Astérisque

THÉORIE DE L'HOMOTOPIE

Colloque CNRS-NSF-SMF au C.I.R.M.

du 11 au 15 juillet 1988

organisé par

Haynes R. MILLER (M.I.T.), Jean-Michel LEMAIRE (Nice)

Lionel SCHWARTZ (Paris-sud), Michel ZISMAN (Paris VII)

INTRODUCTION

On trouvera dans ce volume les articles issus du colloque international sur la théorie de l'homotopie qui s'est tenu au Centre International de Rencontres Mathématiques de Luminy du 11 au 15 juillet 1988.

La théorie de l'homotopie a connu dans les années 80 des développements spectaculaires; le but du colloque était de faire le point sur un certain nombre de ces développements et leurs perspectives: deux thèmes principaux ont été particulièrement évoqués pendant le colloque et se retrouvent dans la plupart des articles de ce volume:

- l'étude de l'homotopie des espaces fonctionnels, notamment lorsque la source est l'espace classifiant d'un groupe fini ou de Lie, dont la solution des conjectures de Segal et de Sullivan a montré la richesse. Au plan des méthodes, l'accent a été mis sur les suites spectrales d'Adams instables et sur les modules instables sur l'algèbre de Steenrod.

- l'utilisation et la construction de modèles algébriques de l'homotopie, particulièrement performants en homotopie rationnelle, mais aussi à présent avec des coefficients plus généraux, à savoir des corps quelconques ou des sous-anneaux appropriés de $\mathbf{Q}$.

Ce colloque a permis de réunir une bonne part des spécialistes américains et européens du sujet, et a bénéficié à ce titre d'un soutien important de la National Science Foundation et du Centre National de la Recherche Scientifique dans le cadre de leur accord de coopération; il a également bénéficié de l'aide financière des Universités de Paris VII, Paris-Sud et Nice, et bien entendu de celle de la SMF sous la forme de la subvention accordée par le Conseil Scientifique du CIRM et de la publication du présent volume.

Les organisateurs tiennent à exprimer leur reconnaissance à toutes ces institutions, ainsi qu'au personnel du C.I.R.M., grâce auquel cette semaine a été - comme toujours - aussi riche en échanges scientifiques et humains qu'agréable sur le plan matériel.

INTRODUCTION

This volume contains papers submitted at the international conference on Homotopy Theory held at the C.I.R.M. in Marseille-Luminy from July 11 to 15, 1988.

Striking progress has been achieved in homotopy theory during the eighties. The purpose of this conference was to survey some of these achievements and their prospects: two main subjects were especially discussed during the conference and can be found in most contributions to this volume:

-The study of the homotopy type of function spaces, in particular when the domain is the classifying space of a finite group or a Lie group, whose richness has been revealed by the proofs of Segal's and Sullivan's conjectures. Among the methods used in this field, unstable Adams spectral sequences and unstable modules over the Steenrod algebra were emphazised.

-The use and construction of algebraic models of homotopy types, which have proven themselves especially fruitful in rational homotopy theory, but also now with more general coefficient rings, namely arbitrary fields and suitable subrings of **Q**.

A fair number of the experts in the field from America and Europe had the opportunity to meet at this conference, which was indeed largely supported by the NSF-CNRS cooperation agreement. Financial support was also provided by the Universities of Paris VII, Paris-Sud and Nice, and by the Mathematical Society of France through the grant distributed by the Scientific Committee of the C.I.R.M. and through the publication of this volume of Astérisque.

The organizers of this conference are pleased to express their warmest thanks to all sponsoring institutions, and to the C.I.R.M. staff who did their usual best to make that week a most profitable and pleasant one.

TABLE DES MATIÈRES

RÉSUMÉS DES ARTICLES

ANICK David J. & DROR-FARJOUN Emmanuel - *On the space of maps between R-local CW complexes.*
　　Over a subring R of the rationals, we construct a simplicial skeleton for the space of pointed maps between two R-local simply-connected CW complexes. The construction makes use of an R-local DG Lie algebra model for spaces.

