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COLLOQUE PAUL LÉVY SUR LES PROCESSUS STOCHASTIQUES

(22-26 juin 1987, Ecole Polytechnique, Palaiseau)

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COLLOQUE PAUL LÉVY SUR LES PROCESSUS STOCHASTIQUES

(22-26 juin 1987, Ecole Polytechnique, Palaiseau)

A l'occasion du 100e anniversaire de la naissance de Paul Lévy, un colloque international sur les processus stochastiques a été organisé à l'Ecole Polytechnique du 22 au 26 juin 1987, sous la présidence du Professeur L. SCHWARTZ.

Les articles originaux qui font l'objet de ce volume reproduisent les textes de la plupart des conférences invitées.

Ce colloque a été très bien accueilli par la communauté probabiliste internationale et un ensemble exceptionnellement riche de contributions y a été présenté. Les thèmes les plus porteurs du développement actuel de la théorie des processus stochastiques présentent de remarquables connexions avec l'oeuvre de Paul Lévy, comme l'ont montré de nombreux exposés.

L'aide du Centre National de la Recherche Scientifique (C.N.R.S.), celle de la National Science Foundation (N.S.F.) et le support de l'Ecole Polytechnique ont été déterminants pour l'organisation de ce colloque.

L'Académie des Sciences nous a accordé son patronage. Nous remercions aussi Electricité de France et Framatome pour leur soutien.

Les Editeurs, Secrétaires du Colloque
M. METIVIER, J. NEVEU, S.R.S. VARADHAN

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CONFÉRENCES INVITÉES

Laurent SCHWARTZ : *"Quelques réflexions et souvenirs sur Paul Lévy"*(*)

Kai-Lai CHUNG, Stanford University, Department of Mathematics, STANFORD, CA 94305
"Reminiscences of some of Paul Lévy's ideas in Brownian Motion and in Markov Chains".(*)

Dominique BAKRY, Université Louis Pasteur, Dépt. de Math. 7 rue René Descartes, F-67084 STRASBOURG.
"La propriété de sous-harmonicité des processus de diffusions".

Martin BARLOW, University of Cambridge, Statistical Laboratory, 16 Mill Lane, CAMBRIDGE CB2 1SB, UK.
"Brownian motion on the Sierpinski gasket".

Daniel BENNEQUIN, Université de Strasbourg, Dépt de Math., 7 rue René Descartes, F-67084 STRASBOURG Cedex.
"Problème de Riemann-Hilbert : conjecture du genre et sphères d'homotopie".

Jean-Michel BISMUT, Université de Paris-Sud, Département de Mathématiques, Bât. 425, F-91405 ORSAY Cedex
"Formules de localisation et formules de Paul Lévy".(*)

J. BOURGAIN, I.H.E.S., 35 Route de Chartres, 91440 BURES SUR YVETTE
"Sur le support de la mesure harmonique dans R^n".

Krzysztof BURDZY, Jim W. PITMAN and Marc YOR
- University of Washington, Department of Mathematics SEATTLE, WA 98195, USA.
- University of California, Department of Statistics, BERKELEY, CA 94720
- Université de Paris 6, Laboratoire de Probabilités, Tour 56, 4 place Jussieu, F-75252 PARIS Cedex 05.
"Some asymptotic laws for crossings and excursions"(*)

Donald L. BURKHOLDER, Univ. of Illinois, Dept of Math., 1409 West Green Street, URBANA, IL 61801, USA.
"Sharp inequalities for martingales and stochastic integrals".(*)

Vladimir DOBRIC, Michael B. MARCUS, Michel WEBER
- Lehigh University, Department of Mathematics, BETHLEHAM, PA 18015, USA.
- The City College of CUNY, Department of Mathematics, NEW YORK, NY 10031, USA
- University Louis Pasteur, 7 rue René Descartes, F-67084 STRASBOURG Cx.
"The distribution of large values of the supremum of a Gaussian process".(*)

Lester E. DUBINS and Gideon SCHWARZ,
- University of California, Department of Statistics, BERKELEY, CA 94720, USA.
- The Hebrew University of Jerusalem, Department of Statistics, IL - 95101 JERUSALEM, Israël.
"A sharp inequality for semi-martingales and stopping-times"(*)

Eugene B. DYNKIN, Cornell University, Department of Mathematics, White Hall, ITHACA, NY 14853-7901, USA.
"Representation for functionals of superprocesses by multiple stochastic integrals, with applications to self-intersection local times". (*)

Xavier FERNIQUE, Université Louis Pasteur, IRMA, 10 rue du Gal Zimmer, F-67084 STRASBOURG.
"Vecteurs gaussiens et mesures majorantes"

Hans FÖLLMER and Marianne ORT, ETH - Zentrum, Mathematikdepartement, CH-8092 ZÜRICH
"Large deviations and surface entropy for Markov fields". (*)

Masatoshi FUKUSHIMA, Osaka University, College of General Education, Toyonaka, OSAKA, 560 Japon.
"On skew products of symmetric diffusions and their local properties".

