COMPOSITIO MATHEMATICA

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Compositio Mathematica, tome 48, nº 1 (1983), p. 25-34
<http://www.numdam.org/item?id=CM_1983__48_1_25_0>

COMPOSITIO MATHEMATICA, Vol. 48, Fasc. 1, 1983, pag. 25–34

THE STRUCTURE OF IDEALS IN THE BANACH ALGEBRA OF LIPSCHITZ FUNCTIONS OVER VALUED FIELDS

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Introduction

The aim of the paper is to present a study (§3) of the structure of closed ideals of the Banach algebra of Lipschitz functions defined on a complete ultrametric space. We start with the extension problem of Lipschitz functions (§1) followed by general theory of Gelfand ideals of a non-archimedean weakly regular (Definition 4.1, [4]) Banach algebra (§2).

We recall that the space $\mathrm{Lip}(X, d)$ of all Lipschitz functions (functions for which both $\|f\|_\infty$, $\|f\|_d$ are finite) on an ultrametric space (X, d) (we continue to assume without loss of generality that (X, d) is complete and $d \leq 1$ as in [4]) is a Banach algebra with pointwise multiplication and the norm $\|\cdot\|$ defined by $\|f\| = \max(\|f\|_\infty, \|f\|_d)$, where $\|f\|_\infty = \sup\{|f(x)| : x \in X\}$ and $\|f\|_d = \sup\{|f(x) - f(y)|/d(x, y) : x, y \in X, x \neq y\}$. $\mathrm{lip}(X, d) = \{f \in \mathrm{Lip}(X, d) : \lim_{d(x,y) \to 0} (|f(x) - f(y)|/d(x, y)) = 0\}$ is a closed subalgebra of $\mathrm{Lip}(X, d)$. The notation and terminology are as in [4].

The material forms a portion of the author's Ph.D. thesis approved by the University of Madras. The author is thankful to Dr. G. Rangan for many useful suggestions.

§1. The Extension Problem

The problem under consideration is about the extension of a Lipschitz function on a subspace (Y, d), to a Lipschitz function on (X, d) without altering the norms. The solution is similar to that of the Hahn–Banach extension problem by Ingleton [6]. For any positive real number k, define the truncation $(T_k f)$ of f at k by

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$$(T_k f)(x) = \begin{cases} f(x) & \text{if } |f(x)| \le k \\ 0 & \text{otherwise.} \end{cases}$$

For any $x, y \in X$, we have

$$|(T_k f)(x) - (T_k f)(y)| \le |f(x) - f(y)|,$$

i.e. the truncation $(T_k f)$ is a Lipschitz function whenever f is so.

DEFINITION 1.1: A non-archimedean valued field F is said to have "Lipschitz extension property" if, given a subspace (Y, d) of (X, d) and any $f \in \mathrm{Lip}(Y, d)$, there exists $f^* \in \mathrm{Lip}(X, d)$ such that i) $f^*(y) = f(y)$, $y \in Y$; ii) $\|f^*\|_\infty = \|f\|_\infty$ and iii) $\|f^*\|_d = \|f\|_d$.

The following theorem characterizes the fields possessing Lipschitz extension property.

THEOREM 1.2: *F has Lipschitz extension property if and only if F is spherically complete.*

PROOF: Let F be spherically complete, (Y, d) be a subspace of (X, d) and $f \in \mathrm{Lip}(Y, d)$. If $x \in X \backslash Y$, consider the nest of closed spheres $\{C_{\varepsilon_y}(f(y)) : y \in Y\}$ in F, where $\varepsilon_y = \|f\|_d\, d(x, y)$. By the spherical completeness of F, we have $\alpha \in \bigcap_{y \in Y} C_{\varepsilon_y}(f(y))$. Define $f_1 : \{Y, x\} \to F$ by

$$f_1(y) = \begin{cases} f(y) & \text{if } y \in Y \\ \alpha & \text{if } y = x. \end{cases}$$

Clearly $|f_1(y) - f_1(y')| \le \|f\|_d\, d(y, y')$, $y, y' \in \{Y, x\}$ and so the truncation of f_1 at $\|f\|_\infty$ gives us an extension of f to a Lipschitz function on $\{Y, x\}$. An application of Zorn's lemma now yields the required extension to the whole of (X, d).