AVRAMOV Luchezar & FÉLIX Yves - *Espaces de Golod.*
　　Nous considérons des fibrations nilpotentes $F \rightarrow E \rightarrow B$, où E et B sont des CW complexes finis simplement connexes. Un espace X est dit *de Golod* s'il existe un entier n tel que le revêtement n-connexe de X ait le type d'homotopie rationnelle d'un bouquet de sphères. Cette notion topologique correspond à celle des anneaux de Golod en algèbre locale. Nous montrons que si la base B de la fibration est un espace de Golod, la série de Poincaré de la fibre F est rationnelle.

BAKER Andrew - *Exotic multiplications on Morava K-theories and their liftings.*
　　For each prime p and (finite) integer $n>0$, there is a ring spectrum K(n) called the n-th Morava K-theory at p. We discuss exotic multiplications upon K(n) and their liftings to certain characteristic zero spectra $\hat{E}(n)$.

BROWN Edward H. & SZCZARBA R.H. - *Continuous homology and real homotopy type II.*
　　In our earlier paper "Continuous homology and real homotopy type", we studied localization of simplicial spaces at the reals and established an equivalence between the category of free nilpotent differential graded commutative algebras of finite type over the reals, and nilpotent simplicial spaces of finite type localized at the reals. In this paper, we extend these results by eliminating the nilpotent condition on the algebraic side, thus proving a conjecture of Sullivan. The main technical work consists in introducing local coefficients into continuous cohomology, continuous de Rham cohomology, the Serre spectral sequence and the constructions involved in real homotopy type.

CRABB Michael C. - *The Fuller index and T-equivariant stable homotopy theory.*
　　In 1967, F.B. Fuller introduced a remarkable index for counting periodic orbits of smooth flows. It has become apparent in recent work of J. Ize and E.N. Dancer that the natural setting for Fuller's index is

$\mathbb{T}$-equivariant homotopy theory, where $\mathbb{T}$ is the circle group. This paper describes their work in the conventional framework of equivariant stable homotopy theory over a base and index theory for fixed-points of maps and zeroes of vector fields.

DROR-FARJOUN Emmanuel & SMITH Jeffrey - *A geometric interpretation of Lannes' functor T.*

In this paper we prove a version of a conjecture of Lannes concerning the mod. p cohomology of the space of maps from $B\mathbb{Z}/p\mathbb{Z}$ to a rather general space X. This gives a topological meaning to an algebraic functor for modules over the Steenrod algebra, defined by Lannes. That functor has proven very useful in understanding spaces of maps from classifying spaces. As a corollary we get new proofs of several results of Lannes.

DWYER William G. & WILKERSON Clarence - *Spaces of null homotopic maps.*

We study the null component of the space of pointed maps from $B\pi$ to X when π is a locally finite group, and other components of the mapping space when π is elementary abelian. Results about the null component are used to give a general criterion for the existence of torsion in arbitrary high dimensions in the homotopy of X.

GOERSS Paul G.- *André-Quillen cohomology and the Bousfield-Kan spectral sequence.*

This paper undertakes to exploit the observation that the non-abelian homological algebra of Quillen and, in particular, the commutative algebra cohomology of André and Quillen provides a framework for discussing the unstable Adams spectral sequence of Bousfield and Kan. We take this observation in a variety of directions; for instance, we show that the long exact "transitivity sequence" in André-Quillen cohomology is related to the homotopy long exact sequence of a fibration, and we show that a product in André-Quillen cohomology can be used to compute the Whitehead product in homotopy.

HENN Hans-Werner. - *Cohomological p-nilpotence criteria for compact Lie groups.*

We introduce the concept of a p-nilpotent compact Lie group and discuss various group theoretical characterisations of such groups. These characterizations are then used to generalize cohomological p-nilpotence criteria for finite groups due to Atiyah and Quillen to the case of compact Lie groups.

