# P. SCHNEIDER <br> J. Teitelbaum <br> Correction to " $p$-adic boundary values" 

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# CORRECTION TO " $p$-ADIC BOUNDARY VALUES" 

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#### Abstract

Correction to the article " $p$-adic boundary values" published in Cohomologies p-adiques et applications arithmétiques (I), Astérisque 278 (2002), p. 51125.


Proposition 5.4 of the paper " $p$-adic boundary values" in the first volume (Cohomologies p-adiques et applications arithmétiques (I), Astérisque 278 (2002), p. 51-125) is incorrect. The proposition asserts that the map

$$
\begin{aligned}
& \nabla_{J}: \mathcal{O}(B)^{\mathfrak{b}_{J}^{>}=0} / \mathcal{O}(B)^{\mathfrak{b}_{J}=0} \xrightarrow{\cong} \mathcal{O}\left(U_{J}^{+} \cap B, M_{J}^{\prime}\right)^{\mathfrak{D}_{J}=0} \\
& f \quad \longmapsto \sum_{\mu \in B(J)}\left[\left(L_{\mu} f\right) \mid U_{J}^{+} \cap B\right] \otimes L_{\mu}^{*}
\end{aligned}
$$

is an isomorphism of Banach spaces. In fact, it is only injective, with dense image. However, its only application, the main theorem of the paper (Theorem 8.6), is still true. In the following we indicate the necessary changes. For the convenience of the reader, in the course of presenting these changes we reproduce here some portions of Proposition 5.4 and Lemma 5.5 from Section 5 of the original paper.

## A.

In section 5 of the paper Prop. 5.4 and Lemma 5.5 have to be replaced as follows:
Proposition 5.4. - The map

$$
\begin{aligned}
\nabla_{J}: \mathcal{O}(B)^{\mathfrak{b}_{J}^{\mathbf{J}}=0} / \mathcal{O}(B)^{\mathfrak{b} J=0} & \longrightarrow \\
f & \longmapsto \sum_{\mu \in B(J)}\left[\left(U_{J}^{+} \cap B, M_{J}^{\prime}\right)^{\mathfrak{D}_{J}=0}\right. \\
& \left.\longmapsto U_{J}^{+} \cap B\right] \otimes L_{\mu}^{*}
\end{aligned}
$$

is a continuous and injective map of Banach spaces.

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Proof. - For $\mathfrak{Z}=\sum_{\nu} \mathfrak{z}_{(\nu)} \otimes L_{\nu} \in \mathfrak{d}_{J} \subseteq U\left(\mathfrak{n}_{J}^{+}\right) \otimes_{K} M_{J}$ we have

$$
\begin{aligned}
\left\langle\mathfrak{Z}, \sum_{\mu}\left[\left(L_{\mu} f\right) \mid U_{J}^{+} \cap B\right] \otimes L_{\mu}^{*}\right\rangle & =\sum_{\mu, \nu} L_{\mu}^{*}\left(L_{\nu}\right) \cdot\left(\mathfrak{z}_{(\nu)} L_{\mu} f\right) \mid U_{J}^{+} \cap B \\
& =\left(\sum_{\nu} \mathfrak{z}(\nu) L_{\nu}\right) f \mid U_{J}^{+} \cap B=0
\end{aligned}
$$

since $\sum_{\nu} \mathfrak{z}(\nu) L_{\nu} \in U\left(\mathfrak{n}_{J}^{+}\right) \cap \mathfrak{b}_{J}^{>}$. Morover for $\mu \in B(J)$ we have $L_{\mu} \in \mathfrak{b}_{J}$. Hence the map $\nabla_{J}$ is well defined. It clearly is continuous. The Banach space on the left hand side of the assertion has the orthonormal basis $\pi^{-\ell(\nu)} f_{\nu} \mid B$ for $J(\nu)=J$. Concerning the right hand side we observe that the above pairing composed with the evaluation in 1 induces an injection

$$
\mathcal{O}\left(U_{J}^{+} \cap B\right) \underset{K}{\otimes} M_{J}^{\prime} \longleftrightarrow \operatorname{Hom}_{K}\left(U\left(\mathfrak{n}_{J}^{+}\right) \underset{K}{\otimes} M_{J}, K\right)
$$

which restricts to an injection

$$
\mathcal{O}\left(U_{J}^{+} \cap B, M_{J}^{\prime}\right)^{\mathfrak{o}_{J}=0} \Longleftrightarrow \operatorname{Hom}_{K}\left(\mathfrak{b}_{J} / \mathfrak{b}_{J}^{>}, K\right) .
$$

