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RAGHAVAN NARASIMHAN

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AUTOMORPHISMS OF BOUNDED DOMAINS IN $\mathbf{C}^n$

by Raghavan NARASIMHAN

[Chicago]

It has been known since the time of Poincaré [13] that the Riemann mapping theorem has no analogue in $\mathbf{C}^n$ when $n \geq 2$. In fact the polydisc $\{(z_1,z_2) \in \mathbf{C}^2 \mid |z_1| < 1, |z_2| < 1\}$ and the ball $\{(z_1,z_2) \in \mathbf{C}^2 \mid |z_1|^2 + |z_2|^2 < 1\}$ are not biholomorphically equivalent. But the very great extent to which domains in $\mathbf{C}^n$, $n \geq 2$, are rigid first appeared in a series of papers of H. Cartan (see in particular the paper [4]). This article will attempt to describe a few of H. Cartan's results, and then to describe recent work of C. Fefferman and S. S. Chern - J. Moser which enable one to attach very strong invariants to a very important class of domains, enabling one to distinguish between them in many cases.

§ 1. Let D be a bounded domain in $\mathbf{C}^n$, and suppose that $0 \in D$. We say that D is circular (with respect to 0) if D is left invariant by the transformation

$$z \mapsto e^{i\theta} z$$

for any real $\theta \in \mathbf{R}$.

If $(p_1,\ldots,p_n)$ are integers which are relatively prime, we call D a $(p_1,\ldots,p_n)$-domain if D is left invariant by the transformation

$$T_\theta^{(p)} : (z_1,\ldots,z_n) \mapsto (e^{i\theta p_1} z_1,\ldots,e^{i\theta p_n} z_n)$$

for any real θ.

Similar definitions can be made when D is an unramified domain over $\mathbf{C}^n$ (i.e. D is a complex manifold of dimension n provided with a holomorphic immersion in $\mathbf{C}^n$).

Cartan's paper [4] is concerned mainly with domains over $\mathbb{C}^2$. The first result showing how rigid circular domains are is the following.

THEOREM 1 (Cartan [4]).- An automorphism of a bounded circular domain D (or a (p_1,p_2)-domain) in $\mathbb{C}^2$ which leaves the origin fixed is the restriction to D of a complex linear transformation of $\mathbb{C}^2$.

The main result in [4] deals with the problem of classifying domains in $\mathbb{C}^2$; it is the following remarkable result.

THEOREM 2 (Cartan [4]).- Let D be an unramified bounded domain over $\mathbb{C}^2$, $0 \in D$, and suppose that there is an infinite group of automorphisms of D which leaves 0 fixed. Then D is analytically equivalent to an unramified (p_1,p_2)-domain over $\mathbb{C}^2$.

This theorem can be reformulated by saying that if the circle S^1 acts as a group of holomorphic transformations of D having a fixed point, then the action of S^1 can be linearised globally by an immersion of D in $\mathbb{C}^2$.

This theorem has been generalised by H. Holmann [9] to manifolds of higher dimension ; however the situation is more complicated. To state his theorem, we introduce a definition.

We say that a complex manifold X of dimension n is a Cartan manifold if there exists an atlas of coordinate neighbourhoods (U_i, φ_i) such that :

(i) $\varphi_i(U_i)$ is a $p^{(i)} = (p_1^{(i)},\dots,p_n^{(i)})$-domain in $\mathbb{C}^n$;

(ii) the coordinate transformations $\varphi_{ij} = \varphi_i \circ \varphi_j^{-1}$ are compatible with the $T_\theta^{(p_i)}$, i.e. $\varphi_{ij} \circ T_\theta^{(p_j)} = T_\theta^{(p_i)} \circ \varphi_{ij}$.

Holmann's theorem is as follows.

THEOREM 3.- Let X be a connected complex manifold on which an infinite compact Lie group G acts effectively (as holomorphic automorphisms of X). Then X is a Cartan manifold.