MARKL Martin - *The rigidity of Poincaré duality algebras and classification of homotopy types of manifolds.*

We prove that Poincaré duality algebras are characterized by a certain rigidity property. As a consequence of this fact, we show that the k-isomorphism class of a Poincaré duality algebra H* of top dimension n is uniquely determined by the factor H*/H^n , provided k is algebraically closed. Using this and usual methods of descent theory, we obtain a description of the set of k-isomorphism classes of Poincaré duality algebras with the same given isomorphism class of H*/H^n, for any field k of characteristic zero. These results are then applied to the study of homotopy types of simply connected Poincaré duality spaces.

MAY J. Peter - *Some remarks on equivariant bundles and classifying spaces.*

A number of results are given on the relationship between equivariant and non-equivariant bundles and their classification. The bundles dealt with are the projections to orbits $E \rightarrow E/\Pi$, where Π is a normal subgroup of a compact Lie group Γ and E is a Π-free Γ-space. The base space has an action by $G = \Gamma/\Pi$, and such bundles are classified by a G-space $B(\Pi, \Gamma)$. Information about the homotopy type of this G-space gives information about the set of equivalence classes of such bundles with base a given G-space X . The bundle theory considerably simplifies when G acts freely on X, and the main theme is the study of the transformation on bundle theories induced by the natural projection $EG \times X \rightarrow X$.

VIGUÉ-POIRRIER Micheline - *Homologie de Hochschild et homologie cyclique des algèbres différentielles graduées.*

Pour toute algèbre différentielle graduée libre (T(V), d) sur un corps commutatif quelconque, nous donnons une description explicite de deux complexes: l'homologie du premier est l'homologie de Hochschild de (T(V), d) et celle du second est l'homologie cyclique de (T(V), d). Ces complexes servent aussi de modèles pour calculer l'homologie (resp. l'homologie équivariante) de l'espace des lacets libres sur un espace simplement connexe.

WOJTKOWIAK Zdzisław - *Maps between p-completions of the Clark-Ewing spaces.*

Let $\mathbf{Z}_p$ denote the ring of p-adic integers. Let $W \subset GL(n, \mathbf{Z}_p)$ be a finite group such that p does not divide the order of W. The group W acts on $K((\mathbf{Z}_p)^n, 2)$. Let $X(W,p,n)_p$ be the p-completion of the space $K((\mathbf{Z}_p)^n, 2) \times_W EW$. We classify homotopy classes of maps between spaces $X(W,p,n)_p$.

ZARATI Saïd - *Derived functors of the destabilization and the Adams spectral sequence*.

In this note we prove the following

Theorem - Let X and Y be two pointed CW complexes such that

(i) $\overline{H}*(X; F_2) \cong \Sigma^2 I$, where I is an injective unstable module

(ii) $\overline{H}*(Y; F_2)$ is gradually finite and nil-closed.

Then the Adams spectral sequence for the group $[S^\infty X, S^\infty Y]$ degenerates at the E_2 term.

This theorem is deduced from the theory of the higher Hopf invariant introduced by Lannes and the author, and from the relationship between the Ext groups and the derived functors of the destabilization.

LISTE DES PARTICIPANTS

AGUADE Jaume

Depart. Matemàtiques
Universitat Autònoma de Barcelona
E-08193 BELLATERRA
ESPAGNE

ANICK David

Department of Mathematics
Massachusetts Institute of Technology
CAMBRIDGE Mass.02139
U.S.A.

ARLETTAZ Dominique

Département de Mathématiques
Ecole Polytechnique Fédérale
Ecublens
CH 1015 LAUSANNE
SUISSE

AUBRY Marc

Département de Mathématiques
Université de Nice
Parc Valrose
06034 NICE - CEDEX
FRANCE

AVRAMOV Luchezar

Institute of Mathematics
Bulgarian Academy of Sciences
ul. "Akad. G. Boncev" blok 8
BG-1113 Sofia
BULGARIE

BAKER Andrew

Department of Mathematics
University of Manchester
MANCHESTER M13 9PL
GRANDE-BRETAGNE

BAUES Hans J.