Suppose F is not spherically complete then F is not pseudo-complete ([10], p. 34). Consider the metric space $X = F \oplus F$, $Y = F$ identified by the map $\alpha \to (\alpha, 0)$ as a subspace of X and the metric on X is defined as in the proof of Ingleton's theorem [6]. Let $\{(\alpha_n, 0)\}$ be the pseudo-cauchy sequence which is not pseudo-convergent in F. If $M > 0$ be such that $d((\alpha_n, 0), (\alpha_m, 0)) \le M$, $n, m \in \mathbb{N}$, define $f: Y \to F$ by

$$f(y, 0) = \begin{cases} y & \text{if } |y| \le M \\ 0 & \text{if } |y| > M. \end{cases}$$

It is easily verified that $f \in \mathrm{Lip}(Y, d)$ with $\|f\|_\infty \leq M$ and $\|f\|_d = 1$. As in the proof of Ingleton's theorem (loc. cit) we can show that f can not be extended as desired. The proof is now complete.

REMARK 1.3: In the context of extension, for the real case (see [12]) we do not require any additional assumptions while for the complex case, the possibility of extension require (as shown by Jenkins [7]) the Lipschitz four point property and Euclidean four point property.

The following theorems, suggested by Prof. van der Put in a private communication to the author, gives sufficient conditions on the metric space for the extension of a Lipschitz function on a subspace to the whole space.

THEOREM 1.4: *If every strictly decreasing sequence of values of d has limit zero, i.e. if $\{d(x_n, y_n)\}$ is strictly decreasing then* $\lim d(x_n, y_n) = 0$, *then every Lipschitz function f on a subspace (Y, d) can be extended to the whole of (X, d) as a Lipschitz function with norms preserved.*

PROOF: We assume, as we may, that Y is closed. By hypothesis if $x \in X \setminus Y$, there exists $y \in Y$ such that $d(x, Y) = d(x, y)$. $f^*: X \to F$ defined by

$$f^*(x) = \begin{cases} f(x) & \text{if } x \in Y \\ f(y) & \text{if } x \notin Y \text{ and } y \text{ is such that } d(x, Y) = d(x, y), \end{cases}$$

is the required extension.

Using the above theorem and a slight modification of the proof of Lemma 3.14(iii), p. 66, [16], we have another theorem on the extension of Lipschitz functions.

THEOREM 1.5: *If a subspace (Y, d) of (X, d) is spherically complete, then every $f \in \mathrm{Lip}(Y, d)$ can be extended to $f^* \in \mathrm{Lip}(X, d)$ with norms preserved.*

§2. Gelfand Ideal Space, Weak Regularity and Silov Ideals

In this section we present first improved versions of certain known results (Theorems 3.5, 3.6 and 4.3 of [4]), in the case of an arbitrary non-archimedean Banach algebra. These results are used to show that the Silov ideals $J(K)$ of a non-archimedean weakly regular Banach algebra are smallest in some sense. In fact the proof of this result is much different from the classical case in the absence of a smooth theory of integration of vector valued analytic functions.

Let X be any set and A any algebra with identity of complex valued bounded functions on X which separate points of X, is self-adjoint (i.e. closed under complex conjugation) and is inversely closed (i.e. $1/f \in A$ if $f \in A$ and $|f(x)| \geq \varepsilon > 0$, $x \in X$). It is known ([8], p. 55) that i) X is dense in the maximal ideal space $\mathcal{M}$ of A with respect to the Gelfand topology and ii) if X is a compact Hausdorff space, then X can be identified with $\mathcal{M}$. In the absence of a process analogous to conjugation we resort to an alternative approach, which as it appears can neither be adapted nor is an adaptation of the classical case.

