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**Positive definite binary quadratic forms and Hilbert modular surfaces (joint work with D. Zagier)**

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POSITIVE DEFINITE BINARY QUADRATIC FORMS AND HILBERT  
MODULAR SURFACES (joint work with D. Zagier)\*

by

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Let  $p$  be a prime  $\equiv 1 \pmod{4}$  and  $\mathcal{O}$  the ring of integers of  $\mathbb{Q}(\sqrt{p})$ . The group  $SL_2(\mathcal{O})$  operates on  $H^2$ . The Hilbert modular surface  $X = H^2/SL_2(\mathcal{O})$  is a non-compact complex surface with finitely many singularities. On  $X$  we define a series of "modular" curves  $F_1, F_2, F_3, \dots$ . All intersections of  $F_M$  and  $F_N$  ( $M \neq N$ ) are transversal and occur at certain distinguished points of  $X$  which we call special. To each special point  $z \in X$  is associated a positive definite binary quadratic form  $\varphi_z$ , and  $F_M$  and  $F_N$  meet in  $z$  if and only if the form  $\varphi_z$  represents both  $M$  and  $N$  primitively. The following questions will be answered: How often does a given form occur as the form  $\varphi_z$  for  $z \in X$ ? How does one calculate the intersection number of  $F_M$  and  $F_N$ ? These questions led to elementary problems concerning the representation of numbers by positive definite binary forms and the representation of forms by forms, which will be discussed. The intersection numbers are related to coefficients of modular forms for  $\Gamma_0(p)$ , cf. the lecture by D. Zagier.

\* F. Hirzebruch and D. Zagier, Intersections numbers of Curves on Hilbert modular surfaces and modular forms of Nebentypus, *Inventiones math.* 36, 57-113 (1976).