Hence the only weights which can occur in the right hand side are those $\nu$ with $J(-\nu)=J$ and the corresponding weight spaces are at most 1-dimensional. Moreover the same argument as after Prop. 2 shows that the occurring weight vectors (scaled appropriately) form an orthonormal basis. Since $\nabla_{J}$ visibly preserves weights the assertion follows once we show that

$$
\nabla_{J}\left(f_{\nu} \mid B\right) \neq 0 \text { for any } \nu \text { with } J(\nu)=J
$$

All that remains to be checked therefore is the existence, for a given $\nu$ with $J(\nu)=J$, of a $\mu \in B(J)$ such that $L_{\mu} f_{\nu}$ does not vanish identically on $U_{J}^{+} \cap B$.

The weight $\nu$ is of the form $\nu=\sum_{j=0}^{d} n_{j} \varepsilon_{j}$ with $n_{j}>0$ for $j \in J$ and $n_{j} \leqslant 0$ for $j \notin J$. We have

$$
\# J \leqslant \sum_{j \in J} n_{j}=-\sum_{j \notin J} n_{j} .
$$

Choose integers $n_{j} \leqslant m_{j} \leqslant 0$ for $j \notin J$ such that $\# J=-\sum_{j \notin J} m_{j}$ and define

$$
\mu:=\sum_{j \in J} \varepsilon_{j}+\sum_{j \notin J} m_{j} \varepsilon_{j} \in B(J) .
$$

Observe that $J(\nu-\mu) \subseteq J$ and $J(\mu-\nu) \cap J=\varnothing$. This means that $L_{\nu-\mu} \in U\left(\mathfrak{n}_{J}^{+}\right)$. It suffices to check that $L_{\nu-\mu} L_{\mu} f_{\nu}(1) \neq 0$. We compute

$$
\begin{aligned}
L_{\nu-\mu} L_{\mu} f_{\nu}(1) & =\operatorname{Res}_{(\bar{C}, 0)} \Xi_{-\nu} \cdot L_{\nu-\mu} L_{\mu} \xi \\
& =-\operatorname{Res}_{(\bar{C}, 0)} \Xi_{-\nu} \cdot L_{\nu-\mu}\left(\Xi_{\mu} \xi\right) .
\end{aligned}
$$

As a consequence of the formula ( + ) in section 4 we have $L_{\nu-\mu}\left(\Xi_{\mu} \xi\right)=m \cdot \Xi_{\nu} \xi$ for some nonzero integer $m$. Hence we obtain

$$
L_{\nu-\mu} L_{\mu} f_{\nu}(1)=-m \cdot \operatorname{Res}_{(\bar{C}, 0)} \xi= \pm m \neq 0 .
$$

In section 8 we will need more precise information about the image of the map $\nabla_{J}$. This means we have to investigate the norms of the elements $\nabla_{J}\left(\pi^{-\ell(\nu)} f_{\nu} \mid B\right)$ for $J(\nu)=J$. We first set some additional notation. Let

$$
I_{J}(\mu):=\left\{\underline{m} \in I(\mu): m_{i j}=0 \text { unless } i \in J \text { and } j \notin J\right\} \subseteq I(\mu)
$$

and set

$$
u(\underline{m}):=\prod_{i \in J, j \notin J} u_{i j}^{m_{i j}} \quad \text { for } \underline{m} \in I_{J}(\mu)
$$

where the $u_{i j}$ are the functions on $U_{J}^{+} \cap B$ which give the matrix entries. It is easy to see that $I_{J}(\mu)$ is nonempty if and only if $J(\mu) \subseteq J$ and $J(-\mu) \cap J=\varnothing$; this is the same as saying that $L_{\mu}$ belongs to $U\left(\mathfrak{n}_{J}^{+}\right)$. Note also that, for $\underline{m} \in I_{J}(\mu)$ with $\mu=\sum_{i=0}^{d} n_{i} \epsilon_{i}$, the "degree" of the monomial $u(\underline{m})$ is given by the formula

$$
\operatorname{deg}(u(\underline{m}))=\sum_{i \in J, j \notin J} m_{i j}=\sum_{i \in J} n_{i} .
$$