We start with the following theorem, more general then Theorem 3, p. 154, [10], which can be easily proved on the same lines.

THEOREM 2.1: *Let X be any compact zero-dimensional Hausdorff space and A any algebra with identity of continuous F-valued functions on X such that A separate points of X strongly and is inversely closed, then X can be identified, in a natural way, with the Gelfand ideal space of A.*

COROLLARY 2.2: *Let X be any set and A any algebra with identity of bounded F-valued functions on X such that A separate points of X strongly and is inversely closed. If F is locally compact, then X is dense in $\mathcal{M}'$ of A with respect to the Gelfand topology.*

PROOF: F being locally compact, $\mathcal{M}'$ is compact. Let $\bar{X}$ denote the closure of X in $\mathcal{M}'$. An application of Theorem 2.1 to the algebra $\tilde{A} = \{\tilde{f} : f \in A,\ \tilde{f} = \hat{f} \mid \bar{X}\}$, $\hat{f}$ is the Gelfand transform of f, completes the proof.

Let $\hat{\beta}(X)$ denote a non-archimedean Stone–Cech compactification of a zero-dimensional Hausdorff space (see [2], p. 121). The following easily proved lemma which is well known in the classical case does not seem to have been stated explicitly in the literature.

LEMMA 2.3: *Let $\hat{\beta}(X)$ be a non-archimedean Stone–Cech compactification of a zero-dimensional Hausdorff space X. Then we have*

i) *X is dense in $\hat{\beta}(X)$;*

ii) *every F-valued bounded continuous function f on X has a unique extension f^* to a F-valued continuous function on $\hat{\beta}(X)$;*

iii) *$\hat{\beta}(X)$ is unique in the sense that if Y is another compact zero-dimensional Hausdorff space possessing properties* i) *and* ii) *then Y and $\hat{\beta}(X)$ are homeomorphic.*

Corollary 2.2 and Lemma 2.3 yield

COROLLARY 2.4: *If X is a zero-dimensional Hausdorff space and F is locally compact, then the Gelfand ideal space $\mathcal{M}'$ of $C(X;F)$ is the non-archimedean Stone–Cech compactification $\beta(X)$ of X.*

Note. $\mathrm{Lip}(X,d)$, $\mathrm{lip}(X,d)$ are inversely closed and separate points of X strongly and so Theorem 3.7 of [4] is immediate from Corollary 2.2. Incidentally Theorem 2.1 and Corollary 2.2 are improvements of Theorems 3.5, 3.6 of [4] and serve as non-archimedean analogues (see [8], p. 55).

We know that a family A of functions on a topological space X is a regular family if A separate closed sets and points outside them. Consider for any F-valued function f on X, the function f_1 defined by

$$f_1(x) = \begin{cases} f(x) & \text{if } |f(x)| < 1 \\ 1 & \text{if } |f(x)| \geq 1. \end{cases}$$

The following theorem which is an improvement of Theorem 4.3 of [4], gives a sufficient condition under which an algebra of F-valued functions on a compact zero-dimensional Hausdorff space is regular.

THEOREM 2.5: *Let X be a compact zero-dimensional Hausdorff space and A an algebra with identity of continuous functions on X such that A separate points of X strongly. If $f_1 \in A$ for each $f \in A$, then A is a regular family of functions on X.*

PROOF: Let $H \subset X$ be closed and $x_0 \in X \backslash H$. By the non-archimedean Stone–Weierstrass theorem ([10], p. 161, Theorem 2) A is dense in $C(X;F)$. X being zero-dimensional there exists a clopen neighbourhood U of x_0 such that $U \cap H = \emptyset$ so that $\chi_{X\backslash U} \in C(X;F)$. By denseness of A, we have $f \in A$ such that $\|\chi_{X\backslash U} - f\|_\infty < 1$. Since $f_1 \in A$, $g = (f_1 - 1)/(f_1(x_0) - 1)$ is also in A and meets the requirements.

