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Astérisque, tome 87-88 (1981), p. 191-205
<http://www.numdam.org/item?id=AST_1981__87-88__191_0>

CONJUGACY CLASSES AND WEYL GROUP REPRESENTATIONS

Hanspeter Kraft

In the celebrated paper [14] T.A.Springer gives an interesting relation between nilpotent conjugacy classes in semisimple Lie algebras and representations of the corresponding Weyl group (cf.also [6],[15]). In this short note we want to describe another construction of Weyl group representations, again starting with nilpotent conjugacy classes, which seems to be strongly related to Springer's construction. Up to now only a few general results are known about these representations; for $\underline{gl}_n$ we have a series of precise conjectures which have been checked for $n \leq 5$.(*)

The starting point of these considerations was a question asked by B. Kostant. I would like to thank him and also W. Borho, C. Procesi and T.A. Springer for helpful discussions.

1. The construction

Let $\underline{g}$ be a complex reductive Lie algebra and G the adjoint group. We consider the adjoint action of G on $\underline{g}$, indicated by $(g,x) \mapsto gx$ for $g \in G$, $x \in \underline{g}$. If X is a variety (or an affine scheme) we denote by $\mathcal{O}(X)$ the $\mathbb{C}$-algebra of global regular functions on X (or the coordinate ring of X). Let $\underline{h} \subset \underline{g}$ be a Cartan subalgebra and W the corresponding Weyl group.

If C is a conjugacy class in $\underline{g}$ and $\overline{C}$ its (Zariski-)closure, we denote by $R_C^{\underline{g}}$ or simply by R_C <u>the coordinate ring of the schematic intersection</u> $\overline{C} \cap \underline{h}$ (a finite scheme!):

$$R_C = \mathcal{O}(\overline{C} \cap \underline{h}) = \mathcal{O}(\underline{h})/I_C ,$$

I_C the image of the ideal

$$J_C := \{f \in \mathcal{O}(\underline{g}) \mid f(x) = 0 \quad \text{for all} \quad x \in \overline{C}\}$$

(*) cf. remark at the end of the paper.

of $\mathcal{O}(g)$ under the canonical projection $\mathcal{O}(\underline{g}) \to \mathcal{O}(\underline{h})$.

Clearly the ideal I_C is stable under W acting in the usual way on $\mathcal{O}(\underline{h})$. In the proposition we collect some properties of this construction; the proofs are easy and left to the reader.

Proposition 1 :(a) R_C is a finite dimensional $\mathbb{C}$ - algebra on which W acts by algebra automorphism.

(b) If C is nilpotent the ideal I_C is homogeneous and R_C has a natural W - stable graduation: $R_C = \bigoplus_{i \geq o} (R_C)_i$.

(c) If C is semisimple (i.e $\overline{C} = C$), $C = Gs$ for some $s \in \underline{h}$, the intersection $\overline{C} \cap \underline{h}$ is reduced and $R_C \underset{W}{\widetilde{\to}} \mathrm{Ind}_{W_s}^{W} \mathbb{C}$, the representation induced from the trivial representation of $W_s := \{w \in W | ws = s\}$.

(d) If $C' \subset \overline{C}$ we have a canonical surjective W - equivariant map $R_C \to R_{C'}$, which is homogeneous in case C nilpotent.

Example: 1) If $C = \{0\}$(or $C = \{z\}$, $z \in$ zent $\underline{g}$) then $R_C = \mathbb{C}$, the trivial representation of W.

2) If C is the regular nilpotent class (i.e the nilpotent class of maximal dimension), the ideal I_C is generated by the homogeneous W - invariants and $R_C =: R_{reg}$ is the regular representation of W ([3] V, § 5, Théorème 2(ii)).

Remark 1 : The construction above works over $\mathbb{Q}$. If the conjugacy class C is defined over $\mathbb{Q}$ then R_C is also defined over $\mathbb{Q}$. In particular for any nilpotent class C the representation of W on R_C is defined over $\mathbb{Q}$.