The spectral norm $\omega_{J}$ on the affinoid algebra $\mathcal{O}\left(U_{J}^{+} \cap B\right)$ is such that, if $\underline{m} \in I_{J}(\nu)$, then, by Lemma 1,

$$
\omega_{J}(u(\underline{m}))=\pi(\underline{m}) \geqslant \ell(\nu) .
$$

Therefore, if a general analytic function

$$
h=\sum_{\nu, \underline{m} \in I_{J}(\nu)} a(\underline{m}) u(\underline{m})
$$

is expanded in the $u(\underline{m})$ on $U_{J}^{+} \cap B$ then

$$
\omega_{J}(h)=\inf _{\nu, \underline{m} \in I_{J}(\nu)}\{\omega(a(\underline{m}))+\pi(\underline{m})\} .
$$

We also need a norm on $\mathcal{O}\left(U_{J}^{+} \cap B, M_{J}^{\prime}\right)$; to make later formulas work nicely, we define

$$
\omega_{J}\left(\sum_{\mu \in B(J)} h_{\mu} \otimes L_{\mu}^{*}\right):=\inf _{\mu}\left\{\omega_{J}\left(h_{\mu}\right)-\ell(-\mu)\right\}
$$

Lemma 5.5. - Given any weight $\nu$ such that $I_{J}(\nu)$ is nonempty, there exists $\underline{m} \in$ $I_{J}(\nu)$ such that $\pi(\underline{m})=\ell(\nu)$.
Proof. - As usual, set $\nu=\sum_{i=0}^{d} n_{i} \epsilon_{i}$. We will proceed by a double induction, on $d$ and on $|\nu|:=\sum_{i=0}^{d}\left|n_{i}\right|=2 \sum_{i \in J} n_{i}$. The base case is $d=1$. If $J=\varnothing$ or $\{0,1\}$, then $\nu=0$ and $\pi((0,0))=\ell(0)=0$. If $J=\{0\}$, then $\nu=m_{01}\left(\epsilon_{0}-\epsilon_{1}\right)$ with $m_{01} \geqslant 0$, and $\pi\left(\left(m_{01}, 0\right)\right)=\ell(\nu)=m_{01}$. If $J=\{1\}$, then $\nu=m_{10}\left(\epsilon_{1}-\epsilon_{0}\right)$ with $m_{10} \geqslant 0$, and $\pi\left(\left(0, m_{10}\right)\right)=\ell(\mu)=0$.

For the general case, consider the sequence of integers

$$
h_{d}(\nu):=n_{d}, h_{d-1}(\nu):=n_{d}+n_{d-1}, \ldots, h_{0}(\nu):=n_{d}+\cdots+n_{1}+n_{0}=0
$$

The integer $-\ell(\nu)$ is the smallest number on this list. We consider three cases: $n_{d}=0$, $n_{d}>0$, and $n_{d}<0$. If $n_{d}=0$, then we may treat the problem as a lower dimensional
one and by the inductive hypothesis we can find $\underline{m} \in I_{J}(\nu)$ with all $m_{i d}=m_{d j}=0$ such that $\pi(\underline{m})=\ell(\nu)$.

If $n_{d}>0$, then $d \in J(\mu) \subseteq J$. Let $i^{\prime}$ be the maximal index between 0 and $d$ such that $n_{i^{\prime}}<0$. Let $j^{\prime}$ be maximal so that $h_{j^{\prime}}(\nu)=-\ell(\nu)$. We claim that $i^{\prime} \geqslant j^{\prime}$. If not, then $-\ell(\nu)=h_{j^{\prime}}(\nu)>0$, which is impossible. Let $\nu_{0}:=\nu+\epsilon_{i^{\prime}}-\epsilon_{d}$. Then $h_{i}\left(\nu_{0}\right)=h_{i}(\nu)$ for all $0 \leqslant i \leqslant i^{\prime}$, and $h_{i}(\nu)-1=h_{i}\left(\nu_{0}\right) \geqslant 0$ for all $i>i^{\prime}$. Therefore $\ell(\nu)=\ell\left(\nu_{0}\right)$. Also, $J\left(\nu_{0}\right) \subseteq J(\nu)$ and $J\left(-\nu_{0}\right) \subseteq J(-\nu)$. We may therefore apply the inductive hypothesis to find $\underline{m}^{0} \in I_{J}\left(\nu_{0}\right)$ so that $\pi\left(\underline{m}^{0}\right)=\ell\left(\nu_{0}\right)$. If we define $\underline{m}$ by setting $m_{d i^{\prime}}:=m_{d i^{\prime}}^{0}+1$, and all other $m_{i j}:=m_{i j}^{0}$, then we obtain an $\underline{m} \in I_{J}(\nu)$ with $\pi(\underline{m})=\pi\left(\underline{m}^{0}\right)$. This solves the problem in this case.

