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Towards a Discretization of Quantum Theory

HANS GRAUERT

Dedicated to the memory of Ennio De Giorgi

Introduction

The aim of this paper is to obtain a united theory for the corpuscle and the wave character of matter. The Schroedinger waves will be replaced by so called discrete waves.

1. The Radon measure. First, we shall consider an unusual mathematical model of space-time $\mathbb{R}^4$. Of course, $\mathbb{R}^4$ carries the Minkowski geometry (i.e. the Pseudo-Euclidean geometry as physicysts like to say). But, here, this geometry is given by a Radon measure ρ in $\mathbb{R}^8$ instead by a metric in $\mathbb{R}^4$. The support of this Radon measure has codimension 1 and it is the union S of all light cones in $\mathbb{R}^4$. Every C^1-differential map $F : \mathbb{R}^4 \simeq \mathbb{R}^4$ gives an isomorphism $(F, F) : \mathbb{R}^8 \to \mathbb{R}^8$. Then F is an isomorphism of Minkowski geometry (a so called Poincaré transformation) if and only if (F, F) leaves ρ invariant. This follows by an old theorem of E. C. Zeeman. So the Radon measure gives the Minkowski geometry as well as the Lorentz metric does.

Moreover it is known: If F maps all the light cones onto light cones, then F is a composition of a Poincaré map and a homothety. Since homotheties are considered to be unimportant in physics, the physicysts just took in their theories the distribution of light cones, only. Here we go the other way: Most important is the measure on the light cones.

The Radon measure ρ is given by the integral over S with respect to the differential form

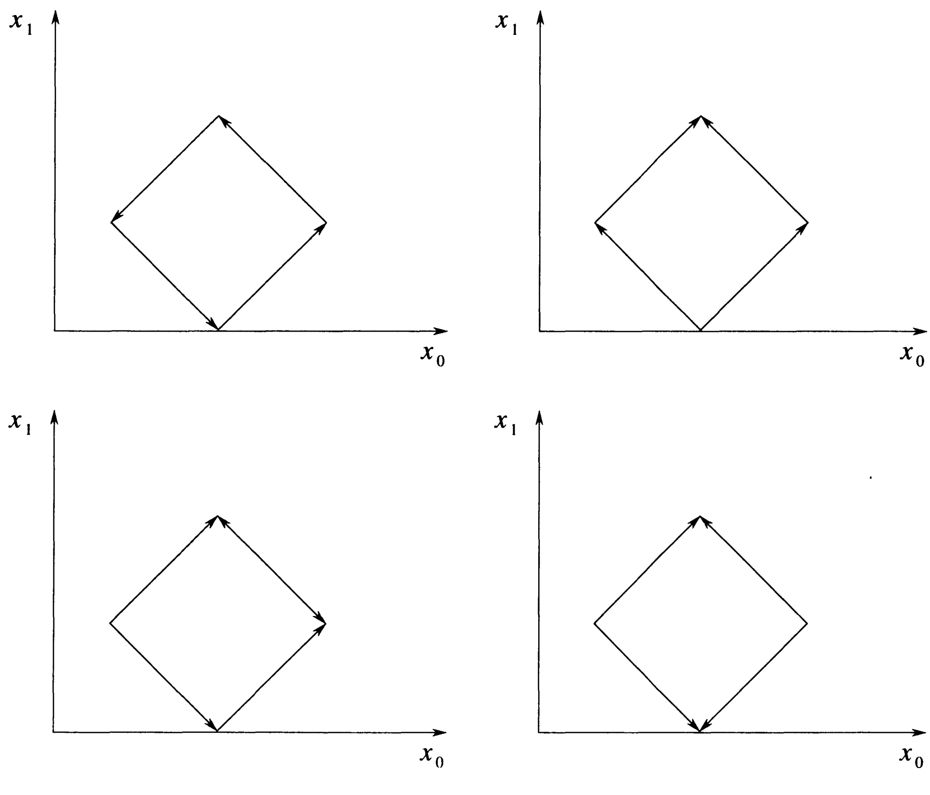
$$\Omega = \frac{dx_1 \wedge \ldots \wedge dx_3}{r} dy_0 \wedge \ldots \wedge dy_3.$$

Here $\vec{x} = (x_0, \ldots, x_3)$ and $\vec{y} = (y_0, \ldots, y_3)$ denote the points of $\mathbb{R}^4$ and r is the Euclidean distance of $(x_1, \ldots, x_3)$ and $(y_1, \ldots, y_3)$ in $\mathbb{R}^3$. This measure is smooth on S.

Now the question is: Why is this definition of Minkowski geometry better than the ordinary one. The reason is that there should be a geometric interpretation of ρ. Since the support of ρ is S, one would think that there is a set $\mathcal{L}$ of points on S, whose Lebesgue measure is ρ in every point. If $\mathcal{L}$ exists it consits of compact line segments in light rays. In mathematics it is better to use (directed) vectors instead of line segments. Of course, the density of these light vectors has to be the same for both directions. - I would like to say here, that $\mathcal{L}$, if it exists, certainly is not Lorentz invariant, but that the measure ρ could be: For the definition of $\mathcal{L}$ we need coordinates, which are determined up to translations and spatial rotations and thus a 4-dimensional Euclidean geometry. In philosophy *time* has a global meaning.

But in Lebesgue theory smooth measures never are the measures of measurable sets, if they are not the maximal measure (i.e. the measure of the full set) or are equal 0. So what can be done?