When X is a Stein manifold and G has a fixed point, Holmann gives a more precise description of X. In this case, let S be a compact 1-parameter subgroup of G. Then there exists a Stein manifold Y of dimension $m < n$, and a holomorphic vector bundle V of rank $r = n - m$ on Y such that X is isomorphic to an open set Ω in V with the property that there exist relatively prime integers $(p_1, \ldots, p_r)$ so that, for any $y \in Y$, $\Omega \cap V_y$ is left invariant by the transformations

$$(z_1, \ldots, z_r) \mapsto (e^{ip_1\theta} z_1, \ldots, e^{ip_r\theta} z_r) \quad , \qquad \theta \in \mathbb{R}$$

where V_y is the fibre of V over y and $(z_1, \ldots, z_r)$ are suitable fibre coordinates on V_y (so that the transition functions of V commute with the $T_\theta^{(p)}$).

[That the hypothesis concerning the Lie structure on the group G and its compactness are natural is shown by another theorem of H. Cartan (see theorem 6 below).]

A consequence of this description is the following result :

A Stein manifold of dimension 2 on which an infinite compact Lie group acts as above and has a fixed point can be immersed holomorphically in $\mathbb{C}^2$ (so that it is an unramified domain over $\mathbb{C}^2$).

One of the essentials in the proofs of these theorems is the following result :

THEOREM 4 (Cartan [4,5]).- Let D be a bounded domain in $\mathbb{C}^n$, $0 \in D$, $f : D \to D$ a holomorphic map. Suppose that $f(0) = 0$ and that $\frac{\partial f_i}{\partial z_j}(0) = \delta_{ij}$ $(f = (f_1, \ldots, f_n))$. Then f is the identity.

Cartan's proof of this beautiful result is so simple and elegant that it is worth repeating.

Let $f(z) = z + P_k(z) + \ldots$ $(k \geq 2)$ be the expansion of f in a series of homogeneous polynomials at 0 ; P_ν is of degree ν. Let $f_p = \underbrace{f \circ \ldots \circ f}_{p\text{-times}}$.

Then the expansion of f_p is

$$f_p(z) = z + pP_k(z) + \dots \quad .$$

Since f_p maps D into D which is <u>bounded</u>, the coefficients of pP_k are bounded, uniformly in p, and so must be 0 since p is arbitrary.

This result is false for unbounded domains in $\mathbb{C}^n$. Bieberbach [2] has given an example of an injective map $f : \mathbb{C}^2 \to \mathbb{C}^2$, $f(0) = 0$, $\frac{\partial f_i}{\partial z_j}(0) = \delta_{ij}$ so that $\mathbb{C}^2 - f(\mathbb{C}^2)$ contains a non-empty open set. This counterexample of Bieberbach's has been used recently to prove the following result, which had been long conjectured :

THEOREM 5 (T. Nishino - C. Watanabe [12]).- The unit disc $\{z \in \mathbb{C} \mid |z| < 1\}$ can be imbedded as a closed submanifold of $\mathbb{C}^2$.

It is easily deduced from Theorem 1 that the following result holds :

Let $P_n = \{(z_1,\dots,z_n) \in \mathbb{C}^n \mid |z_j| < 1\}$ and $B_n = \{z_1,\dots,z_n) \in \mathbb{C}^n \mid \Sigma |z_j|^2 < 1\}$. The connected component of the identity of the group of automorphisms of P_n and B_n leaving 0 fixed are respectively the following :

$$(z_1,\dots,z_n) \mapsto (e^{i\theta_1} z_1,\dots,e^{i\theta_n} z_n) , \qquad \theta_j \in \mathbb{R} ,$$

$$z \mapsto Az , \qquad A \in U(n) \quad \text{(unitary group)} .$$

In particular, P_n and B_n are not analytically isomorphic for $n \geq 2$.

This last result is a consequence of the following theorem of Remmert-Stein [14].

Let D_1, D_2 be bounded domains in $\mathbb{C}^{n_1}$, $\mathbb{C}^{n_2}$ respectively, and Ω a bounded, strongly pseudoconvex domain in $\mathbb{C}^N$ $(N \geq 1)$. Then there is no proper holomorphic map of $D_1 \times D_2$ into Ω.

<u>Proof</u>. Let $(z_1,\dots,z_{n_1})$ be coordinates in $\mathbb{C}^{n_1}$, $(w_1,\dots,w_{n_2})$ coordinates in $\mathbb{C}^{n_2}$ and $f = (f_1,\dots,f_N)$ a proper holomorphic map of $D_1 \times D_2$ into Ω. Choose $w^{(\nu)}$ a sequence of points in D_2 converging to a point $w^{(o)} \in \partial D_2$.