If $n_{d}<0$, then let $i^{\prime}$ be the minimal index between 0 and $d$ such that $n_{i^{\prime}}>0$. Let $j^{\prime}$ be minimal so that $h_{j^{\prime}}(\nu)=-\ell(\nu)$. We claim that $i^{\prime}<j^{\prime}$; otherwise $0=h_{0}(\nu) \leqslant$ $h_{j^{\prime}}(\nu)$, forcing $\ell(\nu)=0$. But we know that $\ell(\nu) \geqslant-n_{d}>0$. Now let $\nu_{0}:=\nu+\epsilon_{d}-\epsilon_{i^{\prime}}$. Since all $i$ such that $h_{i}(\nu)=-\ell(\nu)$ satisfy $d \geqslant i>i^{\prime}$ it follows that $\ell\left(\nu_{0}\right)=\ell(\nu)-1$. Again we have modified $\nu$ to obtain $\nu_{0}$ in such a way that $I_{J}\left(\nu_{0}\right)$ is nonempty, and so we may apply the inductive hypothesis to find a $\underline{m}^{0} \in I_{J}\left(\nu_{0}\right)$ with $\pi\left(\underline{m}^{0}\right)=\ell\left(\nu_{0}\right)$. Defining this time $\underline{m}$ by setting $m_{i^{\prime} d}:=m_{i^{\prime} d}^{0}+1$, and all other $m_{i j}:=m_{i j}^{0}$, we obtain a $\underline{m} \in I_{J}(\nu)$ with $\pi(\underline{m})=\pi\left(\underline{m}^{0}\right)+1$. This completes the proof.

Since the $f_{\nu}$ and $L_{\mu} f_{\nu}$, for $J(\nu)=J$ and $\mu \in B(J)$, are homogeneous of weights $-\nu$ and $\mu-\nu$ respectively, we may expand each of these functions as polynomials in the $u_{i j}$ of the correct weights:

$$
\begin{aligned}
f_{\nu} \mid\left(U_{J}^{+} \cap B\right) & =\sum_{\underline{m} \in I_{J}(\nu)} c_{\nu}(\underline{m}) u(\underline{m}) \\
L_{\mu} f_{\nu} \mid\left(U_{J}^{+} \cap B\right) & =\sum_{\underline{m} \in I_{J}(\nu-\mu)} b_{\nu, \mu}(\underline{m}) u(\underline{m})
\end{aligned}
$$

Lemma 5.6. - The coefficients $c_{\nu}(\underline{m})\left(\underline{m} \in I_{J}(\nu)\right)$ and $b_{\nu, \mu}(\underline{m})\left(\underline{m} \in I_{J}(\nu-\mu)\right)$ are given by

$$
c_{\nu}(\underline{m})= \pm \prod_{j \notin J} \frac{\left(\sum_{i \in J} m_{i j}\right)!}{\prod_{i \in J} m_{i j}!}, \quad b_{\nu, \mu}(\underline{m})= \pm \prod_{j \notin J} \frac{\left(-n_{j}(\nu)\right)!}{\left(-n_{j}(\mu)\right)!\cdot \prod_{i \in J} m_{i j}!}
$$

where $\mu=\sum_{j=0}^{d} n_{j}(\mu) \epsilon_{j}$ and $\nu=\sum_{j=0}^{d} n_{j}(\nu) \epsilon_{j}$ as usual.
Proof. - A simple computation shows that, for $i \in J$ and $j \notin J, L_{i j}$ acts on $\mathcal{O}\left(U_{J}^{+} \cap B\right)$ by

$$
L_{i j} h=\frac{\partial h}{\partial u_{i j}} .
$$

By Taylor's formula,

$$
c_{\nu}(\underline{m})=\left(\left(\prod_{i \in J, j \notin J} \frac{1}{m_{i j}!} L_{i j}^{m_{i j}}\right) f_{\nu}\right)(1) .
$$

Arranging things correctly, formula (+) in section 4 gives

$$
\prod_{i \in J, j \notin J} L_{i j}^{m_{i j}} \equiv \pm\left(\prod_{j \notin J}\left(\sum_{i \in J} m_{i j}\right)!\right) L_{\nu} \bmod \mathfrak{b}
$$