2. The set of light vectors. We take for $\mathcal{L}$ a discrete set of light vectors, which is very dense on S and take a big integer N such that the local integral over $d\rho$ and a bounded open set $U \subset \mathbb{R}^8$ is very near to $2 \cdot A/N$, where A denotes the number of points of $\mathcal{L}$ contained in U. We can make the approximation as good as we like, we only have to require that $\mathcal{L}$ is discrete. The approximation shall be better than any possibility of measurement. We take a fixed inertial system of coordinates belonging to the Minkowski geometry. With respect to this system we can construct 2-dimensional vortices using 4 lightvectors of $\mathcal{L}$ of same Euclidean length always. Look at the following picture:



The vortices are whirls, streams and sources. We say that they have the dimension $0, \ldots, 2$ (in this order). We have to get vortices, which are of purely spatial nature. The length l has to be the same, always and also the edge points of the figures. Therefore, in the beginning already, we take another coordinate, say x_2, and construct the vortices so, that the edges are:

$$(0, 1, 0), (a, 0, 1), (0, -1, 0), (a, 0, -1), \quad a = \pm\sqrt{2}.$$

Now, the number of lightvectors is 8. We get spatial whirls (of any direction: x_1,x_2, x_3 are with equal rights), streams in any spatial direction and sources. By combining a (x_0, x_1)-vortex with a spacial whirl or a source using for $x_1, \ldots, x_3$ arbitrary directions we can obtain the 16 different kinds of vortices in $\mathbb{R}^4$. Every vortex consists of 16 light vectors of same length l and has a dimension. If the dimension is 0 we have a whirl, and for dimension 4 we get the sources. We also can apply the Lorentz transfomations. By this we get the general vortices. We shall consider fields $\mathcal{F}$ of very many vortices such that they are practically dense and have a certain kind of smoothness. We always shall assume that in one inertial system all lightvectors in a field have the same length.

3. The Clifford algebra. In order to study such fields $\mathcal{F}$ we use the so called inner calculus Φ of exterior forms, which is a Clifford algebra. This was considered by E. Kähler: Der innere Differentialkalkül (see: Abh. Math. Sem. Hamburg **25** (1962), 192-205 and Rendiconti di Matematica **21** (1962), 425-523) to replace the spinors in relativistic quantum theory. From now on we have to assume that $\mathbb{R}^4$ is equipped with an orientation, i.e. that the parity is not valid, as was proved by the famous Cobalt-60-experiment.

We put $\gamma_\nu = dx_\nu$ for $\nu = 0, \ldots, 3$. Then, we define $\gamma_\nu \cdot \gamma_\mu = \gamma_\nu \wedge \gamma_\mu = -\gamma_\mu \cdot \gamma_\nu$ if $\nu \neq \mu$ and $\gamma_0^2 = -1$ and $\gamma_\nu^2 = 1$ for $\nu = 1, \ldots, 3$ and obtain the so called inner product. We consider the forms

$$\varphi = \sum_{n=0}^{4} \sum_{0 \leq \iota_1 < \ldots < \iota_n \leq 3} a_{\iota_1 \ldots \iota_n} \gamma_{\iota_1} \cdot \ldots \cdot \gamma_{\iota_n}, \quad a_{\iota_1 \ldots \iota_n} \in \mathbb{C}$$

and obtain a Clifford algebra (see: B. L. van der Waerden: Algebra, Vol. II, Paragraph 93.5, Heidelberger Taschenbuch 23, Springer Heidelberg 1967). The inner product comes from the quadratic form $x_0^2 - x_1^2 - \ldots - x_3^2$. So, if we transform φ like an exterior form, we get that the Clifford algebra $\Phi = \{\varphi\}$ is Lorentz invariant. As said already we use complex coefficients.

We shall replace the spinors by our φ. However, a φ has 16 components and the spinors have just 4. We define $\gamma = \gamma_0 \cdot \ldots \cdot \gamma_3$. Then, we get $\gamma^2 = -1$ and for $\sigma = -i \cdot \gamma$ we obtain $\sigma^2 = 1$. The $*$–operator is well known for exterior forms. It is invariant under those Lorentz transformations, which preserve the orientation. We see immediately that $*\varphi = \varphi \cdot \sigma$. We shall call the forms φ self dual, if $*\varphi = \varphi$. We shall use self dual φ, only.

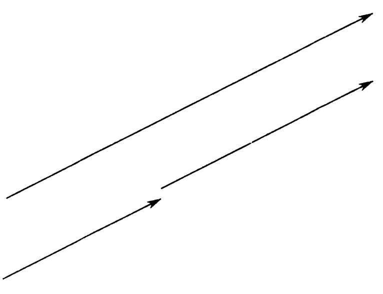
So, from now on, we always shall assume that the $\varphi \in \Phi$ are self dual. Then, for $a \in \mathbb{R}$ we get $*ia\gamma_{\iota_1} \cdot \ldots \cdot \gamma_{\iota_n} = ia \cdot (-i)\gamma_{\iota_1} \cdot \ldots \cdot \gamma_{\iota_n} \cdot \gamma = a\gamma_{\mu_1} \cdot \ldots \cdot \gamma_{\mu_{4-n}}$. So, the imaginary part of a monomial $\gamma_{\iota_1} \cdot \ldots \cdot \gamma_{\iota_n}$ is the real part of the dual monomial. The imaginary parts are superfluous. Therefore in general, we shall write φ with real coefficients. We have 16 real $=$ 8 complex components. This number still is twice as high as in the case of spinors. Later on, we shall prove using the stochastical equilibrium that the reason for this is that the world consists of electric matter and a completely unelectric kind.

4. The stochastical process. We have to get the connection between our field $\mathcal{F}$ and Φ. Consider just the 0-dimensional part $\mathcal{F}_0$ of $\mathcal{F}$. If $P \in \mathbb{R}^4$ is a point, the we denote by A the number of vortices of $\mathcal{F}_0$, which surround (by its convex hull) P (if a vortex is negative, we have to subtract). If U is a small bounded neighborhood of P, then in U this number is practically $= A$ everywhere. Assume that N is a suitable measure factor. Then we define the 0-dimensional part φ_0 of the Clifford form φ belonging to $\mathcal{F}$ so that $\varphi_0(P) = A/N$. In the case of a higher dimension d we have to replace P by a d-dimensional surface and φ_0 by the d-dimensional part of φ. In general the field $\mathcal{F}$ will not be determined by φ. The vortices can be given by very different configurations. But in case of the same φ, there should be a stochastical equilibrium between the configurations (However, I do not know that equilibrium). The local density of these will be defined up to the possibility of mesurement.