Max-Planck Institut für Math.
Gottfried Claren Str. 26
D-5300 BONN 3
REP.FED.ALLEMAGNE

BENSON Dave

Mathematical Institute
University of Oxford
24-29 St. Giles'
OXFORD OX1 3LB
GRANDE-BRETAGNE

BROWN Edward

Department of Mathematics
Brandeis University
Waltham MA 02254
U.S.A

CRABB Michael

Department of Mathematics
University of Aberdeen
ABERDEEN AB9 2TY
GRANDE-BRETAGNE

CROISILLE Rémy   Département de Mathématiques
                                Université de Technologie de Compiègne
                                60206 COMPIEGNE
                                FRANCE

DRECKMANN M.   Max-Planck Institut für Math.
                                Gottfried Claren Str. 26
                                D-5300 BONN 3
                                REP.FED.ALLEMAGNE

DROR FARJOUN Emmanuel   Institute of Mathematics
                                The Hebrew University
                                JERUSALEM 91904
                                ISRAEL

DWYER William   Department of Mathematics
                                University of Notre-Dame
                                NOTRE-DAME IN 46556
                                U.S.A.

FELIX Yves   Institut de Mathématiques Pures
                                Université Catholique de Louvain
                                Chemin du Cyclotron
                                B-1348 LOUVAIN-LA-NEUVE
                                BELGIQUE

FESHBACH Mark   Department of Mathematics
                                University of Minnesota
                                Minneapolis MN 55455
                                U.S.A

FRANJOU Vincent   U.E.R. de Mathématiques
                                Université de Nantes
                                2 Chemin de la Houssinière
                                44072 NANTES CEDEX
                                FRANCE

GOERSS Paul   Department of Mathematics
                                Wellesley College
                                WELLESLEY MA 02181
                                U.S.A.

GOTTLIEB Daniel   Department of Mathematics
                                Purdue University
                                WEST LAFAYETTE IN 47907
                                U.S.A.

HALPERIN Stephen   Department of Mathematics
                                University of Toronto
                                TORONTO Ont. M5S 1A1
                                CANADA

HENN Hans-Werner   Mathematisches Institut der
                                Universität Heidelberg
                                Im Neuenheimer Feld 288
                                D 6900 HEIDELBERG
                                REP.FED.ALLEMAGNE

HUNG Nguyen H. V.

Department of Mathematics
University of Hanoi
DAI HOC TONG HOP
HANOI
VIETNAM

HUSEMOLLER Dale

Department of Mathematics
Haverford College
HAVERFORD Pa. 19041
U.S.A.

JACKOWSKY Stefân

Instytut Matematiky
Uniwersytet Warswazski
Palac Kultury i Nauki XI
PL 00-901 WARSZAWA
POLOGNE

JEANNERET Alain

Institut de Mathématiques
Université de Neuchâtel
Chantemerle 20
CH-2000 NEUCHATEL
SUISSE

JESSUP Barry

Department of Mathematics
York University
4700 Keele Str.
NORTH YORK Ont. M3J 1P3
CANADA

KAROUBI Max

U.E.R. de Mathématiques
Université de Paris VII
Tour 45-55, 2 Place Jussieu
75251 PARIS - CEDEX 05
FRANCE

LANDWEBER Peter

Department of Mathematics
Rutgers University
NEW BRUNSWICK NJ 08903
U.S.A.

LANNES Jean

U.E.R. de Mathématiques
Université de Paris VII
Tour 45-55, 2 Place Jussieu
75251 PARIS - CEDEX 05
FRANCE

LEMAIRE Jean-Michel

Département de Mathématiques
Université de Nice
Parc Valrose
06034 NICE - CEDEX
FRANCE

LESH Kathryn

Department of Mathematics
Massachusetts Institute of Technology
CAMBRIDGE Mass.02139
U.S.A.