Since $L_{\nu} f_{\nu}(1)= \pm 1$, and $f_{\nu}$ is killed by $\mathfrak{b}$, this gives the formula for $c_{\nu}(\underline{m})$.
The formula for $b_{\nu, \mu}(\underline{m})$ is found by a similar method. First we have, for $\underline{m} \in$ $I_{J}(\nu-\mu)$, that

$$
\begin{aligned}
b_{\nu, \mu}(\underline{m}) & =\left(\left(\prod_{i \in J, j \notin J} \frac{1}{m_{i j}!} L_{i j}^{m_{i j}}\right) L_{\mu} f_{\nu}\right)(1) \\
& = \pm \prod_{j \notin J} \frac{\left(\sum_{i \in J} m_{i j}\right)!}{\prod_{i \in J} m_{i j}!} \cdot\left(L_{\nu-\mu} L_{\mu} f_{\nu}\right)(1)
\end{aligned}
$$

But it is easy to see, using formula (+) in section 4 again, that

$$
L_{\nu-\mu} L_{\mu} \equiv \pm\left(\prod_{j \notin J}\binom{-n_{j}(\nu)}{-n_{j}(\mu)}\right) L_{\nu} \bmod \mathfrak{b}
$$

where the binomial coefficient $\binom{-n_{j}(\nu)}{-n_{j}(\mu)}$ should be interpreted as one unless $0 \leqslant$ $-n_{j}(\mu) \leqslant-n_{j}(\nu)$. It remains to note that $\sum_{i \in J} m_{i j}=-n_{j}(\nu)+n_{j}(\mu)$ for $j \notin J$.

Corollary 5.7. - We have the estimate

$$
\omega\left(b_{\nu, \mu}(\underline{m})\right) \leqslant \frac{\omega(p)}{p-1}\left(\sum_{i \in J} n_{i}(\nu)\right)
$$

Proof. - The integer $b_{\nu, \mu}(\underline{m})$ is obtained from a fraction with numerator

$$
\prod_{j \notin J}\left(-n_{j}(\nu)\right)!
$$

so

$$
\omega\left(b_{\nu, \mu}(\underline{m})\right) \leqslant \sum_{j \notin J} \frac{\omega(p)\left(-n_{j}(\nu)\right)}{p-1} \leqslant \frac{\omega(p)}{p-1}\left(-\sum_{j \notin J} n_{j}(\nu)\right)=\frac{\omega(p)}{p-1}\left(\sum_{i \in J} n_{i}(\nu)\right) .
$$

Proposition 5.8. - Suppose that $\nu=\sum n_{i} \epsilon_{i}$ and that $J(\nu)=J$; then

$$
\omega_{J}\left(\nabla_{J}\left(\pi^{-\ell(\nu)} f_{\nu} \mid B\right)\right) \leqslant \frac{\omega(p)}{p-1}\left(\sum_{i \in J} n_{i}\right)
$$

Proof. - Suppose $\mu$ is chosen as in the proof of Prop. 4 so that $I_{J}(\nu-\mu)$ is nonempty. Then, using Lemma 5 and Cor. 7, we have

$$
\begin{aligned}
\omega_{J}\left(\left(L_{\mu} f_{\nu}\right) \mid U_{J}^{+} \cap B\right) & =\inf _{\underline{m} \in I_{J}(\nu-\mu)}\left\{\omega\left(b_{\nu, \mu}(\underline{m})\right)+\pi(\underline{m})\right\} \\
& \leqslant \ell(\nu-\mu)+\frac{\omega(p)}{p-1}\left(\sum_{i \in J} n_{i}\right)
\end{aligned}
$$

Hence

$$
\begin{aligned}
\omega_{J}\left(\nabla_{J}\left(\pi^{-\ell(\nu)} f_{\nu} \mid B\right)\right) & =\inf _{\mu^{\prime} \in B(J)}\left\{\omega_{J}\left(\left(\pi^{-\ell(\nu)} L_{\mu^{\prime}} f_{\nu}\right) \mid U_{J}^{+} \cap B\right)-\ell\left(-\mu^{\prime}\right)\right\} \\
& \leqslant \ell(\nu-\mu)+\frac{\omega(p)}{p-1}\left(\sum_{i \in J} n_{i}\right)-\ell(\nu)-\ell(-\mu) \\
& \leqslant \frac{\omega(p)}{p-1}\left(\sum_{i \in J} n_{i}\right)
\end{aligned}
$$

where for the last inequality we use the subadditivity of the function $\ell$ from the proof of Lemma 1.