Sylvestre GALLOT, Université de Savoie, Département de Mathématiques, BP 1104, F-73011 CHAMBERY
"Some links between isoperimetry, spectrum and topology".(*)

Ronald GETOOR, University of California, Dept. of Mathematics, LA JOLLA, CA 92093, USA.
"Some recent directions in the theory of Markov processes".

Michael GROMOV, I.H.E.S., 35 Route de Chartres, F-91440 BURES SUR YVETTE.
"Paul Lévy isoperimetric inequality and applications".

Takeyuki HIDA, Nagoya University, Department of Mathematics, Chikusa-ku, NAGOYA, 464 Japon.
"Lévy's functional analysis and stochastic calculus".

Robin L. HUDSON, University of Nottingham, Dept. of Math., University Park, NOTTINGHAM NG72RD, UK.
'Quantum stochastic calculus".

Nobuyuki IKEDA, Osaka University, Department of Mathematics, Toyonaka, OSAKA, 560 Japon.
"Limit theorems for stochastic processes".

Kiyosi ITO, RIMS, Kyoto University, KYOTO, 606 Japon.
"Some topics on Malliavin Calculus".

Harry KESTEN, Cornell University, Depart. of Mathematics, White Hall, ITHACA, NY 14853-7901, USA.
"Recent progress in rigorous percolation theory".(*)

Frank KNIGHT, Univ. of Illinois, Department of Mathematics,1409 West Green Str., URBANA, IL 61801, USA.
"Inverse local times, positive sojourns, and maxima for Brownian motion".(*)

Jean-François LE GALL, Univ. de Paris VI, Lab. de Probabilités, Tour 56, 4 pl. Jussieu, F-75252 PARIS Cedex 05.
"Polynomials of the occupation field and asymptotics for the planar Wiener sausage".

Terence J. LYONS and Zheng WEIAN,
- Univ. of Edinburg, Department of Mathematics, Mayfield Road, EDINBURG, EH9 3JZ,UK
- East China Normal University, Department of Mathematical Statistics, SHANGHAI, China
"A crossing estimate for the canonical process on a Dirichlet space and a tightness result".(*)

Paul MALLIAVIN, Univ. de Paris VI, Analyse Complexe et Géométrie, 4 pl. Jussieu, F-75252 PARIS Cedex 05.
"Intégration dans les groupes de lacets".

Benoït B. MANDELBROT, Harvard Univ., Mathematics, PO Box 218, YORKTOWN HEIGHTS, NY 10598, USA.
"Self-affine stochastic processes and other fractals".

Henry McKEAN, Courant Institute, Univ. of New York, 251 Mercer Street, NEW YORK, NY 10012, USA.
"The idea of fluctuations in statistical mechanics".

V.D. MILMAN, School of Mathematical Sciences. The Beverley and Raymond Sackler. Faculty of Exact Sciences. TEL AVIV UNIVERSITY. Israël.
"The heritage of P. Lévy in geometrical functional analysis".(*)

K.R. PARTHASARATHY, Indian Stat. Inst., Delhi Centre, 7 S.J.S Sansanwal Marg, NEW DELHI 110016, India.
"A unified approach to classical, bosonic and fermionic Brownian motions".(*)

Edwin PERKINS, University of Cambridge, Statistical Laboratory, 16 Mill Lane, CAMBRIDGE CB2 1SB, UK.
"Sample path properties of measure-valued diffusions".

Gilles PISIER, Université de Paris VI, Equipe d'Analyse, 4 place Jussieu, F-75252 PARIS Cedex 05.
"p-variation forte des martingales et des séries orthogonales".

