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à Georges Reeb - [Pages préliminaires]

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à Georges REEB

Les organisateurs de cette troisième semaine de Géométrie du Schnepfenried remercient les organismes qui leur ont à nouveau apporté leur soutien : Société Mathématique de France, Université de Haute-Alsace (Mulhouse) et Université Louis Pasteur (Strasbourg).

Ils remercient les conférenciers et les quelques quatre vingt participants, représentant 18 nationalités différentes, qui ont apporté au chalet leur enthousiasme et leur bonne humeur. Ils remercient également Mademoiselle Marianne STEPHAN et Madame Fabienne GARUZ tant pour l'organisation matérielle des journées que pour la préparation du manuscrit.

L'objet de ces journées était de "faire le point, par une large revue, sur la position et les développements actuels de certains problèmes où la géométrie apporte des vues nouvelles". Il s'agissait à la fois de présenter des Mathématiques en train de se faire et d'en donner une vue assez large pour ne pas intéresser les seuls spécialistes pointus de questions plus ou moins étroites : chacun devait parler pour les autres. C'est le contrat que nous avons proposé aux conférenciers, tous invités. A la fin de ces six journées, d'une extrême densité, le but semblait bien prêt d'avoir été atteint..... Au lecteur d'en juger avec un an de recul.

Daniel BERNARD

Théodor HANGAN

Robert LUTZ

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TABLE DES MATIÈRES

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VOLUME 1

- W.T. van EST - Quelques questions de géométrie en rétrospective. 13

One of the striking phenomena in the evolution of geometry and analysis in the last 150 years is the shift in focus from "local problems" to "global" ones. Along with it algebraic topological notions and methods, sometimes typical to the special field, develop. Taking the Serret-Frenet theorem from differential geometry as a starting point this theme is discussed, ramifying eventually into a discussion of some points in foliation theory.

- A. LICHNEROWICZ - Géométrie et cohomologies associées à une variété de contact. 31

A contact manifold $(\hat{W}, \hat{\eta})$ being given, we study the cohomologies of the Lie algebra \hat{L} of the infinitesimal automorphisms of the contact structure, with values in the scalar densities on \hat{W} and corresponding to the Lie derivative. These cohomologies are isomorphic to the Lie algebra N_1 of the homogeneous functions of order 1 on the exact symplectic manifold (W, ω) associated with $(\hat{W}, \hat{\eta})$, the values of which are in the space N_h of the homogeneous functions of order h . We determine then cohomologies for the dimensions 1 and 2. For $h=1$, we obtain that Lie algebra \hat{L} is rigid with respect to the differential deformations. For $h=-1$, we introduce a non-vanishing cohomology 2-class β_1 which is a main invariant of the contact structure. The class β_1 plays an important role in the theory of the pseudodifferential operators of order one, on a manifold M , in agreement with the interesting point of view of Omori and collab.

P. LIBERMANN - Problèmes d'équivalence et géométrie symplectique. 43

In the first part, we study the E. Cartan equivalence problem for foliated symplectic manifolds (we prove Darboux, Jacobie-Lie - Caratheodory and E. Cartan theorems).

In the second part, we study non completely integrable Pfaffian systems on a manifold N , considering them as submanifolds of the symplectic manifold $(T^*M, d\theta)$. We introduce the notion of basic ideal which is used for prolongations.

C.-M. MARLE - Sous-variétés de rang constant d'une variété symplectique. 69

Let N_1 be a submanifold of a symplectic manifold (M_1, Ω_1) , on which the 2-form Ω_{N_1} induced by Ω_1 is of constant rank ($i=1$ or 2). We assume that there exists a diffeomorphism $f : N_1 \rightarrow N_2$, such that $f^* \Omega_{N_2} = \Omega_{N_1}$. We give necessary and sufficient conditions under which f can be extended into a symplectic diffeomorphism of a neighborhood of N_1 in M_1 , onto a neighborhood of N_2 in M_2 . This result, which extends a theorem of A. Weinstein about isotropic embeddings, is used for the study of neighborhoods of orbits of a Hamiltonian action of a Lie group on a symplectic manifold.

D. BENNEQUIN - Entrelacements et équations de Pfaff. 87

The text gives a proof for the existence of non isomorphic contact structures on the 3-dimensional euclidean space. Principal themes : contact topology in dimension 3, linking properties of curves and plane fields, geometry of intersections of a surface with the Reeb foliation. Digressions : theory of braids, and analytic curves in the complex plane.