2. Macdonald representations

Let $\underline{m} \subset \underline{g}$ be a reductive subalgebra containing $\underline{h}$. Consider a system Δ^+ of positive roots of $\underline{m}$. Then the homogeneous function

$$f_{\underline{m}} := \prod_{\alpha\in\Delta^+} \alpha \in \mathcal{O}(\underline{h})$$

of degree

$$d(\underline{m}) := \# \Delta^+ = \frac{1}{2} (\dim \underline{m} - \operatorname{rk} \underline{m})$$

generates an irreducible W-module

$$M_{\underline{m}} = \langle w \cdot f_{\underline{m}} \mid w \in W\rangle \subset \mathcal{O}(\underline{h}),$$

the Macdonald module associated to $\underline{m}$ (cf.[11]). Its character will be denoted by $\mu(\underline{m})$.

Proposition 2 : Let $C_{\underline{m}}$ be the nilpotent conjugacy class in $\underline{g}$ generated by the regular nilpotent class in $\underline{m}$. Then

$$\operatorname{mult}_{\mu(\underline{m})}(R_{C_{\underline{m}}})_i = \begin{cases} 0 & i < d(\underline{m}) \\ 1 & i = d(\underline{m}) \end{cases}$$

Proof: The anti-invariant elements of $\mathcal{O}(\underline{h})$ with respect to W', the Weyl group of $\underline{m}$, are given by $f_{\underline{m}} \cdot \mathcal{O}(\underline{h})^{W'}$ and the image of $f_{\underline{m}}$ in $R^{\underline{m}}_{\overline{reg}}$ is not zero (cf. [11]). The claim now follows since the canonical map $R^{\underline{g}}_{\overline{C}_{\underline{m}}} \to R^{\underline{m}}_{\overline{reg}}$ is W'-equivariant and surjective.

In case $\underline{g} = \underline{gl}_n$ the nilpotent conjugacy classes are in one-to-one correspondence with the partitions of n . If $\lambda = (\lambda_0, \lambda_1, \dots, \lambda_s)$ is a partition of n , i.e. $\lambda_i \in \mathbb{N}$, $\lambda_0 \geq \lambda_1 \geq \dots \geq \lambda_s$ and $\sum_i \lambda_i = n$, the corresponding class C_λ is the conjugacy class of a nilpotent element $x \in \underline{gl}_n$ in Jordan normal form with Jordan blocks of sizes $\lambda_0, \lambda_1, \dots, \lambda_s$. Clearly each class C_λ is of the form $C_{\underline{m}}$: choose $\underline{m} = \underline{m}_\lambda := \underline{gl}_{\lambda_0} \oplus \underline{gl}_{\lambda_1} \oplus \dots \oplus \underline{gl}_{\lambda_s} \subset \underline{gl}_n$.

Furthermore it is know that the associated Macdonald module $M := M_{\underline{m}}$ is the Specht module $S^{\hat\lambda}$, $\hat\lambda = (\hat\lambda_0, \hat\lambda_1, \dots, \hat\lambda_t)$ the dual partition to λ, i.e. $\hat\lambda_i = \#\{j \mid \lambda_j > i\}$ (cf. [11]).
In addition we have

$$d(\lambda) := d(\underline{m}_\lambda) = \sum_i \binom{\lambda_i}{2} = \sum_i i \cdot \hat\lambda_i$$

<u>Proposition 3</u> : <u>Let $C \subset \underline{gl}_n$ be a nilpotent conjugacy class with partition</u> λ. Then $(R_C)_i = 0$ <u>for</u> $i > d(\lambda)$ and the <u>Specht module</u> $S^{\hat\lambda}$ <u>has multiplicity one in</u> R_C <u>and it occurs in the top degree</u> $d(\lambda)$.

<u>Proof</u>: It remains to prove the first assertion, i.e to show that the ideal I_C contains all monomials of degree $> d(\lambda)$. We have $\operatorname{rk} x^k \leq n_k := \sum_{i \geq k} \hat\lambda_i$ for $x \in \overline{C}$ and all $k \in \mathbb{N}$, hence the $(n_k+1) \times (n_k+1)$ - minors of the matrix x^k vanish. Restricting these functions to $\underline{h}$ we obtain some elements in I_C and one easily checks that each monomial of degree $> d(\lambda)$ occurs in the ideal generated by these elements.

<u>Conjecture 1</u> : $(R_{C_\lambda})_{d(\lambda)} \cong S^{\hat\lambda}$.

This holds for $n \leq 5$ (cf. tables) and some other classes (e.g. for partitions of type $(\lambda_0, 1, 1, \dots, 1)$ or $(2,2,2,\dots,1,1,\dots 1)$).

<u>Remark 2</u> : The conjecture above implies that the irreducible representations of S_n are defined over $\mathbb{Q}$ (cf. remark 1).