It is easily seen that $f_1 \in \mathrm{Lip}(X,d)$ ($\mathrm{lip}(X,d)$, $C(X;F)$) for each $f \in \mathrm{Lip}(X,d)$ ($\mathrm{lip}(X,d)$, $C(X;F)$). Consequently we have

COROLLARY 2.6: *If F is locally compact,* i) $\mathrm{Lip}(X,d)$, $\mathrm{lip}(X,d)$ *for any ultrametric space (X,d) and* ii) $C(X;F)$ *for any zero-dimensional Hausdorff space are weakly regular Banach algebras.*

Note. It is not known whether $\mathrm{lip}(X,d)$ is always regular or not in the classical case, while in the non-archimedean case it is always weakly regular if F is locally compact.

Let A be a weakly regular strongly semi-simple (see [4], p. 18) non-archimedean Banach algebra with identity. If K is any closed subset of $\mathcal{M}'$, the Silov ideal $J(K)$ associated to K is defined as the closure of the set of all elements of A whose Gelfand transform vanish in a clopen neighbourhood of K. Clearly $J(K)$ is a closed ideal. For any ideal I of A, let $h_G(I)$ be the collection of all Gelfand ideals containing I. If $M(K) = \{x \in A : \hat{x}(K) = 0\}$, where K is a closed subset of $\mathcal{M}'$, then $M(K)$ is the largest closed ideal with $h_G(M(K)) = K$. We observe that $h_G(J(K)) = K$ and an element $x \in A$ is in $J(K)$ if and only if there is a sequence $\{x_n\}$ in A such that i) $\hat{x}_n = \hat{x}$ in a clopen neighbourhood of K and ii) $\|x_n\| \to 0$, $n \to \infty$. We shall now prove that $J(K)$ is the smallest closed ideal with $h_G(J(K)) = K$. We start with the following interesting lemma.

LEMMA 2.7: *A and $\hat{A} = \{\hat{x} : x \in A\}$ are isomorphic algebras. If, further F is locally compact and A is inversely closed, we have*

i) *all the maximal ideals of $\hat{A}$ are of the form $M_\varphi = \{\hat{f} \in \hat{A} : \hat{f}(\varphi) = 0\}$, $\varphi \in \mathcal{M}'$ of A.*

ii) *if I and $\hat{I}$ are the corresponding ideals of A and $\hat{A}$ and $h_G(I) = K$, then $h(\hat{I}) = K$, where $h(\hat{I})$ is the collection of all maximal ideals containing $\hat{I}$.*

iii) *$\hat{A}$ is a regular non-archimedean normed algebra so that the hull-kernel and Gelfand topology are the same on $\mathcal{M}'$ of $\hat{A}$.*

PROOF: Obviously, the Gelfand transform gives an isomorphism of the algebras A and $\hat{A}$. i) follows from Theorem 2.1. By i) $h(\hat{I}) \subset \mathcal{M}'$. If M is any maximal ideal of $\hat{A}$ containing $\hat{I}$, since $M = M_\varphi$, for some $\varphi \in \mathcal{M}'$, all $\hat{f} \in \hat{I}$ vanish at φ. i.e. $I \subset \varphi$ so that $\varphi \in h_G(I)$. i.e. $h(\hat{I}) \subset h_G(I)$. Again let $\varphi \in h_G(I)$, then $\hat{f}(\varphi) = 0$, $f \in I$. i.e. $I \subset M_\varphi$ so that $h_G(I) \subset h(\hat{I})$, in other words ii) is established. iii) is a consequence of weak regularity of A.

The following theorem which leads to the desired result regarding $J(K)$, can be proved using Lemma 2.7 on lines, similar to that of its classical counter part (see [8], Lemma 25C, p. 84). We omit the proof.