The essential idea of this theory is stochastics. There is a simple stochastical process, which gives the density of $\mathcal{L}$ on S. Here, we do'nt have Lorentz invariance in logic, but within mesurement we will get it. For $\mathcal{L}$ there is a distinguished inertial system. All stochastics will go with respect to this system. We assume moreover that for every lightvector $v \in \mathcal{L}$ there is one lightvector $\bar{a}$ which is very near to a and has opposite direction.

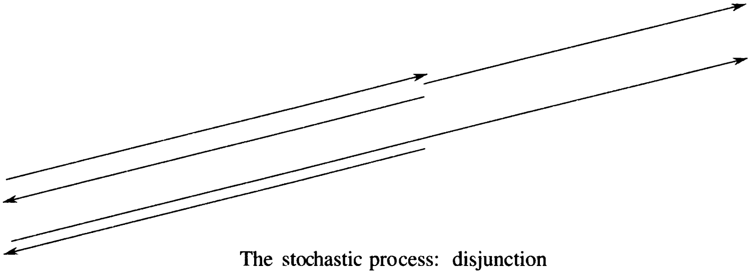
If $v \in \mathcal{L}$ is a lightvector then by time it will move through $\mathbb{R}^4$ (always together with its opposite near vector) step by step such that the projection to the space $\mathbb{R}^3$ is constant and only the time is changed by a constant positive quantity δ in each step. δ depends on the density of light vectors. But there also are exeptions.

1) Assume that v_1, v_2 are two lightvectors of same direction and that the second vector starts nearly (quantity δ) where the first one ends. Then both vectors can join to a light vector $v_1 + v_2$. This has the same direction. Its length is the sum of the lengths.
2) Assume that the vectors v_1, v_2 are of opposite direction and that the second vector v_2 ends nearly at the beginning of the first v_1 and is shorter than v_1.



The stochastic process: junction

Then a division of v_1 into vectors v_1' and v_1'' may happen. The vector v_2 is still there. This process will be the same vice versa if we interchange the end and the beginning.



The stochastic process: disjunction

We assume that these stochastical processes always happen simultaneously for a and $\bar{a}$ such the density of the vectors a and $\bar{a}$ always stays the same in the space-time locally.

5. The elementary length. The relative frequency of the stochastical processes will be given by the product of the densities of the set of those lightvectors which enter in the process. There is an equilibrium in this stochastic process, which consists of junction and disjunction. It is given by an integral equation. Unfortunately, this has more solutions, than that which we need, here. So the question is: What can be done? The answer again is in a another discretization. We have to use the notion of elementary vector and that of elementary length $\epsilon > 0$. Moreover, we have to use our reference system for this purpose. In this reference system every light vector v is the unique junction of $n \in \mathbb{N}$ elementary vectors of same direction (we have to assume that the "same direction" is defined!) All elementary vectors have the same Euclidean length in that reference system, always $\epsilon > 0$. This ϵ should be much larger than the Planck length of 10^{-35} m. However, it has to be much smaller than the length af any wave, which might appear by an elementary particle. Everything has to be smooth within the limits of measure possibilities. This follows from experience! ϵ is an universal constant. The density d of elementary light vectors should be very large. I think of about 10^{200} elementary light vectors per (7-dimensional) volume unit m^7. The quantity δ will be proportional to $d^{1/3}$ and much less than ϵ.

We assume that the junction and the disjunction respect the subdivision in elementary vectors and moreover that a translation of the result by a quantity into a positive time direction happens. This quantity is given by the elementary length. In the case of the junction 1) the vector v_1 (or the vector v_2) is in a translation in negative time direction (quantity δ) against the other. The result $v_1 + v_2$ then is chained directly up to v_2, which will not be translated. In the case of the disjunction 2) we have to follow the drawing. Only the result v_1' will be translated into a positive time direction.

Also if non of these stochatical processes happen the light vectors will be moved through space-time step by step in a similar way. This ordinary movement superposes the movement caused by the stochastics. Also its quantity will be given by δ. Of course, this description of the processes is rather incomplete. It gives just one possibility and others might be possible.

The shift into positive time direction means that the stochastical time equals our usual time. Our usual time has a direction, therefore. There also is a conservation law: The number of the elementary light vectors in the whole space is constant.

By our necessary discretization the integral equation is transformed into a system of very very many quadratic equations. Nevertheless, it can be proved that it has the correct solution, only. See: H. Grauert, Selected Papers, Vol. II, Part X. Springer Heidelberg 1994 and S. Leykum, Thesis 1980.

The stochastics are somewhat different from the usual. Here in a bounded region of space-time only a bounded number of processes can occur. We have to apply the law of big numbers for very big but bounded numbers: The structure has to be fine with respect to the number of processes. If the equilibrium is disturbed, it always will be restored under such a bounded number of processes. If we would take infinitely many processes there once must be a degeneration, which for instance consits in the case that in every light vector the number of elementary ones is even. But the time for this is too long. Even, the collapse of the universe could not be explained in such a way. We see that in our theory time is somewhat more than a coordinate. The Becoming has an essential role.

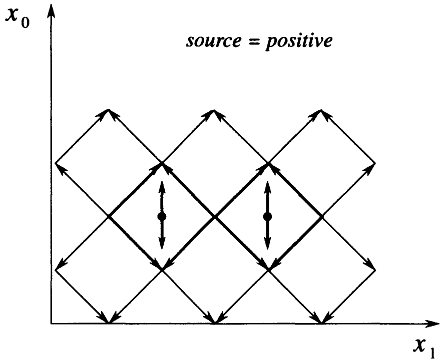
It is easily possible to play the stochastical process on a computer. This comes from the fact that we may assume that there is a maximal number n of elementary light vectors, which can be jont. The distribution of the light vectors with a number not bigger than n then will be the same as in the case $n = \infty$. If, e.g. we take $n = 5$ and a set of 32 elementary light vectors it will be very easy to acchieve degeneration. But already, if we have about 100 light vectors this will become difficult. The equilibrium always will be restored.