<u>Remark 3</u> : For $\underline{g} = \underline{sp}_{2n}$ each nilpotent conjugacy class C is of the form $C_{\underline{m}}$ for some suitable reductive subalgebra $\underline{m} \supset \underline{h}$, but there can be different choices for $\underline{m}$ giving different Macdonald modules in different degrees. E.g. in $\underline{sp}_4$ the class $C_{(2,2)}$ is obtained from $\underline{m}_1 \simeq \underline{gl}_2$ and $\underline{m}_2 \simeq \underline{sp}_2 \oplus \underline{sp}_2$; $M_{\underline{m}_1}$ is the natural representation of W (in degree 1) and $M_{\underline{m}_2}$ is a one dimensional representation (in degree 2).

3. Associated cones and induced representations

Let $\underline{p} \subseteq \underline{g}$ be a parabolic subalgebra with $\underline{h} \subset \underline{p}$ and $\underline{n} \subset \underline{p}$ the nilradical. Since $G\underline{n} = \{gx \mid g \in G, x \in \underline{n}\}$ is an irreducible subset of nilpotent elements it contains a dense conjugacy class $C_{\underline{p}}$, the Richardson class associated to $\underline{p}$. This class depends only on the Levi part $\underline{m}$ of $\underline{p}$. Furthermore the connected component G_x^o of the stabilizer of an element $x \in C_{\underline{p}} \cap \underline{n}$ is contained in the parabolic subgroup P with Lie $P = \underline{p}$. Let us denote by $W_{\underline{p}}$ the subgroup of W corresponding to $\underline{p}$ (i.e the Weyl group of the Levi part $\underline{m} \supset \underline{h}$ of $\underline{p}$).

Proposition 4 : With the notations above assume that

(i) $\overline{C}_{\underline{p}}$ is a normal variety and

(ii) $G_x \subset P$ for $x \in C_{\underline{p}} \cap \underline{n}$.

Then $R_{C_{\underline{p}}}$ contains the induced representation $\mathrm{Ind}_{W_{\underline{p}}}^{W} \mathbb{C}$ of the trivial representation of $W_{\underline{p}}$.

Proof: Consider the Levi decomposition $\underline{p} = \underline{m} \oplus \underline{n}$ with $\underline{h} \subset \underline{m}$ and the center $\underline{z}$ of $\underline{m}$. For a generic element $h \in \underline{h}$ we have $W_h = W_{\underline{p}}$, hence $R_{C'} = \mathcal{O}(\underline{h})/I_{C'} \simeq \mathrm{Ind}_{W_{\underline{p}}}^{W} \mathbb{C}$ for the conjugacy class C' of h (Proposition 1(c)). Let $J' \subset \mathcal{O}(\underline{g})$ be the ideal of functions vanishing on C' and denote by grJ' the ideal generated by the terms of maximal degree of the elements of J'. Then it follows from [2] (Zusatz 3.8 and Theorem A1) that under the assumptions (i) and (ii) $gr\,J'$ is the ideal J of functions vanishing on $C_{\underline{p}}$. Now $I' := I_C$ is the image of $J = grJ'$. This clearly implies that $I \subseteq grI'$ (where grI' is defined in a similar way as grJ'), hence we have a surjective W-equivariant map $\mathcal{O}(\underline{h})/I = R_{C_{\underline{p}}} \twoheadrightarrow \mathcal{O}(\underline{h})/grI'$. Now $\mathcal{O}(\underline{h})/grI'$ is the associated graded algebra with respect to the ascending filtration induced by the degree, hence as W-modules $R_{C'}$ and $\mathcal{O}(\underline{h})/grI'$ are isomorphic.

Remark 4 : In [5] Hesselink describes the Richardson classes in classical Lie algebras and the numbers $[G_x : P_x]$. The papers [8],[13] and [9] are concerned with the normality problem of closures of conjugacy classes in classical Lie algebras. The result for $\underline{g} = \underline{gl}_n$ is the following (cf.[7]):

(a) For any element $x \in \underline{gl}_n$ the stabilizer $\mathrm{Stab}_{GL_n} x$ is connected and the conjugacy class C of x has a normal closure $\overline{C}$.