LODAY Jean-Louis

IRMA
Université de Strasbourg
7 Rue Descartes
67084 STRASBOURG - CEDEX
FRANCE

MAHOWALD Mark

Department of Mathematics
Northwestern University
2033 Sheridan Road
EVANSTON IL 60201
U.S.A.

MARKL Martin

Institute of Mathematics
University of Prague
Zitra 25
166 27 PRAHA 1
TCHECOSLOVAQUIE

MAY J. Peter

Department of Mathematics
University of Chicago
5734 University Avenue
CHICAGO Il 60637
U.S.A.

MCCLURE James

Department of Mathematics
University of Kentucky
LEXINGTON KY 40506
U.S.A.

MILLER Haynes

Department of Mathematics
Massachusetts Institute of Technology
CAMBRIDGE Mass.02139
U.S.A.

MOORE John C.

Department of Mathematics
University of Rochester
ROCHESTER NY 14620
U.S.A.

MOREL Fabien

U.E.R. de Mathématiques
Université de Paris VII
Tour 45-55, 2 Place Jussieu
75251 PARIS - CEDEX 05
FRANCE

NEISENDORFER Joseph

Department of Mathematics
University of Rochester
ROCHESTER NY 14620
U.S.A.

PRIDDY Stewart

Department of Mathematics
Northwestern University
2033 Sheridan Road
EVANSTON IL 60201
U.S.A.

PROUTE Alain

U.E.R. de Mathématiques
Université de Paris VII
Tour 45-55, 2 Place Jussieu
75251 PARIS - CEDEX 05
FRANCE

ROOS Jan-Erik

Matematiska Institutionen
Stockholm Universitet
Box 6701
S-113 85 STOCKHOLM
SUEDE

SCHWARTZ Lionel

Département de Mathématiques
Université Paris-Nord
Avenue J.-B. Clément
93430 Villetaneuse Cedex
FRANCE

SMITH Jeffrey

Department of Mathematics
John Hopkins University
BALTIMORE MD 21218
U.S.A.

SMITH Larry

Mathematisches Institut
Georg August Univ. zu Göttingen
Busenstr. 3/5
D-3400 GÖTTINGEN
REP.FED.ALLEMAGNE

STOVER Christopher

Department of Mathematics
Massachusetts Institute of Technology
CAMBRIDGE Mass.02139
U.S.A.

TANRE Daniel

UFR de Mathématiques
UST Lille-Flandres-Artois
59655 VILLENEUVE D'ASCQ - CEDEX
FRANCE

THOMAS Jean-Claude

UFR de Mathématiques
UST Lille-Flandres-Artois
59655 VILLENEUVE D'ASCQ - CEDEX
FRANCE

VIGUE Micheline

37 Parc d'Ardenay
91120 PALAISEAU
FRANCE

VOGEL Pierre

U.E.R. de Mathématiques
Université de Nantes
2 Chemin de la Houssinière
44072 NANTES CEDEX
FRANCE

WAGNER Mark

Institute of Mathematics
The Hebrew University
JERUSALEM 91904
ISRAEL

WILKERSON Clarence   Department of Mathematics
                            Wayne State University
                            DETROIT MI 48202
                            U.S.A.

WOJTKOWIAK Zdzislaw   Depart. Matemàtiques
                            Universitat Autònoma de Barcelona
                            E-08193 BELLATERRA
                            ESPAGNE

ZARATI Saïd   Département de Mathématiques
                            Université de Tunis
                            Faculté des Sciences
                            1060 TUNIS
                            TUNISIE

ZISMAN Michel   U.E.R. de Mathématiques
                            Université de Paris VII
                            Tour 45-55, 2 Place Jussieu
                            75251 PARIS - CEDEX 05
                            FRANCE