Remark 5.9. - If $\nu=\sum n_{i} \epsilon_{i}$ such that $J(\nu)=\{0, \ldots, j-1\}$ for some $0 \leq j \leq d$ (this is the case of interest in section 8) then $\sum_{i \in J} n_{i}=\ell(\nu)$ so that

$$
\omega_{J}\left(\nabla_{J}\left(\pi^{-\ell(\nu)} f_{\nu} \mid B\right)\right) \leqslant \frac{\omega(p)}{p-1} \ell(\nu)
$$

Using Prop. 8 we will be able to show that "sufficiently overconvergent" $M_{J}^{\prime}$-valued power series on $U_{J}^{+} \cap B$ that are killed by $\mathfrak{d}_{J}$ belong to the image of the map $\nabla_{J}$.

More precisely, let $y_{J}$ be the diagonal matrix in $G$ with $\pi$ in the $i^{\text {th }}$ entry when $i \in J$, and 1 in the other entries. Fix an integer $x>\omega(p) /(p-1)$. Then $y_{J}^{-x}\left(U_{J}^{+} \cap B\right) y_{J}^{x}$ is an affinoid polydisk containing $U_{J}^{+} \cap B$. We let $\omega_{J, x}$ denote the spectral norm on the affinoid algebra $\mathcal{O}\left(y_{J}^{-x}\left(U_{J}^{+} \cap B\right) y_{J}^{x}\right)$ and define on $\mathcal{O}\left(y_{J}^{-x}\left(U_{J}^{+} \cap B\right) y_{J}^{x}, M_{J}^{\prime}\right)$ the Banach norm

$$
\omega_{J, x}\left(\sum_{\mu \in B(J)} h_{\mu} \otimes L_{\mu}^{*}\right):=\inf _{\mu}\left\{\omega_{J, x}\left(h_{\mu}\right)-\ell(-\mu)\right\}
$$

Thus there is a continuous and injective restriction map

$$
r_{x}: \mathcal{O}\left(y_{J}^{-x}\left(U_{J}^{+} \cap B\right) y_{J}^{x}, M_{J}^{\prime}\right) \longrightarrow \mathcal{O}\left(U_{J}^{+} \cap B, M_{J}^{\prime}\right)
$$

We set

$$
E_{J, x}:=r_{x}\left(\mathcal{O}\left(y_{J}^{-x}\left(U_{J}^{+} \cap B\right) y_{J}^{x}, M_{J}^{\prime}\right)\right) \cap \mathcal{O}\left(U_{J}^{+} \cap B, M_{J}^{\prime}\right)^{\mathfrak{d}_{J}=0}
$$

and view it as a $K$-Banach space with respect to the norm induced by $\omega_{J, x}$. Hence the inclusion $E_{J, x} \subset \mathcal{O}\left(U_{J}^{+} \cap B, M_{J}^{\prime}\right)$ is continuous but not a topological embedding.

Lemma 5.10. - The map $\nabla_{J}$ restricts to a bijection $\nabla_{J}^{-1}\left(E_{J, x}\right) \xrightarrow{\sim} E_{J, x}$ whose inverse viewed as a map

$$
E_{J, x} \longrightarrow \mathcal{O}(B)^{\mathfrak{b}_{J}^{\mathfrak{J}}=0} / \mathcal{O}(B)^{\mathfrak{b}_{J}=0}
$$

is continuous.
Proof. - If $h$ is in the image of $r_{x}$, then it has an expansion

$$
h=\sum_{\mu \in B(J)} h_{\mu} \otimes L_{\mu}^{*}
$$

and each $h_{\mu}$ has an expansion

$$
h_{\mu}=\sum_{\nu, \underline{m} \in I_{J}(\nu-\mu)} a_{\mu}(\underline{m}) u(\underline{m})
$$

with

$$
\omega\left(a_{\mu}(\underline{m})\right)+\pi(\underline{m})-x \sum_{i \in J, j \notin J} m_{i j}=\omega\left(a_{\mu}(\underline{m})\right)+\pi(\underline{m})-x \sum_{i \in J} n_{i}(\nu-\mu) \longrightarrow \infty
$$

writing, as before, each weight $\nu=\sum_{i=0}^{d} n_{i}(\nu) \epsilon_{i}$. On the other hand, because $h$ satisfies the differential equations, we know that the homogeneous components of $h$ are multiples of the corresponding $\nabla_{J}\left(\pi^{-\ell(\nu)} f_{\nu} \mid B\right)$ where $J(\nu)=J$. In fact, then, we have

