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Compositio Mathematica, tome 3 (1936), p. 304
<http://www.numdam.org/item?id=CM_1936__3__304_0>

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by

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Szeged

The problem dealt with in the following paper has been raised by P. Turán. To every point of the interval $0 \leqq x \leqq 1$ we adjoin a finite number of points of the same interval. This means, that we define a function $y = \varphi(x)$ for $0 \leqq x \leqq 1$ where $0 \leqq y \leqq 1$, $\varphi(x)$ takes a finite number of values for every x, and the equation $x = \varphi(x)$ is impossible. Two points x and y are called independent if neither of the two equations $y = \varphi(x)$ and $x = \varphi(y)$ holds.

I am going to prove the following theorem. We can find a set of points in the interval $0 \leqq x \leqq 1$ with the power of the continuum, so that any pair of its points is independent. The theorem, that there exist countably many points with the above property, has been proved by Mr. G. Grünwald [1]) in a quite elementary way, using a theorem of Ramsay [2]).

To every point x we adjoin an interval containing x and having endpoints with rational coordinates, in such a manner that all values of $\varphi(x)$ are situated outside of this interval. We assert that there is an interval which is adjoined to a set of points of the power of the continuum. This assertion follows immediately from the fact that intervals with rational endpoints are countably many, and from G. König's theorem, which states that the power of the continuum cannot be the sum of countably many smaller powers. The points to which this interval belongs are evidently independent, for the adjoined values are all outside the interval.

We see that we did not make use of the fact that $\varphi(x)$ has only a finite number of values for every x, but only of the fact that x is not a condensation point of the values of $\varphi(x)$.

(Received, October 11th, 1934.)

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[1]) The paper of Mr. G. Grünwald is to be published in a subsequent issue of the Matematikai és fizikai Lapok.

[2]) F. R. Ramsey, Collected papers. On a problem of formal logic, 82—111.