6. Consequences. The theory has some interesting results.

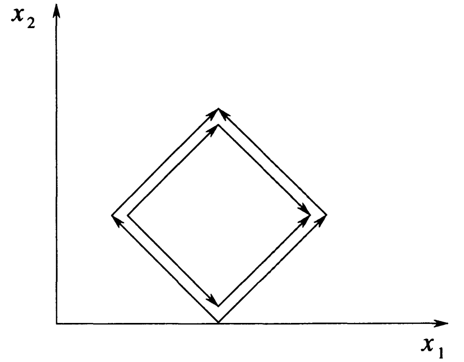
FIRST: We can derive the Dirac equation geometrically. It consits just of a statement of stochastical equilibrium.

We first have to do it in $\mathbb{R}^2 = \{(x_0, x_1)\}$. We take 4 light vectors in $\mathbb{R}^2$ and form a square, which stands on a vortex. If the orientation of the vectors is suitable, we get vortices $\pm 1, \pm\gamma_0, \pm\gamma_1 \pm \gamma_0 \cdot \gamma_1$ of dimension 0 to 2. Then, a vortex in $\mathbb{R}^4$ is constructed by linking vortices in orthogonal 2 dimensional planes. We shall consider lattices of vortices, which always are of the same kind.

We start with $\mathbb{R}^2$. We consider fields $\mathcal{F}$ in $\mathbb{R}^2$ with coordinates x_0, x_1. The $\mathcal{F}$ are constant in x_1-direction but are increasing in x_0-direction. We can look a the following pictures:



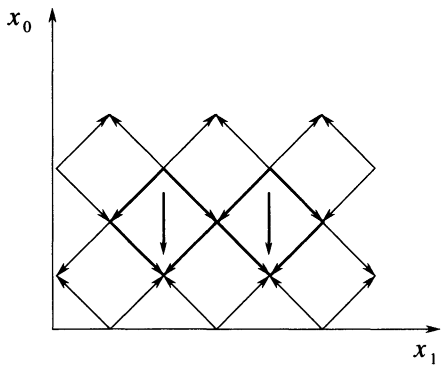
Dirac equation for γ_1



Disjunction of the vortices into its vectors by spin

First we have the γ_1-vortices in the upper and the lower line. The particle has to have a spin in x_3-direction. If this is $\frac{1}{2}$ and the particle is simple, we must have $i\gamma_1 \cdot \gamma_2 \cdot \varphi = \varphi$, if we have choosen the complex style (to write the wave function). We assume this. We can obtain the vortices in the upper and the lower line also by linking a x_2-stream ($= \gamma_1$-vortex) in the (x_1, x_2)-plane with a 0-dimensional vortex in the (x_0, x_3)-plane (with a whirl). Then, because of the spin there are also linkings of x_1-streams with 0-dimensional (x_0, x_3)-vortices in the middle. These cause there a division of the x_2-streams into their 8 vectors. And these can junct again there. We get lattices of linkings of sources in the (x_0, x_1)-plane with 0-dimensional vortices in the (x_2, x_3)-plane, that means a lattice of $\gamma_0\gamma_1$-vortices.

In order that all this works, we see that not only one lattice is sufficient but a whole family of subsequent latttices whose distance may be around δ: also in the middle such stream lattices have to be present.



Dirac equation for $\gamma_0\gamma_1$

Assume, conversely, that $\gamma_0\gamma_1$-vortices are given in the upper and lower line. Then, in the same way, a field of γ_1-vortices is formed in the middle.

In the first case positive objects are generated, in the second case negative ones. Making the fields so dense that we get differentiability, it follows that we get a wave equation. This leads to the matter waves.

So we shall consider very dense fields φ, which are the superposition of many lattices such that exact sinus–vibrations are obtained. We denote the wave length by $l = \frac{1}{m}$ and by $D = \gamma_0 \frac{\partial}{\partial x_0} + \ldots + \gamma_3 \frac{\partial}{\partial x_3}$ the Dirac operator. In the drawn case, and as it can be seen rather easily, also in general the Dirac equation $D\varphi = m \cdot \varphi$ is valid. So we got a geometric deduction of this equation. It is important that for every lattice we have the spin lattice, which has to be determined very exactly. So 4 real components are fixed. We call them the real part of φ.

SECOND: Also the imaginary part is essential for the particle. Its components are given by the complementary monomials. The imaginary part decides if we have a particle or an anti-particle. But, what is the proper sense of the imaginary part? The real part and the imaginary part consist of lattices with four components. It can happen that at a given time there are exactly two such lattices, one of the real and one of the imaginary part, which go together as is the case in a vernier scale: the magnitude of the real and the imaginary lattices may differ by a lenth around δ (of course, here should be more exactness!). In time the place ot these 2 lattices may move through the particle wave, in general, continuously but even with a speed exceeding the speed of light. There are reasons to presume that at such places the particle wave always is generated. So we shall speak of *generating points*. Outside of these generating points an attenuation will occur, since the recombination, which has to take place by the Dirac equation, not always will be successful. The wave itself leads the movement of the generating points. So we get propositions on the probability of the place of residence of the particle. This place always is a generating point, we can measure nothing else, since the action of two particles onto each other is a tensor product in generating points.

THIRD: If there are two particles, it can happen that the lattices link in generating points, or better that they generate a united wave there. First, that means a real tensor product (in each point). If for instance

$$\varphi = \sin(\omega_1 \cdot t) \cdot \gamma_0 \text{ and } \psi = \cos(\omega_2 \cdot t) \cdot \gamma_0, \text{ so}$$

$$\varphi \otimes \psi = \sin(\omega_1 \cdot t) \cdot \cos(\omega_2 \cdot t) \cdot \gamma_0 \otimes \gamma_0 = \frac{1}{2} \sin\Big((\omega_1 + \omega_2)t\Big) \cdot \gamma_0 \otimes \gamma_0$$

will be generated. But, space-time is oriented. There is a maximal field of 4-dimensional vortices. Using this, we can give reasons for a complex product as it is necessary in physics. Only the components in even dimension are used. Also the time direction is essential. The opinion that an anti-particle moves into negative time direction is a total nonsense. It moves in ordinary direction. All this means that the tensor product is Hermitian for anti particles. We get the conversation of energy–momentum.