By Montel's theorem, there is a subsequence $\{\nu_p\}$ so that for $1 \le j \le N$,

$$f_j(z\,,w^{(\nu_p)})$$

converges, uniformly on compact subsets of D_1, to a holomorphic function $g_j : D_1 \to \mathbb{C}$.

Since f is proper, $(g_1,\ldots,g_N) = g$ maps D_1 into $\partial\Omega$. Let p be a strongly plurisubharmonic function in a neighbourhood U of $\partial\Omega$ so that $\Omega \cap U = \{x \in U \mid p(x) < 0\}$. Then $p(g(z)) \equiv 0$ on D_1. Now, for $1 \le k\,,\ell \le n_1$, we have

$$0 = \frac{\partial^2 p(g(z))}{\partial z_k \partial \bar{z}_\ell} = \sum_{1 \le i\,,\,j \le N} \frac{\partial^2 p}{\partial x_i \partial \bar{x}_j}(g(z)) \cdot \frac{\partial g_i}{\partial z_k}\overline{\frac{\partial g_j}{\partial \bar{z}_\ell}} .$$

Since the Levi form of p is positive definite by hypothesis, this implies that

$$\frac{\partial g_i}{\partial z_k} \equiv 0\,, \qquad 1 \le i \le N\,, \qquad 1 \le k \le n_1 .$$

Hence, for any $w^{(\nu)} \to w^{(o)}$, there is a subsequence (ν_p) so that

$$\frac{\partial f_i}{\partial z_k}(z\,,w^{(\nu_p)}) \to 0 .$$

Hence

$$\frac{\partial f_i}{\partial z_k}(z,w) \to 0 \qquad \text{as } w \to w^{(o)} .$$

By the maximum principle, $\frac{\partial f_i}{\partial z_k}(z,w) \equiv 0$, $w \in D_2$, and clearly f is not proper.

One of the most important of H. Cartan's theorems on automorphisms of bounded domains is the following :

THEOREM 6 (H. Cartan [5,6]).- Let D be a bounded domain, and let $G = \mathrm{Aut}(D)$ be the group of holomorphic automorphisms of D with the topology of compact convergence. Then

(i) for any compact set $K \subset D$, the set $\{g \in G \mid G(K) \cap K \neq \emptyset\}$ is compact (so that G is locally compact and acts properly on D) ;

(ii) G carries a natural structure of Lie group so that the map

$$G \times D \to D \ , \quad (g,z) \mapsto g(z)$$

is real analytic ;

(iii) G has real dimension $\leq n(n+2)$.

(A proof is given also in [11].) .

This theorem, apart from being a major contribution to the solution of Hilberts 5^{th} problem, has been of great influence in many fields. Bochner-Montgomery used Cartan's ideas to prove that a locally compact group acting as a group of diffeomorphisms of a smooth manifolds is a Lie group. Cartan's work also led to the famous classification by E. Cartan of the bounded symmetric domains in $\mathbf{C}^n$. The theorem has been generalized in various directions (see W. Kaup [10] and the references given there). One of Kaup's results is an addendum to part (iii) of Cartan's theorem : $\dim G = n(n+2)$ if and only if D is analytically isomorphic to the unit ball B_n in $\mathbf{C}^n$.

There are many more important results obtained by Cartan about the structure of mappings of bounded domains into themselves (in particular automorphisms). For these, see Cartan [4,5], Behnke-Thullen [1] and the references given there. There are also many extensions of these results. Of these, I shall mention only the paper of Remmert-Stein [14] dealing with the rigidity of analytic polyhedra, in particular simplices in $\mathbf{C}^n$.

There are also several open questions in this circle of ideas. What is the nature of proper holomorphic maps of B_n into itself ? Are they automorphisms ? If not, are they at least given by rational functions ?