Pal RÉVÉSZ, Technische Universität Wien, Institut für Statistik und Wahrscheinlichkeitstheorie, Wiedner Hauptstrasse 8-10/107, A-1040 WIEN
"In random environment the local time can be very big".(*)

Fraydoun REZAKHANLOU and S. James TAYLOR
- Courant Institute, 251 Mercer Street, NEW YORK, NY 10012, USA.
- Univ. of Virginia, Depart. of Math., Math-Astr. Building, CHARLOTTESVILLE, VA 22903-3199, USA.
"The packing measure of the graph of a stable process". (*)

Michael J. SHARPE, University of California, Department of Mathematics, LA JOLLA, CA 92093, USA.
"Foundations of the theory of right processes".

Anatoli Vla. SKOROHOD, Mathematical Institute Ul. Repina n.3, 252601 KIEV, URSS.
"On the currents of σ-algebras generated by the martingales".

Daniel W. STROOCK, M.I.T., Department. of Mathematics, CAMBRIDGE, Mass. 02139, USA.
"Chaotic monomials".

D. SURGAILIS, Lithuanian SSR. Academy of Sciences. 232000 VILNIUS. MTP-1. Lenino pr.3. URSS.
"Polygonial fields : a new class of Markov fields on the plane".

Alan-Sol SZNITMAN, Université de Paris VI, Lab. de Probabilités, Tour 56, 4 pl. Jussieu, F-75252 PARIS Cedex 05.
"A trajectorial representation for certain nonlinear equations".(*)

Michel TALAGRAND, Univ. de Paris VI, Equipe d'Analyse, UA au CNRS 754, Tour 46, F-75252 PARIS Cedex 05.
"Majorizing measures and the sample continuity of stochastic processes".

Srinivasa R.S. VARADHAN, Courant Institute, N.Y.U., 251 Mercer Street, NEW YORK, NY 10012, USA.
"Bulk diffusion and large deviations".

Shinzo WATANABE, Kyoto University, Department of Mathematics, KYOTO, 606 Japon.
"Point processes with values in path spaces".

(*) Conférences publiées dans le présent volume.

RÉSUMÉS DES CONFÉRENCES PUBLIÉES

J.-M. BISMUT, Université de Paris-Sud, Département de Mathématiques, F-91405 ORSAY Cx

"Formules de localisation et formules de Paul Lévy".

Abstract. In this paper, we prove localization formulas and asymptotic localization formulas in equivariant cohomology. These formulas exhibit in a finite dimensional context remarkable local cancellations. Following ideas of Atiyah and Witten, we show the relations of localization formulas to Index Theory. We then relate asymptotic localization formulas to our proof of the asymptotic Morse inequalities of Demailly.

Krzysztof BURDZY, Jim W. PITMAN and Marc YOR

- University of Washington, Department of Mathematics, SEATTLE, WA 98195, USA.
- University of California, Department of Statistics, BERKELEY, CA 94720, USA.
- Université de Paris 6, Laboratoire de Probabilités, Tour 56, 4 place Jussieu, F-75252 PARIS Cx 05.

"Some asymptotic laws for crossings and excursions"

Abstract. A et C désignant deux sous-ensembles de l'espace d'états d'un processus de Markov X, les auteurs étudient la distribution asymptotique de $N_{AC}(t)$, nombre de visites de A, via C, effectuées par X jusqu'à l'instant t. Si X est le mouvement brownien plan, alors, sous des conditions adéquates, la loi asymptotique est exponentielle et a pour paramètre un multiple universel de Cap(A,C), la capacité logarithmique de A relative à C. Ces résultats peuvent être étendus à l'étude asymptotique du processus de comptage pour différents types d'excursions ; cette étude est liée à la mesure de Palm associée à un processus stationnaire d'excursions. Enfin, dans le cas du mouvement brownien plan, des résultats différents sont obtenus concernant la distribution asymptotique de $N_{AC}(t)$ lorsque $Cap(A,C)=\infty$.

Donald L. BURKHOLDER, Univ. of Illinois, Dept of Math, 1409 West Green Str. URBANA, IL 61801 USA.

"Sharp inequalities for martingales and stochastic integrals".

Abstract. Differentially subordinate martingales taking values in a real or complex Hilbert space H are studied and sharp exponential, L^p, and weak-type inequalities are obtained. These sharp inequalities are new even for $H=\mathbb{C}$ and lead to the best constants for some inequalities between stochastic integrals in which either the martingale integrators or the predictable integrands are H-valued. In addition, they yield new information about the square-function inequality for H-valued martingales in the case $H=\mathbb{R}$.