J. GONZALO et F. VARELA - Modèles globaux des variétés de contact. 163

There are some compact manifolds characterized by the fact of possessing a differentiable function defined in the large (an element of zero degree of the exterior algebra) and satisfying some additional conditions (as an example, Reeb's theorem).

It is shown in this paper that certain additional conditions on form of degree 1 (being a contact form and having a specific global expression) determines the manifold up to a diffeomorphism.

- F. VARELA - Sur l'orientation définie par une forme de Pfaff de classe maximale. 169

We point out in this paper the two following facts :

- a) The set of those contact forms which give rise to a given orientation is an open subset of the contact forms space in the C^0 -topology.
- b) The subset of the contact forms on \mathbb{R}^{2n+1} giving a chosen orientation is not a C^0 open set, for $n \geq 2$, of the set of all contact forms.

- R. LUTZ - Structures de contact et systèmes de Pfaff à pivot. 175

We show that in some sense the Liouville-Cartan contact-structures on the projective cotangent bundles are universal among all contact structures which admit an integral fiber bundle of maximal dimension called here a "pivot". A classification of such structures in the compact case follows ; we discuss also the general case of Pfaffian systems of constant class with a pivot, including the case of r -contact structures which occurs naturally in this context.

- Th. HANGAN et R. LUTZ - Champs d'hyperplans totalement géodésiques sur les sphères. 189

A sub-bundle F of the tangent bundle of a riemannian manifold is called totally geodesic (t.g.) if a geodesic tangent to F at some point is tangent to F at each of its points. One proves that on the odd dimensional spheres the kernels of a Pfaff's form ω define a t.g.-sub-bundle iff ω is projectively equivalent to the standard contact form.

Integral formulas by Brito-Langevin-Rosenberg are then used in order to discuss a problem of E. Cartan concerning the prolongation of local t.g. sub-bundles to global ones on the projective space.

Developpements to partially t.g. sub-bundles and to the hyperbolic space are finally considered.

- R. LANGEVIN - Feuilletages, énergies et cristaux liquides. 201

The goal of this article is to recall some results of integral geometry concerning foliations and to present some possible definitions of the energy of a foliation. One of these comes naturally from the study of liquid crystals.

Then come natural questions about the integral geometric relations satisfied by the energy of a foliation.

- N.H. KUIPER et W. MEEKS III - Sur la courbure des surfaces nouées dans \mathbb{R}^3 . 215

The total absolute curvature $\tau = \int |K| d\sigma / 2\pi$ of a knotted (= not isotopic to standard) closed surface of genus $g \geq 1$ in \mathbb{R}^3 , is $\tau \geq 2g + 6$, and the infimum $\tau = 2g + 6$ is attained for $g \geq 3$, and not for $g \leq 2$.

- T.J. WILLMORE - Commutative and related metrics . 219

Commutative spaces are defined as Riemannian spaces in which the members of a sequence of globally defined differential operators commute. D'Atri spaces are those in which the geodesic symmetries are volume preserving maps. The paper examines relations between commutative spaces, d'Atri spaces and naturally reductive homogeneous spaces.

- P. MOLINO - Connexions adaptées à un système différentiel extérieur et prolongements d'Estabrook-Wahlquist. 229

Following ideas of Estabrook-Wahlquist, this paper considers Cartan-Ehresmann's connections which are adapted to a given exterior differential system. Moreover, using a notion of polarization for such a system, one obtains an intrinsic and more general formalisation of Estabrook-Wahlquist's calculus.

- G. HECTOR - Architecture des feuilletages de classe C^2 . 243

Let (M, F) be a codimension one foliation of class C^2 on a compact manifold M . We show that for any leaf $L \in F$, there is a very strong correlation between :

- i) the endset of L (i.e. its topological type) ;
- ii) the growth of L (i.e. its quasi-isometry type) ;
- iii) the level of L (which describes the embedding $j : L \rightarrow M$).

- M. CHAPERON - Quelques outils de la théorie des actions différentiables . 259

This paper is meant as an introduction to some basic ideas in the theory of differentiable actions. It consists in an exposition of more or less classical results in the field, with proofs.