If you measure the place of a particle T, you put a small particle S into the wave region of T. Then you know the place of T if there is a reaction with S. This reaction can be arbitrary small. It just means a tensor product between

T and S. After the reaction the particle T will spread from S. All other pats will underlie a damping.

FORTH: We shall consider our waves as to be really existent. They are different from the Schroedinger waves. But these should be the average under a set of many experiments. In this theory the odd German Idealism (as it is in the Copenhagen model) is replaced by chance. So also the waves of the electron and the μ-particle will be totally different. You can see why the μ-particle is so much heavier than the electron. The wave function of the μ-particle gives its decay.

FIFTH: What is the aim of the whole essay? We wish to construct a mathematical object, which contains a model for the corpuscle nature and the wave nature of matter simultaneously. But unfortunately it is not possible to give a system of axioms and everything can be derived from these axioms as is done in mathematics. So the contents may not be called mathematics. On the other hand we use the discrete waves, which are nearer at the reality than the Schroedinger waves are. But these cannot be pictured by experiments like the Schroedinger waves. So they are not objects of physics. We just use then in order to get a complete logic. So we must say: all what was done is philosophy.

1. – Proof of some statements

Radon measure and Minkowski geometry, the dircete set $\mathcal{L}$ generated by the stochastical process, the generating point, the complex tensor product, the electric charge.

1. The Radon measure. First, we have to prove that our Radon measure ρ gives the Minkowski geometry, precisely.

We denote by S_O the union set of all light rays through $O \in \mathbb{R}^4$ and call S_O the *light cone* in O. There is a light cone S_x in every point $x \in \mathbb{R}^4$, correspondingly.

Assume that $F : \mathbb{R}^4 \xrightarrow{\sim} \mathbb{R}^4$ is an invertible continuously differentiable map. Then, we put $\hat{F}(\tilde{x}, \tilde{y}) = \Big(F(\tilde{x}), F(\tilde{y})\Big) : \mathbb{R}^8 \xrightarrow{\sim} \mathbb{R}^8$. Also this map is invertible continuously differentiable. Then $\rho \circ \hat{F}$ again is a Radon measure in $\mathbb{R}^8$.

THEOREM 1.1. *The Radon measure ρ is Lorentz invariant. That means, we always have $\rho \circ \hat{F} = \rho$ for any Poincaré transformation F.*

PROOF. We have $x = F(\tilde{x}) = L(\tilde{x}) + x_0$. In this L is a Lorentz transformation and x_0 is a constant vector. The translations do not change anything. Therefore, we may assume $x_0 = O$ and $y = \tilde{y} = O$ The jacobian of L is 1. So we get in $y = O$:

$$\begin{aligned}\rho \circ \hat{F} &= \left(\frac{dx_1 \wedge \ldots \wedge dx_3}{r}\right) \circ L \wedge (dy_0 \wedge \ldots \wedge dy_3) \circ L \\ &= \frac{d\tilde{x}_1 \wedge \ldots \wedge d\tilde{x}_3}{r} \wedge d\tilde{y}_0 \wedge \ldots \wedge d\tilde{y}_3 = \rho.\end{aligned}$$

PROPOSITION 1.2. *The points x, y are on the same light ray if and only if $(x, y) \in S$ and that if and only if $\rho(x, y) \neq 0$.*

PROOF. Since the Radon measure is defined by integration on S and we have a positive volume element on S we get $\rho(x, y) \neq 0$ if and only if $(x, y) \in S$.

THEOREM 1.3. *Assume that $F : \mathbb{R}^4 \xrightarrow{\sim} \mathbb{R}^4$ is invertibly differentiable and that the map $\hat{F}$ leaves ρ invariant. Then F is a Poincaré transformation.*

PROOF. We may again assume that $F(O) = O$. Now, $\tilde{x}$, O are on the same light ray if and only if $\rho(\tilde{x}, O) \neq 0$. Then the same is true for $x = F(\tilde{x})$, O. So, every light cone is mapped into a light cone. Assume that $\tilde{x}_1$ is on the line segment connecting O and $\tilde{x}$ and that $x_1 = F(\tilde{x}_1)$. Then x is in the intersection $F(S_O) \cap F(S_{\tilde{x}_1}) = S_O \cap S_{x_1}$. That means that also x_1 is in the line segment from O to x. Every light ray through O is mapped into a light ray through O. Then, for F this is true for any light ray. So by a theorem of Zeeman (see Borchers and Hegerfeld: Nachr. Akad. Wissenschaften Göttingen 10, 1972) the map F is the composition of a Poincaré transformation with a homothety. But the homotheties change the measure ρ, if they are different from the identity. So F is a Poincaré transformation.

COROLLARY 1.4. *The Pseudo-Euclidean geometry is given by the Radon measure ρ as well as by the Minkowski metric.*

2. The discrete set $\mathcal{L}$. Second, we have to prove that the discrete set $\mathcal{L}$ is the equilibrium of a stochastical process.

We take a fixed positive integer k and two elementary objects o_+, o_-. We say that the first one has the oposite direction of the second one. For every integer i, $0 < i \leq k$ we define sets of i-tupels of o_+ and denote the number of its elements by $h(i)$. We do the same with o_- and assume the condition that here the number of elements also is $h(i)$. – Of course, we think that the elementary objects o_+ are elementary light vectors on a light ray such that the direction of o_+ is in positive time and that of o_- is in negative. We assume that the i-tupels are lightvectors consisting of i elementary ones of the described direction. Every light vector has a begin and an end. – Since the stochastical process works for lightvectors v and its very near opposite lightvector $\bar{v}$ simultaneouyly we will have the condition on $h(i)$ for these lightvectors. For description of the stochastical processes we use an infinitesimally small positive number η.

