Cours de l'institut Fourier

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Preface

Cours de l'institut Fourier, tome 23 (1995), p. I-II
<http://www.numdam.org/item?id=CIF_1995__23__A1_0>

Preface

These notes are based on courses given in the fall of 1992 at the University of Leiden and in the spring of 1993 at the University of Grenoble. These courses were meant to elucidate the Mori point of view on classification theory of algebraic surfaces as briefly alluded to in [P].
The material presented here consists of a more or less self-contained advanced course in complex algebraic geometry presupposing only some familiarity with the theory of algebraic curves or Riemann surfaces. But the goal, as in the lectures, is to understand the Enriques classification of surfaces from the point of view of Mori-theory.
In my opininion any serious student in algebraic geometry should be acquainted as soon as possible with the yoga of coherent sheaves and so, after recalling the basic concepts in algebraic geometry, I have treated sheaves and their cohomology theory. This part culminated in Serre's theorems about coherent sheaves on projective space.
Having mastered these tools, the student can really start with surface theory, in particular with intersection theory of divisors on surfaces. The treatment given is algebraic, but the relation with the topological intersection theory is commented on briefly. A fuller discussion can be found in Appendix 2. Intersection theory then is applied immediately to rational surfaces.
A basic tool from the modern point of view is Mori's rationality theorem. The treatment for surfaces is elementary and I borrowed it from [Wi]. The student doesn't need all of the material in Chapter 4 to understand it, but at some point, it is very useful to have the Stein factorisation at one's disposal. This is the main reason to insert Chapter 4 before the material on the rationality theorem.
Right from the beginning I have adopted a dual point of view. A complex projective variety can be studied both from the complex-analytic as well from the commutative algebra point of view. For instance, I have treated coherent sheaves and their cohomology from the algebraic point of view, since this is the most elementary way to do. On the other hand, sometimes it is useful to be able to look at smaller sets than just affine open sets and then the complex topology is more natural. For instance, if you have a morphism $f : X \to Y$ between say smooth complex projective varieties, $f_*\mathcal{O}_X \cong \mathcal{O}_Y$ if and only if all fibres of f are connected, but this is hard to prove in the algebraic context (one needs the formidable theory of formal functions), but relatively elementary in the complex analytic context. It is in chapter 4 that the fruits of the dual point of view are reaped. The construction of the normalisation of a projective variety is easy from an algebraic point of view, but the proof of Zariski's main theorem etc. is much easier if you use complex topology. The subsequent treatment of Kodaira dimensions is not too hard and follows [U]. I also profited from Otto Forster's exposition on this subject in Bologna (I cherish my notes of the course he gave in Italian; I made use of the lecture delivered on 'Venerdi Santo 1980').
Of course, one must pay a price for this flexibility: the basic GAGA theorems have to be assumed so that one can switch between the two approaches at will. I have stated these theorems in an Appendix (without proofs).
Besides the rationality theorem one needs a few other specific aspects from the theory of surfaces that deal with fibrations and with families of curves. In §15 some general facts are treated and then, in the next section, an elementary treatment is given for the so-called canonical bundle formula for elliptic fibration (avoiding the use of relative duality; the latter is used for instance in [B-P-V] to arrive at the canonical bundle formula). Section 17 is the most sketchy one. The reason is that I did not have the time to treat deformation theory of curves in greater detail so that I had to invoke the local-triviality theorem of Grauert-Fischer instead. The 'Grand Final' is presented in section 18, a proof of the Enriques Classification theorem. After all the preparations

the proof becomes very short indeed.
It should be clear that most of the material presented is not very original. Chapter 3 has a large overlap with Arnaud Beauville's book [Beau]. Chapter 2 is adapted from [Ha], but I tried to simplify the treatment by restricting to projective varieties. This is rewarding, since then one does not need the abstract machinary of derived functors which, in my opininon, makes [Ha] hard to digest at times. For instance I have given a very elementary proof for the fact that the cohomology of coherent sheaves on a variety vanishes beyond its dimension. The final chapter borrows from [B-P-V], but again with simplifications as mentioned before. Needless to say I did not have time to treat the topic of surfaces exhaustively. Surfaces of general type and their geography could not be treated, nor the beautifully detailed theory of K3-surfaces and Enriques surfaces. Non algebraic surfaces all as well as phenomena particular to non zero characteristics are almost completely absent (I only give the Hopf surface as an example of a non-Kähler surface).
From the preceding description of the content of the course one might conclude that it nevertheless has been rather demanding for the pre-graduate students it was aimed at. I am glad they not only stayed untill the very end, but also contributed much to improve on this written exposition. I want to thank all of them, but in particular Robert Laterveer who very carefully read first drafts of this manuscript. I also want to thank José Bertin, Jean-Pierre Demailly and Gerardo Gonzalez-Sprinberg for useful conversations.

Grenoble, September 4, 1995

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