THEOREM 2.8: *Let F be locally compact and K_1, K_2 be two disjoint closed subsets of $\mathcal{M}'$. If A is inversely closed, there exists $x \in A$ such that $\hat{x}(K_1) = 1$ and $\hat{x}(K_2) = 0$. In fact, any closed ideal I of A with $h_G(I) = K_2$ contains such an x.*

COROLLARY 2.9: *If K is a closed subset of $\mathcal{M}'$, then $J(K)$ is the smallest closed ideal such that $h_G(J(K)) = K$, when F is locally compact.*

PROOF: It suffices to prove that $J(K)$ is the smallest. Let I be any other closed ideal such that $h_G(I) = K$. If x be such that $\hat{x}$ vanish in a clopen neighbourhood of K, then support C of $\hat{x}$ is compact and $C \cap K = \varnothing$. By Theorem 2.8, there exists $e \in I$ such that $\hat{e}(C) = 1$ and $\hat{e}(K) = 0$. i.e. $(ex - x)\hat{} = 0$. A being strongly semisimple $ex = x$, i.e. $x \in I$. The proof is complete.

REMARK 2.10: Corollary 2.9 is the non-archimedean version of Silov's theorem [13].

An ideal I of a non-archimedean Banach algebra is said to be G-primary (G-primary at $\varphi \in \mathscr{M}'$) if $h_G(I)$ is a singleton (if $h_G(I) = \{\varphi\}$). If $h_G(I) = K$, the G-primary component of I at $\varphi \in K$, is defined as the smallest closed G-primary ideal at φ containing I and is denoted by $P_G(I, \varphi)$. If $I^* = \bigcap_{\varphi \in K} P_G(I, \varphi)$, then I^* is a closed ideal containing I and $h_G(I^*) = K$.

DEFINITION 2.11: A non-archimedean weakly regular Banach algebra A is said to have the G-ideal intersection property if $I = I^*$ for every closed ideal I of A.

We give below some examples of non-archimedean Banach algebras having G-ideal intersection property.

EXAMPLE 1: Let X be a compact zero-dimensional Hausdorff space and F be locally compact. $C(X; F)$ has the G-ideal intersection property (see [10], p. 154, Theorem 3, p. 155, Theorem 4).

EXAMPLE 2: If $p = \{p_k\}$ be a bounded sequence of real numbers, then $c_0(p) = \{x = \{x_k\} : x_k \in F,\ |x_k|^{p_k} \to 0,\ k \to \infty\}$ is a metric linear space, where the metric is defined by the paranorm $\|x\| = \sup\{|x_0|, |x_k|^{p_k}, k \geq 1\}$. $c_0(p)$ has G-ideal intersection property.

EXAMPLE 3: This is due to Schikhof and it has no classical analogue (see [8], 37.3, p. 149). For further details see [11]. Let H be a locally compact, zero-dimensional Hausdorff topological group and $C_\infty(H \to F)$ be the collection of all μ-integrable functions on H (μ is the left (right) Haar integral). If $f, g \in C_\infty(H \to F)$, then $f * g(x) = \int f(xy) g(y^{-1}) d\mu(y)$ defines a convolution product on $C_\infty(H \to F)$ making it into a Banach algebra. This Banach algebra has G-ideal intersection property.

§3. Ideals in lip(X, d), Lip(X, d)

We shall assume throughout this section that F is locally compact. Suppose $f \in \text{lip}(X, d)$ is such that $K = \{x \in X : f(x) \neq 0\}$ is non-empty, then there exists (as can be easily seen) a sequence $\{f_n\}$ in lip(X, d) such that a) $f_n = f$ in a clopen neighbourhood of K and b) $\|f_n\| \to 0$, $n \to \infty$. Consequently we have i) for any closed ideal I with $h_G(I) = K \subset X$, $J(K) = I = M(K)$ and ii) if (X, d) is compact, every closed ideal of lip(X, d) is of the form $M(K)$ for a suitable subset K of (X, d). It may be noted here that ii) is known in the classical case for lip(X, d) only when lip(X, d) is regular (see [12], p. 251).