Assume that $i + j \leq k$. Then in every stochastical step the i-tupels of o_+ and the j-tupels of o_+ may link to $(i + j)$-tupel. The number of events will be

$A_1 = \eta \cdot h(i) \cdot h(j)$. The same is true for o_-. A light vector of $i + j$ components can be decomposed by a i-lightvector of opposite direction coming from the end and by a j-lightvector of opposite direction beginning in the first point of the lightvector. The number of events will be $A_2 = \eta \cdot h(i + j) \cdot (h(i) + h(j))$. We put $A_{i,j} = A_1 - A_2$.

The state of our system is given by the k-tupel $q = (q_1 = 1 \cdot h(1), \dots, q_k = k \cdot h(k))$. The changement of our system means adding the k-tupel

$$R_{i,j} = (0, \dots, -i, \dots, -j, \dots, i + j, \dots, 0) \cdot A_{i,j}(h),$$

where only the components with ordinal numbers $i, j, i + j$ are different from 0. In the case $i = j$ we have to replace the $(i = j)$-th component by $-2 \cdot i$, of course. We see that the number $N = \sum q_i$ stays constant. It is the number of elementary objects o_+. The whole changement of the systen is $R(q) = \sum R_{i,j}$. We denote by $R_{i,j}^{(n)}(q)$ respectively $R^{(n)}(q)$ the n-th component of $R_{i,j}(q)$ respectively $R(q)$. If all $q_n = N/k$, we get $R_{i,j}(q) = 0$ and hence $R(q) = 0$. That means, our system is in equilibrium. We denote this state by q_0.

We take a k-tupel $p = (p_1, \dots, p_k)$ of numbers p_n, which are infinitesimal, i.e. very very small with respect to N/k and denote by Q the quadratic form $\sum_{n,m} p_n \cdot R^{(m)}(q_0 + p)$. It is the sum of the quadratic forms $Q_{i,j} = \sum_{i,j} p_n \cdot R_{i,j}^{(m)}(q_0 + p)$. These have non zero components only for ordinal numbers $i, j, i + j$. We have

$$A_{i,j}(q) = \eta \cdot \left(\frac{q_i \cdot q_j}{i \cdot j} - \frac{q_{i+j} \cdot (iq_j + jq_i)}{(i + j) \cdot ij} \right).$$

Therefore the non zero parts of $Q_{i,j}$ are given by the following matrices, which have to be multiplied by $-\eta \cdot \frac{1}{ij(i+j)}$:

$$\begin{pmatrix} i^2, & i \cdot j, & -i(i + j) \\ i \cdot j, & j^2, & -j(i + j) \\ -i(i + j), & -j(i + j), & (i + j)^2 \end{pmatrix}.$$

In the case $i = j$ we get a 2 dimensional matrix, however:

$$\begin{pmatrix} 4i^2, & -4i^2 \\ -4i^2, & 4i^2 \end{pmatrix}.$$

All these matrices are positive semi definit. We get that our quadratic forms $Q_{i,j}$ are negative semidefinit. It follows that also Q ist negative semidefinit. If for a vector p the value of Q is 0, it can be proved that p is a multiple of q_0. But this cannot occur since N will not be changed under the stochastical processes. So Q is negative definit. But this means that any small change of the state q_0 will return to q_0. Of course, essential is the lattitude of the neighborhood of q_0, in which this will happen. We call this neighborhood the region of stability. I hope that it is the full open (awfully high dimensional) standard simplex of all possible k-tupels q. Unfortunately, I could not prove anything. It is only known by Leykum that q_0 is the only equilibrial state.

The theory can be applied to the light vectors. It is possible to derive the distribution of the local densities in $\mathcal{L}$. The light cones S_x are asumed to be given.

As said before, we have to do the stochastics in a fixed distinguished inertial system. Here we have the Euclidean distance. We consider 2 lightvectors as to have the same length and direction, if the distance of the initial points and also that of the end points is less than the length δ. We do the stochastical process for every direction and assume that the sets of elementary lightvectors for every direction are isomorphic and especially that the numbers N and k always are the same. Then by these processes we obtain the set $\mathcal{L}$ with the desired properties.

3. The generating points. Third we have to prove the existence of generating points. We look at a field $\varphi \in \Phi$, where Φ is the Clifford algebra again. The vortices γ of φ have to be opend by the spin vortices. Otherwise the wave would not be there. There should be pairs of such vortices at the same place very often. Moreover, we have for the γ the dual vortices $\gamma^* = \gamma\sigma$ which can be considered as its imaginary part. Here γ and γ^* should be at the same place in very exceptional cases, only. There is no reason for the contrary. We may assume that for every time moment there is exactly one configuration of 4 real vortices γ such that their place and that of the corresponding γ^* is exactly the same. We call this place the generating point of φ. Of course, the notion of *time moment* is problematic. The distance of a time moment and the next one should be around δ. The existence of these generating points comes from an effect of vernier scale. They will move through the particle mostly continously, in general by a speed beyond that of light.

What happens in a generating point? We have the orientation of space time. It consists of a constant field $\mathcal{P}$ of 0-dimensional vortices. The question morover is: how can a vortex in 4-dimensional space time be configurated? It must be done by a (orthogonal) linking of 2 vortices v_1, v_2 of a $\mathbb{R}^2$. If you replace v_1, v_2 by $-v_1, -v_2$ you get the same vortex in $\mathbb{R}^4$. We denote it by $v_1 \otimes v_2 = (-v_1) \otimes (-v_2)$. For every vortex v_i in $\mathbb{R}^2$ there is the complimentary vortex v_i' in $\mathbb{R}^2$. If v_i is 0-dimensional then v_i' is 2-dimensional and vice versa. The complement of a stream is a stream in orthogonal direction. For $v_1 \otimes v_2$ the vortex $v_1' \otimes v_2'$ is its dual. If $\gamma_1 \otimes \gamma_2 \in \mathcal{P}$ is suitable, there should be an equilibrium between $v_1 \otimes v_2$ and $v_1 \otimes \gamma_2 + \gamma_1 \otimes v_2$, similarily for the complements.