$$
\sum_{\mu \in B(J)}\left(\sum_{\underline{m} \in I_{J}(\nu-\mu)} a_{\mu}(\underline{m}) u(\underline{m})\right) \otimes L_{\mu}^{*}=c_{\nu} \nabla_{J}\left(\pi^{-\ell(\nu)} f_{\nu} \mid B\right)
$$

for some $c_{\nu} \in K$, whereas the other coefficients $a_{\mu}(\underline{m})$ vanish. Now we see that

$$
\begin{aligned}
\omega\left(c_{\nu}\right)= & \inf _{\mu \in B(J)}\left(\inf _{\underline{m} \in I_{J}(\nu-\mu)}\left\{\omega\left(a_{\mu}(\underline{m})\right)+\pi(\underline{m})\right\}-\ell(-\mu)\right)-\omega_{J}\left(\nabla_{J}\left(\pi^{-\ell(\nu)} f_{\nu} \mid B\right)\right) \\
\geqslant & \inf _{\mu \in B(J)}\left(\inf _{\underline{m} \in I_{J}(\nu-\mu)}\left\{\omega\left(a_{\mu}(\underline{m})\right)+\pi(\underline{m})-\frac{\omega(p)}{p-1} \sum_{i \in J} n_{i}(\nu)\right\}-\ell(-\mu)\right) \\
= & \inf _{\mu \in B(J)}\left(\inf _{\underline{m} \in I_{J}(\nu-\mu)}\left\{\omega\left(a_{\mu}(\underline{m})\right)+\pi(\underline{m})-\frac{\omega(p)}{p-1} \sum_{i \in J} n_{i}(\nu-\mu)\right\}-\ell(-\mu)\right) \\
& -\frac{\omega(p) \cdot \sharp J}{p-1} \\
\geqslant & \inf _{\mu \in B(J)}\left(\inf _{\underline{m} \in I_{J}(\nu-\mu)}\left\{\omega\left(a_{\mu}(\underline{m})\right)+\pi(\underline{m})-x \sum_{i \in J} n_{i}(\nu-\mu)\right\}-\ell(-\mu)\right) \\
& -\frac{\omega(p) \cdot \sharp J}{p-1} \\
& \left.-\infty \text { as } \nu \longrightarrow \infty \text { (i.e. as } \sum_{i \in J} n_{i}(\nu) \longrightarrow \infty\right) .
\end{aligned}
$$

Then $f:=\sum_{J(\nu)=J} c_{\nu} \pi^{-\ell(\nu)} f_{\nu} \mid B$ converges in $\mathcal{O}(B)^{\mathfrak{b}_{J}^{>}=0}$ and satisfies $\nabla_{J}(f)=h$. Moreover

$$
\omega_{B}(f)=\inf _{J(\nu)=J} \omega\left(c_{\nu}\right) \geq \omega_{J, x}(h)-\frac{\omega(p) \cdot \sharp J}{p-1}
$$

Hence the map $\nabla_{J}^{-1}\left(E_{J, x}\right) \xrightarrow{\nabla_{J}} E_{J, x}$ is a bijection whose inverse is continuous.
We observe that, since the $\left(L_{\mu} f_{\nu}\right) \mid U_{J}^{+} \cap B$ are polynomials in the $u_{i j}$, the weight vectors $\sum_{\mu \in B(J)}\left[\left(L_{\mu} f_{\nu}\right) \mid U_{J}^{+} \cap B\right] \otimes L_{\mu}^{*}$, for $J(\nu)=J$, do lie in $E_{J, x}$.

As a consequence of this discussion (mainly Prop. 3 and Lemma 10) we in particular have the following map.

Lemma 5.11. - There is a unique continuous linear map

$$
D_{J}: E_{J, x} \longrightarrow\left[\Omega_{b}^{d}\left(U^{0}\right)^{j} / \Omega_{b}^{d}\left(U^{0}\right)^{j+1}\right]^{\prime}
$$

where $j:=\# J$, which sends the weight vector $\sum_{\mu \in B(J)}\left[\left(L_{\mu} f_{\nu}\right) \mid U_{J}^{+} \cap B\right] \otimes L_{\mu}^{*}$, for $\nu$ with $J(\nu)=J$, to the linear form $\lambda_{\nu}(\eta):=\operatorname{Res}_{(\bar{C}, 0)} \Xi_{-\nu} \eta$.

