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Abstract

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ABSTRACT

We develop a theory of wave-functions, i.e. of functions ψ which have the following properties :

i) $\int_{\mathbb{R}^n} |\psi(x)|^2 \, dx = 1$

ii) for every $X \in \mathbb{R}^n$ which is infinitesimal $\int |\psi(x+hX)-\psi(x)|^2 dx \simeq 0$

iii) for every $A>0$ infinitely large $\int_{|x|>A} |\psi(x)|^2 \, dx \simeq 0$

(the sense of "infinitesimal" or "infinitely large" is that of non-standard analysis). We prove that such functions are the most general type of ψ-functions of the wave-mechanics, and that the quantic behaviour of such wave-functions results at the macroscopic level in the classical laws of newtonian mechanics. The main concept of our study is the "dynamic spectrum" of a wave-function: it is a closed set of the phase-space, the elements of which are precisely the points and momenta which can be reached by the particle with non-infinitesimal probability. This dynamical spectrum looks like the well-known notions of "wave front set" or "singular spectrum", but is different (in the last two chapters, we study these differences). We develop also a calculus for our wave-functions, especially for "wave-packets", which are the special case of wave-functions concentrated (in space and momentum) at a single point.