Vortices $\gamma_1 \otimes v_2$, $\gamma_1 \otimes v_2'$ can distroy each other. But instead of destroyment a pair of very near negative and a positive vortex will be produced with some probability. This comes from a stochastical equilibrium.

This described process causes a wave in the real (an the dual also in the imaginary part) like that is done by the spin. Therefore, it is not a real destroyment.

We get $\gamma_1 \otimes v_2 + \gamma_1 \otimes (-v_2)$, $\gamma_1 \otimes v_2' + \gamma_1 \otimes (-v_2')$. The same is true, if both components of the pair are interchanged. Using the equilibrium we finally obtain, that $v_1 \otimes v_2$, $v_1' \otimes v_2'$ passes over in $v_1 \otimes v_2 + (-v_1) \otimes (-v_2)$, $v_1' \otimes v_2' + (-v_1') \otimes (-v_2')$. So we have generation, since a real destroyment took not place.

This generation will lead to a chain process. The generating vortices produce the vortices of the wave. These will be spread into space. The generating vortices might be destroyed by such processes. But the described chain process will go on until the normal density of generating vortices is reached again. - The generation center will be a superposition of very many vortices, very narrow around a line. Their distance should be about δ.

4. The complex tensor product. We consider a monomial $\gamma \in \Phi$. The squares γ^2, $(\gamma^*)^2$ with $\gamma^* = \gamma\sigma$ are real numbers. One of them is negative, one of them positive: We assume that the dimension is even. If γ is given, then the field of orientation $\mathcal{P}$ produces the monomial γ^*. We define $\gamma \otimes \gamma = (\gamma, \gamma) - (\gamma^*, \gamma^*)$, since γ, γ^* give a connection with themselves and the product in the dual case is negative. Next we get $\gamma\sigma \otimes \gamma\sigma$. We take for this the negative of the first result. So we have $\gamma\sigma \otimes \gamma\sigma = -\gamma \otimes \gamma$. The multiplication by σ is like the multiplication with $i = \sqrt{-1}$. We moreover define $\gamma \otimes \gamma\sigma = -(\gamma, \gamma) - (\gamma^*, \gamma^*) = \gamma\sigma \otimes \gamma$. We have the complex law for $\alpha = (a + ib)\gamma$, $\beta = (c + id)\gamma$: $((e + if)\alpha) \otimes \beta = \alpha \otimes ((e + if)\beta)$.

The definition of the tensor product is not yet completely correct. It depends on the fact, if the factors belong to a particle or to an anti particle. In the case, where a factor is an anti particle we first have to take its conjugate complex and then to form the old product. Then this is the genuine result. Now the product becomes Hermitian in the case of factors of mixed type. I do not know the reason how this construction really is performed. But the multiplication by a complex number should the developement of the wave in time.

Probably, the tensorproduct is just the binding force. In the generating points of the tensorproduct the generating part of the old particles still will be there. If we desire a special aime (e.g. the production of an electric field) the old particle waves may be in suitable forms, which are allowed by the tensorproduct. – If there is a spin direction, we speak of weak force, which can be very strong, however, if the spin directions are sharp. If the spin is 0 (like in the case a a π-particle) we have the strong force. Moreover there is the electric interaction: We shall consider it in the next paragraph. – Also the gravitation can be explained, since the particles make the free set of light vectors smaller: they bind some of them. It can be seen: If the density of $\mathcal{L}$ gets smaller, the particles will get bigger. The structure of $\mathcal{L}$ in space-time may change from point to point a little bit. We may have a curvature.

5. The electric charge. Assume that $\varphi \in \Phi$ is a wave, which satisfies the Dirac equation with a mass $m = 1$, which morover is resting and has spin 1/2. Every (simple) wave with $m = 1$ and spin 1/2 is superposition of transforms of such waves.

But we shall discuss resting waves only. We choose the complex style:

$$\text{I)} \qquad \varphi = e^{ix_0} \cdot [\gamma_1 - i\gamma_2 + i\gamma_0\gamma_1 + \gamma_0\gamma_2 + i\gamma_0\gamma_2\gamma_3 - \gamma_0\gamma_1\gamma_3 + \gamma_2\gamma_3 + i\gamma_1\gamma_3]$$

$$\text{II)} \qquad \psi = e^{ix_0}[1 + i\gamma_1\gamma_2 + i\gamma_0 - \gamma_0\gamma_1\gamma_2 - i\gamma_0\gamma_1\gamma_2\gamma_3 - \gamma_0\gamma_3 - \gamma_1\gamma_2\gamma_3 + i\gamma_3].$$

If we consider φ there are always streams into direction of a vector v of the (x_1, x_2)-plane and orthogonal to it 2-dimensional vortices in that plane, which is spanned by the orthogonal to v in the (x_1, x_2)-plane and by x_3. These generate 3-dimensional vortices in the (x_1, x_2, x_3)-plane. That is a current $a \cdot \gamma_0$ and the potential of an electric field.

If we multiply the mass by a factor $r > 0$, then the length of the entering lightvectors is multiplied by $1/r$. Since this length became smaller, the probability that 2 vortices react to a vortex of type $a \cdot \gamma_0$ gets greater. It also will be multiplied by r. So the potential of the electric field at the boundary of the generating part of the particle will be multiplied by r. But the diameter is $1/r$-times the old one. That means that the electric charge of the particle will be the same. May be one can obtain from this that the elementary charge is independent of the particle.

If we take the basis wave ψ, no such electric field is generated. – If we would use the terminology of spinors, we would have only one basis wave. Here we get two of them and they are very different.

Interpretation. In classical quantum theory in a point $x \in \mathbb{R}^4$ spinors have just 4 complex components. The set of spinors is an irreducible representation of the Lie group $SU(2)$, which is simply connected. This group is the 2-sheeted universal covering of the Lie group $SO(3)$. Instead of the spinors we use here the Clifford algebra. It is a representation of the $SO(3)$. But this representation is not irreducible. It is the direct sum of two representations of the complex dimension 4. That is related to the two waves φ und ψ. We think φ to be the description of electric matter and ψ a description of totally unelectric matter (like the neutrinos).

