FRIEDRICH HIRZEBRUCH

Positive definite binary quadratic forms and Hilbert modular surfaces (joint work with D. Zagier)

Astérisque, tome 41-42 (1977), p. 229
<http://www.numdam.org/item?id=AST_1977__41-42__229_0>

Société Mathématique de France
Astérisque 41-42 (1977) p.229

POSITIVE DEFINITE BINARY QUADRATIC FORMS AND HILBERT MODULAR SURFACES (joint work with D. Zagier)*

by

F. HIRZEBRUCH

Let p be a prime $\equiv 1 \bmod 4$ and $\mathcal{O}$ the ring of integers of $\mathbb{Q}(\sqrt{p})$. The group $SL_2(\mathcal{O})$ operates on H^2. The Hilbert modular surface $X = H^2/SL_2(\mathcal{O})$ is a non-compact complex surface with finitely many singularities. On X we define a series of "modular" curves $F_1, F_2, F_3, \ldots$ All intersections of F_M and F_N $(M \neq N)$ are transversal and occur at certain distinguished points of X which we call special. To each special point $z \in X$ is associated a positive definite binary quadratic form φ_z, and F_M and F_N meet in z if and only if the form φ_z represents both M and N primitively. The following questions will be answered : How often does a given form occur as the form φ_z for $z \in X$? How does one calculate the intersection number of F_M and F_N ? These questions led to elementary problems concerning the representation of numbers by positive definite binary forms and the representation of forms by forms, which will be discussed. The intersection numbers are related to coefficients of modular forms for $\Gamma_o(p)$, cf. the lecture by D. Zagier.

* F. Hirzebruch and D. Zagier, Intersections numbers of Curveson Hilbert modular surfaces and modular forms of Nebentypus, Inventiones math. 36, 57-113 (1976).