(b) If $\underline{p} \subseteq \underline{gl}_n$ is a parabolic with Levi part $\underline{m} \simeq \underline{gl}_{\lambda_0} \oplus \underline{gl}_{\lambda_1} \oplus \dots \oplus \underline{gl}_{\lambda_s}$, $\lambda = (\lambda_0, \lambda_1, .., \lambda_s)$ a partition of n , then $C_{\underline{p}} = C_{\hat{\lambda}}$ and $W_{\underline{p}} = S_\lambda :=$ $S_{\lambda_0} \times S_{\lambda_1} \times .. \times S_{\lambda_s} \subseteq S_n$.

From this we get the following corollary.

Corollary: Let $C \subset \underline{gl}_n$ be a nilpotent conjugacy class with partition λ . Then R_C contains the induced representation $\mathrm{Ind}_{S_{\hat{\lambda}}}^{S_n} \mathbb{C}$.

Conjecture 2 : $R_{C_\lambda} \simeq \mathrm{Ind}_{S_{\hat{\lambda}}}^{S_n} \mathbb{C}$

Again this holds for $n \leq 5$ and some other classes.

On the other hand the class $C_{(3,3)}$ of $\underline{sp}_6$ is the Richardson class associated to the parabolic $\underline{p}$ with Levi part isomorphic to $\underline{gl}_2 \oplus \underline{sp}_2$ and satisfies the assumptions of proposition 4 , but $R_{C_{(3,3)}}$ is strictly bigger than the induced representation $\mathrm{Ind}_{W_{\underline{p}}}^{W} \mathbb{C}$ (see tables).

Remark 5 : For two nilpotent conjugacy classes C_λ and C_μ in $\underline{gl}_n$ one has $\overline{C}_\lambda \supset C_\mu$ if and only if $\lambda \geq \mu$ i.e $\sum_{i=o}^{k} \lambda_i \geq \sum_{i=o}^{k} \mu_i$ for all k (cf.[5]Theorem 3.10).

The conjecture above would imply one implication of the following know result (sometimes called Snapper conjecture) :

$\mathrm{Ind}_{S_\lambda}^{S_n} \mathbb{C}$ is contained in $\mathrm{Ind}_{S_\mu}^{S_n} \mathbb{C}$ if and only if $\lambda \geq \mu$.

Remark 6 : In the paper [10] G. Lusztig and N. Spaltenstein introduce the concept of inducing conjugacy classes(cf.[1]): Starting from a Levi subalgebra $\underline{m} \subseteq \underline{g}$ they associate to any conjugacy class C in $\underline{m}$ a class $\mathcal{C} = \mathrm{Ind}_{\underline{m}}^{\underline{g}} C$ in $\underline{g}$: $\mathcal{C}$ is the dense class in $G(C+\underline{n})$, where $\underline{n}$ is the nilradical of a parabolic $\underline{p}$ in $\underline{g}$ with Levi part $\underline{m}$ .Generalizing proposition 4 one can prove that under similar conditions (e.g. $\mathcal{C}$ has a normal closure and the stabilizer of any $x \in \mathcal{C}$ is connected)$R_{\mathcal{C}}^{\underline{g}}$ contains the induced representation $\mathrm{Ind}_{W'}^{W} R_{C}^{\underline{m}}$, W' the Weyl group of $\underline{m}$.

4. Relation with the theory of Springer([6],[14],[15])

Let $\mathcal{B}$ be the variety of Borel subgroups of G and $x \in \underline{g}$ a nilpotent element. Springer's construction yields a representation of W on the cohomology $H^*(\mathcal{B}_x, \mathbb{C})$, where

$$\mathcal{B}_x := \{B \in \mathcal{B} \mid \mathrm{Lie}\, B \ni x\} \quad .$$

There is a canonical isomorphism

$$R_{reg}^{\underline{g}} \xrightarrow{\sim} H^*(\mathcal{B}, \mathbb{C})$$

of graded $\mathbb{C}$-algebras, hence by restriction a $\mathbb{C}$-algebra homomorphism

$$\varphi_x : R_{reg}^{\underline{g}} \to H^*(\mathcal{B}_x, \mathbb{C}).$$

It turns out that after twisting Springer's representation with the sign representation ε this map becomes W-equivariant (cf.[6]). The twisted representation will be denoted by $H^*(\mathcal{B}_x, \mathbb{C})_\varepsilon$.

In case $\underline{g} = \underline{gl}_n$ the map φ_{x_*} is always surjective and, as remarked by Macdonald, $H^*(\mathcal{B}_x, \mathbb{C})_\varepsilon$ is an induced representation.