If a particle has a larger mass in rest and is sufficiently stable, this should come from the electricity. The wave φ should be the wave of the electron. There will be an instable equlibrium between the wave and the electric field. But the particle will send energy into the vacuum. On the other hand there are virtual particles in the vacuum. These are particles, which have a generating center, but whose wave is not developed. They have an energy and might bring them to the particle. Probably, they are photons. So an equilibrium will be produced. It will bring the latitude of the electron. Of course this latitude comes from the elementary length and the density of $\mathcal{L}$. A particle in rest with wave ψ should not exist.

2. – Elementary particles

Particles of mass 0, μ-particle

1. Particles of mass 0. We define again

$$\psi = e^{ix_0}[1 + i\gamma_1\gamma_2 + i\gamma_0 - \gamma_0\gamma_1\gamma_2 - i\gamma_0\gamma_1\gamma_2\gamma_3 - \gamma_0\gamma_3 - \gamma_1\gamma_2\gamma_3 + i\gamma_3].$$

We accelerate ψ in x_3-direction going towards the velocity of light. We obtain:

$$\psi_{\mathrm{I}} = e^{i(x_0-x_3)} \cdot [-i\gamma_0 + \gamma_0\gamma_1\gamma_2 + \gamma_1\gamma_2\gamma_3 - i\gamma_3]$$

with spin $\frac{1}{2}$ and

$$\psi_{\mathrm{II}} = e^{i(x_0-x_3)} \cdot [1 - i\gamma_1\gamma_2 - i\gamma_0\gamma_1\gamma_2\gamma_3 + \gamma_0\gamma_3]$$

with spin $-\frac{1}{2}$. The anti-particles have the opposite spin. So the spin is given by direction of spreading. We have the so called helicity. It is possible to see that the waves ψ_{I} belong to the electron neutrinos and the ψ_{II} the μ-neutrinos. The neutrinos are not invariant under parity transfomations.

We can do the same thing in the case of the electron. We obtain:

$$\varphi_{\mathrm{I}} = e^{i(x_0-x_3)}[\gamma_1 - i\gamma_2 + i\gamma_0\gamma_2\gamma_3 - \gamma_0\gamma_1\gamma_3]$$

and

$$\varphi_{\mathrm{II}} = e^{i(x_0-x_3)}[i\gamma_0\gamma_1 - \gamma_0\gamma_2 + \gamma_2\gamma_3 - i\gamma_1\gamma_3].$$

But the case $m = 0$ can occur only, if the electric field vanishes. So we have to form the tensor product of φ_{I} and the anti-particle of φ_{II}. We obtain a neutral particle with spin ± 1. That is the photon. In this case we have particle = anti-particle. There is no helicity, any longer.

2. The μ-particle. We shall define the wave of an electric particle with spin $\frac{1}{2}$. We define $\psi = \varphi_1 + *\varphi_1$ with $\varphi_1 = (x_1 - ix_2)\gamma_1 + (ix_1 + x_2)\gamma_2 + i(x_1 - ix_2)\gamma_0\gamma_1 + i(ix_1 + x_2)\gamma_0\gamma_2$. This (quasi-) particle is circular symmetric with respect to x_1, x_2. Its spin is $\frac{1}{2}$. But it satisfies the time part of the Dirac operator only. The rest is given by $D' = \frac{\partial}{\partial x_1}\gamma_1 + \ldots + \frac{\partial}{\partial x_3}\gamma_3$. So we have $D = \frac{\partial}{\partial x_0}\gamma_0 + D'$. The wave $D'\varphi_1$ consists of $\varphi_2 = 2 + 2i\gamma_1\gamma_2$ and $\varphi_3 = -2i\gamma_0 + 2\gamma_0\gamma_1\gamma_2$. The spin is $\frac{1}{2}$, too.

The whole situation can be interpreted so: ψ is an electron with spin $-\frac{1}{2}$, which is moved with spin 1 around the origin of the (x_1, x_2)–plane. Then, there is an interaction of the wave in all points symmetric to O. This is a force, which produces the large mass of the particle in rest.

ψ should really give an electron and $D'\psi$ its motion in space. Unfortunately, the Dirac equation is not satisfied for $\psi + D'\psi$. But it is for $\psi + D'\psi - \frac{1}{2}D'\psi$. This means that we have to take the tensor product at φ_2 with a μ-neutrino (or anti neutrino) and at φ_3 with an e-neutrino (or an e-anti–neutrino). Both must have the spin $\frac{1}{2}$. One of the neutrinos has to be a particle, the other one an anti particle. We have to add these two particles. In the decay we will get back all three parts. The electron will move into one x_3-direction, the neutrinos both into the opposite. Then, it can be seen that the e-neutrino must be an anti-particle. So, the kind of decay will be determined by our theory.

REFERENCES

[1] H. Grauert, *Statistische Geometrie. Ein Versuch zur geometrischen Deutung physikalischer Felder*, Nachr. Akad. Wiss. Göttingen Math. Phys. Kl. **2** (1976).

[2] H. Grauert, *Statistical geometry and spacetime*, Commun. Math. Phys. **49** (1976).

[3] H. Grauert – S. Leykum, *Die pseudoeuklidische Geometrie als statistisches Gleichgewicht*, Math. Ann. **255** (1981).

[4] H. Gauert, "Selected Papers", Volume II, p. 907-913. Springer, Heidelberg, 1994.

[5] H. Grauert, *Discrete Geometry*, Nachr. Akad. Wiss. Goettingen, II. Math-Phys. Kl. **6** (1966), 343-362.

[6] S. Leykum, Ein Satz aus der statistischen Geometrie, Thesis 1980, University of Goettingen.

[7] E. C. Zeeman, *Causality implies the Lorentz Group*, J. Math. Phys. **5** (1964), 490-493.

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