VOLUME 2

L. MÁRKUS - Lie Dynamical Systems.

A Lie Dynamical System is a smooth action of a Lie group $G=K \times \mathbb{R}$, where K is a connected compact Lie group, on a differentiable n -manifold M . In the special case where K reduces to a single point, the flow of \mathbb{R} on M yields a classical dynamical system.

The author generalizes the classical theorems of Poincaré-Bendixon on S^2 , and Denjoy on T^2 , to Lie Dynamical Systems with a non-stationary orbit of codimension 1 in M .

E.M. de JAGER - Introduction to the theory of singular perturbations.

A review of the theory of singular perturbations is presented. Although it is not well possible to cover in a self contained text the whole theory - including the latest important developments - , essential aspects are emphasized, some methods described in detail and proofs have been given. In particular initial and boundary value problems for second order linear and semi-linear ordinary differential equations have been treated ; the generalization to partial differential equations have been indicated.

R. LUTZ - L'intrusion de l'Analyse non Standard dans l'étude des perturbations singulières.

We consider the problem of singular perturbations of differential equations from the point of view of Non Standard Analysis ; it appears as the search for external properties of non-standard flows. Some very simple ideas, following the initial intentions of G. Reeb, prove to yield efficient tools that we apply in much particular cases. We study first pure fast fields of type $\frac{1}{\epsilon} Y$ (ϵ infinitesimal, Y standard), whose integral curves admit free layers that may be located in non-degenerate cases (or degenerate cases with a prime integral) ; then we discuss the general fast fields of type $\frac{1}{\epsilon} Y + Z$ (Y and Z nearly standard), using a very powerful "stroboscopy technic" which, in particular, allows an easy description of fast oscillations the shape of which evolves slowly. A lot of differential equations may easily be studied within this frame ; the paper ends with some general examples.

- T. SARI - Sur la théorie asymptotique des oscillations non stationnaires.

This paper is a contribution to the foundations of the asymptotic method for non linear differential equations. A new approach of the well-known theory of Krylov-Bogolioukov-Mitropolski is given, using the Nonstandard Analysis.

Furthermore, a construction procedure for the higher approximation is presented and applications to other problems show that the new approach can be of valuable help in the study of many problems of non linear oscillations.

- E. BENOIT - Systèmes lents-rapides dans \mathbb{R}^3 et leurs canards.

Let (E) be a slow-fast vector field in \mathbb{R}^3 . We obtain the "reduced" vector field (\tilde{E}) when the parameter of the corresponding singular perturbation takes the value zero. We show, with non standard methods, how the collection of trajectories of (E) converge to that of (\tilde{E}) , in many generic cases.

- I. VAN DEN BERG - Un principe de permanence général.

We present a permanence principle in nonstandard analysis containing Robinson's lemma and the Fehrele principle as special cases. The principle makes it possible to distinguish several classes of external sets and to classify external Dedekind cuts of ordered groups.

- I. VAN DEN BERG - Un point de vue nonstandard sur les développements en série de Taylor.

We describe the convergence behaviour of the sequence of Taylor polynomials for some analytic functions by means of local approximation of the remainders. An important tool in our calculations is a nonstandard version of Lebesgue's dominated convergence theorem.

- J.L. CALLOT - Groupes vus au microscope.

I give a non standard presentation of Gromov's proof that every finitely generated group of polynomial growth has a nilpotent subgroup of finite index.

This illustrates the method of macroscopy : the shadow of a macroscopic view of a discrete group is, in good cases, a continuous metric space.

J. HARTHONG - Éléments pour une théorie du continu.

We propose a theory of real numbers which has no recourse to uncountable cardinalities and requires only classical combinatorics, this means the formal theory of finite sets. Such a theory needs an external intuitionistic point of view upon the formal theory of finite sets. We give a classical example of integration which illustrates first, how classical formulas can be interpreted in our framework, and second, how methods of combinatorics can be underlying well-known classical techniques.

M. GOZE - Etude locale des courbes algébriques planes.

In order to describe the branches of an algebraic curve of \mathbb{C}^2 , with center M_0 , we apply a sequence of quadratic transformations entirely determined by a point M , infinitely close to M_0 (M_0 standard and $M \neq M_0$). The important fact is that this sequence of blowing-up is completely fixed when the point M is given. The process gives us the desingularization of the curve (after a standard number of steps ; i.e. after a standard number of transformations associated to M), and all the branches (with their multiplicity) with center M_0 .