],$$

which holds in general, for any σ, ρ. On the other hand, the statement of Theorem 2, which only concerns the largest pole $s = \beta_\varphi$, does not require any modification. Insofar as the proof of the theorem is concerned, the following lemma is needed to give a complete proof of Theorem 2.

LEMMA. *Let $s = \rho$ be a pole for which [some irreducible component of $\mathscr{S}(\sigma, \rho)] \cap \{Q = 0\} = \emptyset$. Then $\rho < \beta_\varphi$.*

Proof. The (+) condition on P implies that

$$\mathscr{S}(\sigma, \rho) \cap [\{Q = 0\} - \{x_1 \ldots x_n = 0\}] \cap \Phi_{\theta'}(1/a') = \emptyset.$$

Assume now that $\mathscr{C}$ is a (non-empty) component of $\mathscr{S}(\sigma, \rho) \cap [\{x_1 \ldots x_n = 0\} - \{Q = 0\}]$. Then in the notation of (5.5), one sees that there exists a cone $\theta = \langle \alpha_1, \ldots, \alpha_n \rangle$ in a refining partition, used to construct the manifold X_Γ, so that

$$\rho = \left\{\frac{e + |\alpha_j| + \vec{d}\cdot\alpha_j}{A_j}\right\}, \tag{1}$$

for some $e = 0, -1, -2, \ldots$ and $j = 1, \ldots, n$.

On the other hand, if $\mathscr{S}(\sigma, \rho) \cap \{x_1 \ldots x_n = 0\} \cap \{Q = 0\} \neq \emptyset$, then ρ has the form

$$\rho = \left\{\frac{e + |\alpha'_k| + \vec{d}\cdot\alpha'_k}{A'_k - B'_k}\right\}, \tag{2}$$

for some $e = 0, -1, -2, \ldots$ and $k = 1, 2, \ldots, n$, and where $\theta' = \langle \alpha'_1, \ldots, \alpha'_n \rangle$ is a possibly different cone in the partition. However, now $B'_k > 0$ because the monomial M (cf. (2.8)) is not in the support of P. This follows from the assumption $Q(0) = 0$. Thus, M lies strictly above the Newton polyhedron $\Gamma_\infty(P)$ in the following sense. For any covector α, the plane $\mathscr{M}(\alpha)$

$$\alpha \cdot x = M \cdot \alpha$$

lies strictly above the support plane $\mathscr{S}(\alpha)$

$$\alpha \cdot x = M(\alpha).$$

Let $\bar{1} = (1, \ldots, 1)$. The ratios

$$t_1(\alpha) = \frac{M(\alpha)}{\alpha \cdot (\bar{1} + \vec{d})} \text{resp. } t_2(\alpha) = \frac{M \cdot \alpha}{\alpha \cdot (\bar{1} + \vec{d})}$$

are the parameter values at which the line $t \cdot (\bar{1} + \vec{d})$ meets $\mathscr{S}(\alpha)$resp. $\mathscr{M}(\alpha)$. For each α one then has $t_1(\alpha) < t_2(\alpha)$. One now observes that the pole ρ, with the expression (1), at most equals $t_2(\alpha_j)$, whereas the pole ρ, with the expression (2), at most equals $t_1(\alpha'_k)$.

For $i = 1, 2$ let $\alpha^{(i)}$ be a convector at which $\max_\alpha 1/t_i(\alpha)$ is assumed. It follows that

$$1/t_1(\alpha^{(1)}) > 1/t_2(\alpha^{(2)}).$$

Since $\beta_\varphi = 1/t_1(\alpha^{(1)})$, one concludes that β_φ is strictly larger than any pole ρ for which some component of $\mathscr{S}(\sigma, \rho)$ is disjoint from $\{Q = 0\}$. This proves the lemma.

A consequence of the lemma and the arguments given in pp. 113–117 is the following.

COROLLARY. *Assume* (5.9) *holds for Q and the* $(+)$ *condition holds for P. Then the identity*

$$\mathrm{Pol}_{s=\rho}\, \mathscr{D}_P(s, \varphi) = \mathrm{Pol}_{s=\rho}\, J_0(s, \varphi)$$

holds for any pole ρ for which

$$\mathscr{S}(\sigma, \rho) \subset \Phi_{\theta'}(1/a') \cap [\{x_1 \ldots x_n = 0\} \cap \{Q = 0\}].$$