Conjecture 3 : If $x \in \underline{gl}_n$ is nilpotent with partition λ, the two quotient maps $\varphi_x : R_{reg}^{\underline{gl}_n} \to H^*(\mathcal{B}_x, \mathbb{C})$ and $R_{reg}^{\underline{gl}_n} \to R_{C_{\hat{\lambda}}}$ are the same.

Remark 7 : There is some duality between the Springer representation ρ_x on $H^*(\mathcal{B}_x, \mathbb{C})$ and the representations considered here. E.g. $\rho_x \otimes \varepsilon$ is the trivial representations for a regular nilpotent element x and $\rho_x \otimes \varepsilon$ is the regular representation. The conjecture above states that for $\underline{g} = \underline{gl}_n$ this duality comes from the duality on the nilpotent conjugacy classes given by $C_\lambda \mapsto C_{\hat{\lambda}}$. It is not clear to me what happens for the other simple Lie algebras, since there is no such duality on the nilpotent classes.

5. Some tables

In the following tables we list the irreducible representations of W and give the decomposition of the representations $R_\lambda = R_{C_\lambda}$, λ a partition, in the various degrees $(R_\lambda)_i$ for $\underline{gl}_n$ $(n \leq 5)$ and sp_{2m} $(m \leq 3)$. A reductive subalgebra $\underline{m} \subseteq \underline{g}$ of the same rank as $\underline{g}$ is given by its type X (e.g. $X = A_1 \times A_2$ or $C_1 \times A_2$) and $\mu(X)$ denotes the character of the (irreducible) Macdonald representation $M_{\underline{m}}$ (cf. section 2). The character of the sign representation is indicated by ε .
The tables have been calculated using the decomposition of $R_{reg}^{\underline{g}}$ (for $\underline{gl}_n$, $n \leq 6$, given in [12] p. 126-127) and the results of section 2 and 3.

Type $A_1 = C_1$ ($\underline{gl}_2$ and $\underline{sp}_2$)

R_λ \ deg	0	1
(2)	1	ε
(1,1)	1	

Type A_2 ($\underline{gl}_3$)

$\chi = \mu(A_1)$ natural representation, dim $\chi = 2$

R_λ \ deg	0	1	2	3
(3)	1	χ	χ	ε
(2,1)	1	χ		
(1^3)	1			

Type A_3 ($\underline{gl}_4$)

$\chi_1 = \mu(A_1)$ natural representation, dim $\chi = 3$

$\chi_2 = \mu(A_1 \times A_1) = \varepsilon\chi_2$, dim $\chi_2 = 2$

$\chi_3 = \mu(A_2) = \varepsilon\chi_1$, dim $\chi_3 = 3$

R_λ \ deg	0	1	2	3	4	5	6
(4)	1	x_1	x_1+x_2	x_1+x_3	x_2+x_3	x_3	ε
(3,1)	1	x_1	x_1+x_2	x_3			
(2,2)	1	x_1	x_2				
(2,1,1)	1	x_1					
(1^4)	1						

Type A_4 ($\underline{gl}_5$)

$x_1 = \mu(A_1)$ natural representation, $\dim x_1 = 4$

$x_2 = \mu(A_1 \times A_1)$, $\dim x_2 = 5$

$x_3 = \mu(A_2) = \varepsilon x_3$, $\dim x_3 = 6$

$x_4 = \mu(A_1 \times A_2) = \varepsilon x_2$, $\dim x_4 = 5$

$x_5 = \mu(A_3) = \varepsilon x_1$, $\dim x_5 = 4$

R_λ \ deg	0	1	2	3	4	5	6	7	8	9	10
(5)	1	χ_1	$\chi_1+\chi_2$	$\chi_1+\chi_2+\chi_3$	$\chi_1+\chi_2+\chi_3+\chi_4$	$\chi_2+2\chi_3+\chi_4$	$\chi_2+\chi_3+\chi_4+\chi_5$	$\chi_3+\chi_4+\chi_5$	$\chi_4+\chi_5$	χ_5	ε
(4,1)	1	χ_1	$\chi_1+\chi_2$	$\chi_1+\chi_2+\chi_3$	$\chi_2+\chi_3+\chi_4$	$\chi_3+\chi_4$	χ_5				
(3,2)	1	χ_1	$\chi_1+\chi_2$	$\chi_2+\chi_3$	χ_4						
(3,1,1)	1	χ_1	$\chi_1+\chi_2$	χ_3							
(2,1,1)	1	χ_1	χ_2								
$(2,1^3)$	1	χ_1									
(1^5)	1										