## B.

In section 8 of the paper in the proof of Theorem 8.6 the six sentences on p. 120 starting "We rephrase ..." have to be replaced as follows:

Finally, in order to be able to apply Lemma 5.11, we have to reduce to the case where $\varepsilon$ is "sufficiently overconvergent". Fix an integer $x>\omega(p) /(p-1)$ and find coset representatives $v_{i}$ for $U_{\underline{j}}^{(0)} / U_{\underline{j}}^{(x)}$. Then

$$
B P_{\underline{j}}=\left(U_{\underline{j}}^{+} \cap B\right) P_{\underline{j}}=\bigcup_{i} v_{i} y^{x}\left(U_{\underline{j}}^{+} \cap B\right) P_{\underline{j}}
$$

Now define

$$
f_{i}:=\left(\left(v_{i} y^{x}\right)^{-1} f\right) \mid B P_{\underline{j}} \text { extended by zero to } G
$$

Then

$$
f=\sum_{i} v_{i} y^{x} f_{i}
$$

Now, if $u \in U_{\underline{j}}^{+} \cap B$, then

$$
f_{i}(u)=f\left(v_{i} y^{x} u\right)=\pi^{j x} f\left(v_{i} y^{x} u y^{-x}\right)=\pi^{j x} \varepsilon\left(v_{i} y^{x} u y^{-x}\right) \otimes \varphi_{\mathrm{o}}
$$

If we write $f_{i} \mid U_{\underline{j}}^{+} \cap B=\varepsilon_{i} \otimes \varphi_{\mathrm{o}}$, then $\varepsilon_{i}(u)=\pi^{j x} \varepsilon\left(v_{i} y^{x} u y^{-x}\right)$ is not only analytic, it is visibly the restriction of the analytic function $\pi^{j x} \varepsilon\left(v_{i} y^{x} \cdot y^{-x}\right)$ from the larger disk $y^{-x}\left(U_{\underline{j}}^{+} \cap B\right) y^{x}$. Hence $\varepsilon_{i} \in E_{\underline{j}, x}$. Treating the $f_{i}$ separately we are reduced to considering a function $f$ such that
$-f$ is supported on $B P_{\underline{j}}$ with $f \mid U_{\underline{j}}^{+} \cap B=\varepsilon \otimes \varphi_{\mathrm{o}}$ for some $\varepsilon \in E_{\underline{j}, x}$.
We rephrase the above discussion in the following way. We have the linear map

$$
\operatorname{Ext}_{\underline{j}}: \mathcal{O}\left(U_{\underline{j}}^{+} \cap B, M_{\underline{j}}^{\prime}\right)^{)_{\underline{\mathfrak{j}}}=0} \longrightarrow C^{\mathrm{an}}\left(G, P_{\underline{j}} ; \operatorname{Hom}_{K}\left(M_{\underline{j}}, \mathrm{St}_{d+1-j}\right)\right)^{\mathfrak{o}_{\underline{j}}=0}
$$

defined by

$$
\operatorname{Ext}_{\underline{j}}(\varepsilon)(g):= \begin{cases}h^{-1}\left(\varepsilon(u) \otimes \varphi_{\mathrm{o}}\right) & \text { for } g=u h \text { with } u \in U_{\underline{j}}^{+} \cap B, h \in P_{\underline{j}} \\ 0 & \text { otherwise }\end{cases}
$$

Starting with the natural bilinear map

$$
\operatorname{Hom}_{K}\left(M_{\underline{j}}^{\prime}, \operatorname{Hom}_{K}\left(M_{\underline{j}}, \mathrm{St}_{d+1-j}\right)\right) \times M_{\underline{j}}^{\prime} \longrightarrow \operatorname{Hom}_{K}\left(M_{\underline{j}}, \mathrm{St}_{d+1-j}\right)
$$

an argument analogous to the proof of [Fea] 4.3.1 shows that Ext $\underline{j}_{\underline{j}}$ is continuous. The image $\operatorname{Ext}_{\underline{j}}\left(E_{\underline{j}, x}\right)$ generates the target of $\operatorname{Ext}_{\underline{j}}$ (algebraically) as a $G$-representation.

On the other hand, in section 6 after Lemma 4 we had constructed a continuous linear map

$$
D_{\underline{j}}: E_{\underline{j}, x} \longrightarrow\left[\Omega^{d}(X)^{j} / \Omega^{d}(X)^{j+1}\right]^{\prime}
$$

The surjectivity of $I^{[j]}$ therefore will follow from the identity

$$
\operatorname{Ext}_{\underline{j}}=I^{[j]} \circ D_{\underline{j}} .
$$

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