§ 2. Cartan's work enables one to distinguish between two domains (in certain cases) because of the behaviour of analytic automorphisms in the interior of these domains. As early as 1907, Poincaré suggested that one should look at boundary points, which raises the question of knowing if automorphisms of bounded domains can be extended to the boundary, and if so, what degree of smoothness these extensions have. This question has proved very difficult and there were almost no results till recently, when C. Fefferman solved this problem in a very important case. His theorem runs as follows :

THEOREM 7 (Fefferman [8]).- Let D_1 and D_2 be bounded strongly pseudoconvex domains in $\mathbf{C}^n$, and let $f : D_1 \to D_2$ be an analytic isomorphism. Then f extends to a diffeomorphism $\tilde{f}$ of $\bar{D}_1$ onto $\bar{D}_2$.

There are two major steps in the proof of Fefferman's theorem, each of them of great difficulty.

The first step is an asymptotic expansion for the Bergman kernel function of a strongly pseudoconvex domain D in $\mathbf{C}^n$. Recall that if H_D is the Hilbert space of square integrable holomorphic functions on D with the L^2 norm, the kernel function $K_D(z,w)$ is defined to be

$$K_D(z,w) = \sum_{\nu=1}^{\infty} \varphi_\nu(z)\,\overline{\varphi_\nu(w)}$$

where $\{\varphi_\nu\}$ is an orthonormal basis of H_D.

Let U be a neighbourhood of ∂D and p a C^∞ function on U such that $U \cap D = \{x \in U \mid p(x) > 0\}$, the hermitian form $\sum_{k,\ell} \frac{\partial^2 p}{\partial z_k \partial \bar{z}_\ell} \alpha_k \bar{\alpha}_\ell$ is negative definite on U and $\operatorname{grad} p \neq 0$ on U.

The first step in the proof of Theorem 7 is the following.

THEOREM 8.- There is a neighbourhood $V \subset U$ of ∂D and C^∞ functions Φ, $\tilde{\Phi}$ on V so that $\Phi(z) \neq 0$, $z \in \partial D$ and

$$K_D(z,\bar{z}) = \frac{\Phi(z)}{[p(z)]^{n+1}} + \tilde{\Phi}(z)\log p(z)\,, \qquad z \in V \cap D\,.$$

The basic idea of the proof of this theorem goes as follows. Let $a \in \partial D$. Then one can find a domain $\Omega_a \subset D$ which is tangent to ∂D at a of third order and an explicit analytic isomorphism of Ω_a with the unit ball (given by polynomials). Using the fact that the kernel function of the ball is defined in the complement of any tangent hyperplane and the fact (to be proved) that the high degree of tangency of Ω_a to D implies good approximation of the kernel of Ω_a to that of D (near a) in a suitable sense, Fefferman constructs operators K^o, K^1 on $L^2(D)$ so that

$$K^o(L^2(D)) \subset H_D \quad , \quad K^1(H_D) = 0 \quad , \quad \|E\| < 1$$

where $E = I - K^o - K^1$ and the norm is the operator norm in $L^2(D)$. Then it is easily seen that K_D, considered as a projection of $L^2(D)$ onto H_D, is given by

$$K_D = K^o + K^oE + K^oE^2 + \dots \ .$$

Fefferman computes these iterates explicitly and estimates their norms to prove Theorem 8. These computations are of extraordinary difficulty. They involue results from the theory of singular integrals and deep theorems relating to the $\bar{\partial}$-Neumann problem, both in the interior and on the boundary.

I have been told that Boutet de Monvel and Sjöstrand have obtained a proof of this theorem based on completely different ideas.

Theorem 8 has some applications. The kernel function of the unit ball is given by

$$\frac{\text{const}}{(1 - |z|^2)^{n+1}} \ .$$

Also, it is not hard to show that the presence of the logarithmic term in Theorem 8 ($\tilde{\Phi} \not\equiv 0$) is invariant under holomorphic isomorphisms. This term is present in many cases, e.g.

$$|z_1|^2 + |z_2|^2 < 1 + \varepsilon|z_2|^8 \quad , \qquad \varepsilon > 0 \text{ small}$$

(also in most cases when one starts with the unit ball and perturbs the boundary in a small neighbourhood of one point). These domains are therefore not isomorphic to the unit ball. The above example also shows that there are regular, holomorphic families of Stein manifolds such that one of the fibres is the unit ball, but there are fibres arbitrarily close to it which are not isomorphic to it.