Type C_2 ($\underline{sp}_4$)

$\chi_1 = \mu(C_1) = \mu(A_1) = \varepsilon\chi_1$ natural representation, $\dim \chi_1 = 2$
$\chi_2 = \mu(C_1 \times C_1)$, $\dim \chi_2 = 1$
$\chi_3 = \varepsilon\chi_2$, $\dim \chi_3 = 1$ (not a Macdonald representation)

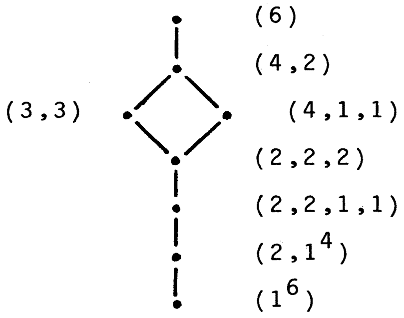
R_λ \ deg	0	1	2	3	4
(4)	1	χ_1	$\chi_2+\chi_3$	χ_1	ε
(2,2)	1	χ_1	χ_2		
(2,1,1)	1	χ_1			
(1^4)	1				

Type C_3 ($\underline{sp}_6$)

$\chi_1 = \mu(A_1) = \mu(C_1)$ natural representation, $\dim \chi_1 = 3$
$\chi_2 = \mu(A_1 \times C_1)$, $\dim \chi_2 = 3$
$\chi_3 = \mu(A_2) = \varepsilon\chi_2$, $\dim \chi_3 = 3$
$\chi_4 = \mu(C_2) = \varepsilon\chi_1$, $\dim \chi_4 = 3$
$\rho = \mu(C_1 \times C_2)$, $\dim \rho = 2$
$\varepsilon\rho$ natural representation of $W/\mathbb{Z}_2^3 \cong S_3$, $\dim \varepsilon\rho = 2$
$\tau = \mu(C_1 \times C_1 \times C_1)$, $\dim \tau = 1$
$\varepsilon\tau$ sgn representation of $W/\mathbb{Z}_2^3 \cong S_3$

<u>One has</u>: $\tau\rho = \varepsilon\rho$, $\tau\chi_1 = \chi_2$;
$\varepsilon\rho$, $\varepsilon\tau$ not Macdonald representations

Diagram of inclusions of closures of nilpotent conjugacy classes:



R_λ \ deg	0	1	2	3	4	5	6	7	8	9
(6)	1	χ_1	$\chi_2+\varepsilon\rho$	$\chi_1+\chi_3+\tau$	$\chi_2+\chi_4+\varepsilon\rho$	$\chi_1+\chi_3+\rho$	$\chi_2+\chi_4+\varepsilon\tau$	$\chi_3+\rho$	χ_4	ε
(4,2)	1	χ_1	$\chi_2+\varepsilon\rho$	$\chi_1+\chi_3+\tau$	$\chi_2+\chi_4$	ρ				
(3,3)	1	χ_1	$\chi_2+\varepsilon\rho$	$\chi_3+\tau$						
(4,1,1)	1	χ_1	$\chi_2+\varepsilon\rho$	$\chi_1+\chi_3+\tau$	χ_4					
(2,2,2)	1	χ_1	χ_2	τ						
(2,2,1,1)	1	χ_1	χ_2							
$(2,1^4)$	1	χ_1								
(1^6)	1									

Remark :

The classes $C_{(6)}$, $C_{(4,2)}$, $C_{(3,3)}$, $C_{(2,2,2)}$, $C_{(2,2,1,1)}$, $C_{(2,1^4)}$ are Richardson classes; $R_{(6)}$, $R_{(4,2)}$, $R_{(2,2,2)}$, $R_{(2,1^4)}$ are induced representations, $R_{(2,2,1,1)}$ does not contain the corresponding induced representation (condition (ii) of proposition 4 is not satisfied), $R_{(3,3)}$ is strictly bigger than the corresponding induced representation (here condition (i) and (ii) are both satisfied).

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After the preparations of this manuscript I was informed by C. De Concini and C. Procesi that they have proved all the conjectures stated in this paper.

De Concini, C., Procesi, C.: Symmetric functions, conjugacy classes and the flag variety. Inventiones 64 (1981) 203-220

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