Vladimir DOBRIC, Michael B. MARCUS, Michel WEBER

- Lehigh University, Department of Mathematics, BETHLEHAM, PA 18015, USA.
- The City College of CUNY, Department of Mathematics, NEW YORK, NY 10031, USA.
- University Louis Pasteur, 7 rue René Descartes, F-67084 STRASBOURG Cx.

"The distribution of large values of the supremum of a Gaussian process".

Abstract. Ce travail étudie les processus gaussiens centrés $X=\{X(t), t\in T\}$ dont la variance $E X^2(t)$ atteint son maximum en un nombre fini seulement de points de T; il donne des évaluations précises de $P(\sup_{t\in T} X(t)\geq u)$ pour les grandes valeurs de u qui étendent des résultats récents de Talagrand. Ces évaluations sont utilisées pour étudier le comportement asymptotique de $P((\sum |\sigma_k \lambda_k|^p)^{1/p}>u)$, $u\to\infty$, où $\{\lambda_k\}$ est une suite de v.a. gaussiennes indépendantes centrées et réduites.

Lester E. DUBINS and Gideon SCHWARZ,

- University of California, Department of Mathematics, BERKELEY, CA 94720, USA.
- The Hebrew University of Jerusalem, Department of Statistics, IL - 95101 JERUSALEM, Israël.

"A sharp inequality for sub-martingales and stopping-times"

Abstract. The expectation of the maximum of a mean-zero martingale is at most the L_2-norm of the martingale. For nonnegative submartingales, the bound is $\sqrt{2}$ times its L_2 norm. These bounds are sharp. The results follow from the solution of some optimal-stopping problems for the simple symmetric random walk.

Eugene B. DYNKIN, Cornell University, Dept. of Mathematics, White Hall, ITHACA, NY 14853-7901, USA.

"Representation for functionals of superprocesses by multiple stochastic integrals, with applications to self-intersection local times".

Abstract. The representation of functionals of a gaussian process by the multiple Wiener-Ito integrals plays an important role in stochastic calculus. We establish a similar representation for a certain class of non-Gaussian measure-valued Markov process. A process X of this class can be associated with every Markov process ξ and we call X a superprocess over ξ .The existence of local times and self-intersection local times for X depends on the behaviour of the transition density of ξ as $t \to 0$.

Hans FÖLLMER and Marianne ORT, ETH - Zentrum, Mathematikdepartement, CH-8092 ZÜRICH.

"Large deviations and surface entropy for Markov fields".

Abstract. We introduce the surface entropy of a random field indexed by Z^d and prove the corresponding Shannon-McMillan theorem. For a certain class of Markov fieds, this allows us to obtain a new bound for large deviations of the empirical field in the case of a phase transition.

Sylvestre GALLOT, Université de Savoie, Département de Mathématiques, BP 1104, F-73011 CHAMBERY.

"Isoperimetric inequalities based on integral norms of Ricci curvature".

Abstract. We give an estimate of the volume of tubes (or balls) in Riemannian manifolds in terms of the average of the p/2-th power of Ricci curvature in the inside of the tube (for any $p \in]n,+\infty]$). Estimates of isoperimetric constants, analytic (spectrum, heat kernel, Sobolev constants,...) and topological (Betti numbers, index of Dirac operators,...) invariants are obtained ; they are uniformly computable on the set of Riemannian manifolds whose diameter is bounded and such that the part of Ricci curvature which is not bounded from below has small $L^{p/2}$ norm. These assumptions are almost non improvable (cf. Appendix). We also bound the topological norms of M. Gromov (such as the simplicial volume) and the infimum of the spectrum of non compact manifolds in terms of the average of the (p/2)-th power of Ricci curvature.

Harry KESTEN, Cornell University, Department of Mathematics, White Hall, ITHACA, NY 14853-7901, USA.

"Recent progress in rigorous percolation theory".

Abstract.We discuss some recent results in (Bernoulli) percolation. In particular these include

(i) $p_T = p_H$This equality (proved in [1] and [21]) between two differently defined critical probabilities solves one of the major problems of the subject.

(ii) Uniqueness of infinite clusters. With probability one there exists at most one infinite open cluster ; this holds in independent models in any dimension.

(iii) Scaling relations for two -dimensional percolation. Some relations between the singularities of various quantities near the critical probability can be proven ; these show that in two dimensions most of the conjectured scaling relations between critical exponents have to be true, provided the critical exponents exist.

Frank B. KNIGHT, Univ. of Illinois, Dept of Mathematics, 1409 West Green Street, URBANA, IL 61801, USA.

"Inverse local times, positive sojourns, and maxima for Brownian motion".