The second main step in Fefferman's proof of Theorem 8 is the study of geodesics in the Bergman metric

$$ds^2 = \sum_{k,\ell} \frac{\partial^2 \log K_D(z,z)}{\partial z_k \, \partial \bar{z}_\ell} dz_k \, d\bar{z}_\ell$$

near the boundary. Let $z^{(o)} \in D$ and $w \in S^{2n-1} \subset \mathbf{C}^n$, and let $t \longmapsto X(t, w, z^{(o)})$ be the geodesic, parametrised by arc length, starting at $z^{(o)}$ in the direction w. We call $(z^{(o)}, w^{(o)}) \in D \times S^{2n-1}$ k-transversal if

$$\pi_{z^{(o)}}(w) = \lim_{t \to +\infty} X(t, w, z^{(o)})$$

exists for all w close to $w^{(o)}$ and if $\pi_{z^{(o)}}$ is a C^k-diffeomorphism of a neighbourhood of $w^{(o)}$ onto one of $\pi_{z^{(o)}}(w^{(o)}) \in \partial D$.

Moreover, we call a geodesic $t \longmapsto X(t)$ k-transversal if $(X(t), w_X(t))$ is k-transversal for all t large enough where $w_X(t) =$ unit vector in the direction $\frac{dX(t)}{dt}$.

THEOREM 9.- Any geodesic whose closure in D is not compact is k-transversal for every k. Moreover any point in ∂D is in the image of $\pi_{z^{(o)}}$ for some $z^{(o)} \in D$.

The proof of this result is of very great ingenuity, and, according to Fefferman, inspired by work (Siegel, Moser) on celestial mechanics.

In terms of real coordinates $(x_1, x_2, \ldots, x_{2n-1}, x_{2n})$, let J be the automorphism of $\mathbf{R}^{2n}$ corresponding to multiplication by $\sqrt{-1}$ on $\mathbf{C}^n$. Choose a basis $\nu_1(X), \ldots, \nu_{2n}(X)$ of $\mathbf{R}^{2n}$, depending smoothly on X so that

$$\nu_{2k-1}(X) = J\nu_{2k}(X) \quad , \quad \nu_2(X), \ldots, \nu_{2n}(X) \in T_X(\partial D)$$

(this for X near a boundary point of D), and let $\rho(X)$ be an equation for ∂D (D is given by $\rho > 0$). Let $w_1(X) = \rho^2(X)\nu_1(X)$, $w_j(X) = \rho(X)\nu_j(X)$ for $j \geq 2$ and define P_j by

$$Y = \sum_j P_j(X,Y) w_j(X) \qquad \forall\, Y \in \mathbf{R}^{2n} .$$

Introduce a new "time variable" τ by $\frac{d\tau}{dt} = \rho(X(t))$. Then, the equations for a geodesic can be written

$$\frac{dX}{d\tau} = \sum_j P_j \cdot \frac{w_j(X)}{\rho(X)} \ , \qquad \frac{dP_j}{d\tau} = \sum_{r,s} \frac{A^{(j)}_{r,s}(X)}{\rho(X)} \cdot P_r \cdot P_s \ .$$

In the case when the Bergman kernel has no logarithmic term, the coefficients in this system are smooth up to ∂D, and the theorem follows easily from standard facts on ordinary differential equations. However, in general, the coefficients are still singular at ∂D . Fefferman takes as a new time variable the function ρ , which is also chosen as the first coordinate function Y_1 in a system of coordinates $Y_1,\dots,Y_{2n}$. The equations then become :

$$\frac{dY_i}{d\rho} = \sum_{j=1}^{2n} \omega_{ij}(\rho\,, Y_2,\dots,Y_{2n}) \frac{P_j}{P_2} \ , \qquad i \geq 2 \ ,$$

$$\frac{dP_i}{d\rho} = \sum_{r,s} \Omega^{(i)}_{r,s}(\rho\,, Y_2,\dots,Y_{2n}) \frac{P_r P_s}{P_2} \ , \qquad i \geq 1 \ ,$$

where the ω_{ij} are smooth, and the $\Omega^{(i)}_{r,s}$ are smooth functions of $Y_2,\dots,Y_{2n}$, ρ , $\rho^{n-1}\log\rho$. Although solutions of this system are not necessarily smooth in ρ , they are, for fixed ρ , smooth in the initial data, and this is just sufficient for the proof of Theorem 9.

