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PHASE TRANSITIONS IN FERROMAGNETIC SPIN SYSTEMS  
AT LOW TEMPERATURES

J. Slawny

In a joint work of W. Holsztyński and the author a criterium of the existence of phase transitions in ferromagnetic spin systems at low temperatures has been given. It involves properties of an algebra canonically associated with any such system:

Let  $\mathbb{F}_2[\mathbb{Z}^V]$  be the group algebra of the group  $\mathbb{Z}^V$  of translations with coefficients in the two-element field  $\mathbb{F}_2 = \{0,1\}$  ; with a ferromagnetic interaction  $J$  one associates an ideal  $\bar{B}(J)$  of  $\mathbb{F}_2[\mathbb{Z}^V]$  . It then appears that if this ideal is not a principal one the system exhibits phase transitions at low enough temperatures.

As far as the calculations are concerned, to apply this criterium one has to find the greatest common divisor of few polynomials (with coefficients in  $\mathbb{F}_2$ ) and to find out if this g.c.d. belongs to  $\bar{B}(J)$  . This is a rather easy task.

The criterium can be developed into a method of calculation of all ergodic equilibrium states at low temperatures. One then easily produces examples of models with a large number of ergodic equilibrium states and with varying mixing properties.

The details and a number of examples can be found in:

- [1] W. Holsztyński, J. Slawny, Phases of Ferromagnetic Spin Systems at Low Temperatures, Lett. N.C. 1975.
- [2] W. Holsztyński, J. Slawny, In preparation.
- [3] J. Slawny, Ferromagnetic Spin Systems at Low Temperature, to appear in Commun. Math. Phys.

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