For any ideal I, I^2 stands for $\{\sum_{i=1}^{n} \alpha_i f_i g_i : f_i, g_i \in I, \alpha_i \in F, n \in \mathbb{N}\}$. I^2 and so $\bar{I}^2$ are ideals. For the study of the ideals $J(K)$ in Lip(X, d) we first observe that $\{f \in \text{Lip}(X, d)$: i) $f(x) = 0$, $x \in K$ and ii) $|f(x) - f(y)|/d(x, y) \to 0$ as $(x, y) \to K \times K\}$ is a closed subset of Lip(X, d), where K is a compact subset of (X, d). Using this observation we present a simpler proof of the following theorem concerning $J(K)$.

THEOREM 3.1: *Let K be a compact subset of (X, d). Then $f \in \text{Lip}(X, d)$ is in $J(K)$ if and only if f satisfies*

i) $f(x) = 0$, $x \in K$;

ii) $|f(x) - f(y)|/d(x, y) \to 0$ *as* $(x, y) \to K \times K$.

PROOF: If $f \in J(K)$, the observation implies that f satisfies both i) and ii). Let now $f \in \text{Lip}(X, d)$ and satisfy i) and ii). For each positive integer n, let U_n be a clopen neighbourhood of K such that 1) $|f(x)| < 1/n$, $x \in U_n$ and 2) $|f(x) - f(y)|/d(x, y) < 1/n$, $x, y \in U_n$, $x \neq y$. By compactness of K, there exist spheres $S_1, \ldots, S_{m(n)}$ such that $K \subset \bigcup_{i=1}^{m(n)} S_i \subset U_n$. Let $f'_n = f \cdot \sum_{i=1}^{m(n)} \chi_{S_i}$. On $\bigcup_{i=1}^{m(n)} S_i$, clearly $f'_n = f$ and $\|f'_n\|_\infty$, $\|f'_n\|_d$ are both less than $1/n$. Now f'_n can be extended to f_n on the whole of X by Theorem 1.2 such that $\|f_n\| < 1/n$. The sequence $\{f_n\}$ satisfy i) and ii) stated in the paragraph preceeding Lemma 2.7. The proof is complete.

The following result is important for the study of G-primary ideals.

THEOREM 3.2: *If characteristic of $F \neq 2$, then for each compact subset K of (X, d), $J(K) = \overline{M(K)^2}$.*

PROOF: That $\overline{M(K)^2} \subset J(K)$ can be proved as in the classical case

[12]. For the reverse inclusion, let $f \in J(K)$ be such that $\hat{f} \equiv 0$ in a clopen neighbourhood U of K. K being compact, there exists $g \in \mathrm{Lip}(X, d)$ such that $g = 0$ on K and $g = 1$ on $X \backslash U$. Clearly $g \in M(K)$ and so $g^2 \in M(K)^2$. Further $g^2 = 1$ on $X \backslash U$. Therefore $fg^2 = f$. As $M(K)^2$ is an ideal it follows that $f = fg^2 \in M(K)^2$. i.e. $J(K) \subset \overline{M(K)^2}$.

Consequently we have i) the closed primary ideals of $\mathrm{Lip}(X, d)$ at $x \in X$ are precisely the closed linear subspaces between $J(x)$ and $M(x)$; ii) if K is a closed subset of a compact ultrametric space then $J(K) = \bigcap_{x \in K} J(x)$ and iii) if (X, d) is a compact ultrametric space and I is any ideal of $\mathrm{Lip}(X, d)$ such that $h_G(I)$ is a clopen subset of (X, d), then I is the intersection of G-primary ideals at $x \in K$.

REMARK 3.3: From the results mentioned in the first paragraph of this section, it follows that if (X, d) is compact, $\mathrm{lip}(X, d)$ has G-ideal intersection property. It has been proved in the classical case by Lucien Waelbroeck [17], that when (X, d) is compact $\mathrm{Lip}(X, d)$ has ideal intersection property. The analogous result for the non-archimedean case is not known.

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(Oblatum 19-III-1980 & 1-VII-1981)

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