Although the work of Fefferman enables one to distinguish between certain domains, its relevance to the classifying problem comes from the work of Chern-Moser. E. Cartan [3] carrying out the program initiated by Poincaré [13], had given a complete set of invariants to decide when two germs of real analytic hypersurfaces in $\mathbf{C}^2$ can be transformed into each other by a local holomorphic automorphism of $\mathbf{C}^2$ (with certain mild restrictions on the hypersurfaces).

Given two hypersurfaces in $\mathbf{C}^n$ (passing through 0) of the form

$$\operatorname{Re} z_1 = f(\operatorname{Im} z_1, z_2,\dots,z_n) \quad \text{and} \quad \operatorname{Re} z_1 = g(\operatorname{Im} z_1, z_2,\dots,z_n) \ ,$$

we say that they are equivalent to the N^{th} order if there is a local automorphism h of $\mathbf{C}^n$ at 0 so that the Taylor series of $g \circ h$ of order N coincides with that of f . The work of Chern-Moser consists in computing the orbits of this equivalence relation, at least in the case when f , g are real analytic and are subject to certain mild restrictions.

Let $z \in \mathbf{C}^n$, $w \in \mathbf{C}$, $w = u + iv$. If f is a germ of real analytic function at $0 \in \mathbf{C}^n \times \mathbf{R}$, $f = f(z,\bar{z},v)$, we write

$$f(z,\bar{z},v) = \sum_{k,\ell \geq 0} f_{k\ell}(z,\bar{z},v) ,$$

where

$$f_{k\ell}(\lambda z , \mu \bar{z} , v) = \lambda^k \mu^\ell f_{k\ell}(z,\bar{z},v) \qquad \text{for } \lambda , \mu \in \mathbf{C} .$$

THEOREM 10 (Chern-Moser [7]).- Given a germ of analytic hypersurface

$$u = f(z,\bar{z},v)$$

at 0 in $\mathbf{C}^{n+1}$, such that its Levi form is nondegenerate, there is a local holomorphic automorphism of $\mathbf{C}^n$ at 0 taking it into a hypersurface of the form

$$u = \langle z,\bar{z} \rangle + g(z,\bar{z},u)$$

where $\langle \, , \, \rangle$ is a bilinear form and g satisfies

$$g_{k\ell} = 0 \quad \text{if} \quad \min(k,\ell) \leq 1 .$$

(The actual theorem of Chern-Moser is a little more precise than this and ensures uniqueness of the normal form.)

THEOREM 11 (Chern-Moser [7]).- Let M be a real analytic hypersurface in $\mathbf{C}^{n+1}$ with nondegenerate Levi form. There is canonically associated a principal bundle $Y = Y(M)$ on M and a set of $(n+2)^2 - 1$ 1-forms $\omega_\nu(Y)$ on Y with the following property :

Two such germs of hypersurfaces M , M' are biholomorphically equivalent if and only if there exists a real analytic diffeomorphism of $Y(M)$ onto $Y(M') = Y'$ taking the $\omega_\nu(Y)$ to $\omega_\nu(Y')$.

This result sets up a system of second order differential equations whose solvability is equivalent to the existence of a biholomorphic map of $\mathbf{C}^{n+1}$ taking M to M' .

In principle, the 1-forms $\omega_\nu(Y)$ can be calculated explicitely and provide very strong necessary conditions for the equivalence of two real analytic hypersurfaces in the form of systems of differential equations.

From this work of Chern-Moser, it appears that if D is a strongly pseudoconvex domain in $\mathbb{C}^n$ with a smooth boundary, then there is a family of curves on ∂D covering ∂D which, as a family, is left invariant by any holomorphic automorphism of D . Fefferman has shown how such a family can be constructed using the behaviour of the kernel function.

This construction of Fefferman has some interesting applications, for instance the following result.

Let D be a strongly pseudoconvex domain analytically isomorphic to a ball. If it coincides with a ball in the complement of a sufficiently small neighbourhood of a boundary point, then D is a ball.

Raghavan NARASIMHAN

Department of Mathematics
University of Chicago
5734 University Avenue
CHICAGO - Ill. 60637
U.S.A.

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