Abstract. On étudie la loi conjointe de $M^+(\alpha)$ et $S^+(\alpha)$, où $M^+(\alpha)$ est le maximum de $S^+(\alpha)$ le séjour positif d' un mouvement Brownien, avant le temps $T(\alpha)$ quand le temps local en 0 atteint $2\alpha > 0$.

Terence J. LYONS and Zheng WEIAN,

- University of Edinburg, Department of Mathematics, Mayfield Road, EDINBURG, EH9 3JZ, UK.
- East China Normal University, Department of Mathematical Statistics, SHANGHAI, China.

"A crossing estimate for the canonical process on a Dirichlet space and a tightness result".

Abstract. A uniformly elliptic operator in divergence form has a diffusion process associated with it even if the coefficients are only bounded and measurable. However the associated process is not a semi-martingale and the Ito calculus is not appropriate for integration along paths. This paper explains how an extension of Stratanovich's integral can be defined and works rather smoothly. We also established tightness results for these processes which would allow one to construct them through approximating sequences of processes associated with operators having smooth coefficients.

V.D. MILMAN, School of Math. Sciences. The Beverley and Raymond Sackler. Faculty of Exact Sciences.
TEL AVIV UNIVERSITY, Israël.

"The heritage of P. Lévy in geometrical functional analysis".

Abstract. This survey discusses the concept of concentration of measure ("Levy families") and some applications of this idea to some geometric and topological problems, to infinite dimensional integration and others. We give a lot of examples of Levy families and classify them by general methods which analyze concentration property. The main methods are an isoperimetric inequality approach, a martingale approach and eigenvalues of a suitable Laplace operator. We also sketch the concept of spectrum which is the principal underlying idea in geometric applications of concentration property.

K.R. PARTHASARATHY, Indian Stat. Ins., Delhi Centre, 7 S.J.S Sansanwal Marg, NEW DELHI 110016, India

"A unified approach to classical, bosonic and fermionic Brownian motions".

Abstract. Starting from martingale hypotheses the canonical commutation and anticommutation relations for fields of observables in one dimension are derived.

Pal RÉVÉSZ, Technische Universität Wien, Institut für Statistik und Wahrscheinlichkeitstheorie
Wiedner Hauptstrasse 8-10/107, A-1040 WIEN

"In random environment the local time can be very big".

Abstract. Let $\mathcal{E}=\{...,E_{-2},E_{-1},E_0,E_1,E_2,...\}$ *be a sequence of i.i.d.r.v.'s with* $0<E_i<1$. *Define a random walk* $R_0,R_1,...$ *by* $R_0=0$ *and*

$$P\{R_{n+1} = i+1 \mid R_n=i\} = 1-P\{R_{n+1} = i-1 \mid R_n=i\} = E_i$$

the properties of the local time $\xi(x,n) = \#\{k:0\le k\le n, R_k = x\}$ *are investigated. The main result says*

$$\limsup_{n\to\infty} \max_x n^{-1}\ \xi(x,n) \ge \alpha \quad a.s.$$

with some $0<\alpha<1$.

Fraydoun REZAKHANLOU and S. James TAYLOR
- Courant Institute, 251 Mercer Street, NEW YORK, NY 10012, USA.
- Univ. of Virginia, Dept. of Math. Math-Astr. Building, CHARLOTTESVILLE, VA 22903-3199, USA.

"The packing measure of the graph of a stable process".

Abstract. For a monotone function $\phi(s)$, *the* ϕ*-packing measure of a subset S of Euclidean space is the result of maximizing the* ϕ*-content of disjoint small balls centred in S. For a strictly stable Lévy process* X_t *we characterize functions* ϕ *which make the* ϕ*-packing measure of the graph* $G_t=(t,X_t)$ *zero or infinite, and show that* $\phi(s)=s/|\log s|$ *gives a finite positive* ϕ*-packing measure to* G_t *whenever* X_t *is strictly asymmetric Cauchy. This last result is also proved for the trajectory of an asymmetric Cauchy process in* $\mathbb{R}^d (d\ge 2)$.

Alan-Sol SZNITMAN, Univ. Paris 6, Laboratoire de Probabilités, Tour 56, 4 pl. Jussieu, F-75252 PARIS Cx 05.

"A trajectorial representation for certain nonlinear equations".

Abstract. We construct here the limiting tree for collisions between independent particles in the "constant capacity" or "constant mean free path" regime. We then study annihilation directly on this limit object.