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Solution of the $\bar{\partial}$-equation on non-smooth strictly q-concave domains with Hölder estimates and the Andreotti-Vesentini separation theorem

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Solution of the $\overline{\partial}$-equation on Non-smooth Strictly q-concave Domains with Hölder Estimates and the Andreotti-Vesentini Separation Theorem

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0. - Preface

In the present paper we show that the approach from [10] can be used to solve the $\overline{\partial}$-equation with Hölder estimates on strictly q-concave domains and to prove the Andreotti-Vesentini separation theorem with Hölder estimates on non-smooth domains.

A real-valued C^2 function ϱ defined on the domain $U \subseteq \mathbb{C}^n$ will be called $(q+1)$-convex if its Levi form has at least $q+1$ positive eigenvalues in every point on U. A domain $D \subset\subset X$, in some n-dimensional complex manifold X, will be called *strictly q-concave* $(1 \leq q \leq n-1)$, if there exists a $(q+1)$-convex function $\varrho : U \to \mathbb{R}$ defined in some neighbourhood U of ∂D such that

$$D \cap U = \{\varrho > 0\}. \tag{0.1}$$

(We do not assume that $d\varrho(z) \neq 0$ for all $z \in \partial D$.)

For these domains, and for all $1 \leq r \leq q-1$, we prove in the present paper the following

THEOREM 0.1. *The space of forms f, for which $\overline{\partial} u = f$ can be solved on $\overline{D}$ by a continuous $(0, r-1)$-form u, has finite codimension in the space of all $\overline{\partial}$-closed continuous $(0, r)$-forms on $\overline{D}$.*

If the $(q+1)$-convex function ϱ in (0.1) can be chosen even of class $C^{2+\alpha}$, with $0 < \alpha \leq 1/2$, then we prove the following

THEOREM 0.1$'$. *The space of forms f, for which $\overline{\partial} u = f$ can be solved on $\overline{D}$ by an α-Hölder-continuous $(0, r-1)$-form u, has finite codimension in the space of all $\overline{\partial}$-closed continuous $(0, r)$-forms on $\overline{D}$.*

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Theorem 0.1′ was published in 1976 without proof in the case that ∂D is of class C^∞ by N. Øvrelid [8]. In 1979, I. Lieb [6] proved Theorem 0.1′ in the case that ∂D is of class C^∞. In the case that the $(q+1)$-convex function ϱ in (0.1) is of class C^3 and the critical points of ϱ on ∂D are non-degenerate, Theorem 0.1′ was proved in the book [5]. The assertion of Theorem 0.1′ is there formulated under the weaker condition that ϱ is only of class C^2 and with $\alpha = 1/2$. But the proof is correct only if ϱ is of class C^3. It seems to be not clear whether Theorem 0.1′ is right for $\alpha = 1/2$ and $\varrho \in C^2$. In Section 2 we give an example which shows that the approach from [5] and of the present paper does not give such a result.

In Section 4 we prove a version of the Andreotti-Vesentini separation theorem with Hölder estimates. The main result can be formulated as follows:

THEOREM 0.2. *Let $D \subset\subset X$ be a strictly q-concave domain in an n-dimensional compact complex manifold X, $1 \leq q \leq n-1$, such that the defining $(q+1)$-convex function in* (0.1) *can be chosen of class $C^{2+\alpha}$, with $0 \leq \alpha \leq 1/2$. Then the space of $\bar\partial$-closed continuous $(0,q)$-forms f on $\overline{D}$, for which $\bar\partial u = f$ can be solved with an α-Hölder-continuous form u on $\overline{D}$, is topologically closed with respect to the max-norm.*

Theorem 0.2 was proved in the book [5] under the condition that the $(q+1)$-convex function ϱ in (0.1) is of class C^3 and has only non-degenerate critical points on ∂D; the case $q = n-1$ and $d\varrho(z) \neq 0$, $z \in \partial D$, has been proved in [3].

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1. - Local solution of $\bar\partial u = f_{0,r}$ on strictly q-concave domains with $1 \leq r \leq q-1$

First we recall some definitions concerning q-convexity (according to the terminology used in [5]).

Let $U \subseteq \mathbb{C}^n$ be an open set, and $\varrho : U \to \mathbb{R}$ a C^2 function. Then the function

$$F_\varrho(z,\zeta) = 2\sum_{j=1}^{n} \frac{\partial \varrho}{\partial \zeta_j}(\zeta_j - z_j) - \sum_{j,k=1}^{n} \frac{\partial^2 \varrho(\zeta)}{\partial \zeta_j \partial \zeta_k}(z_j - \zeta_j)(z_k - \zeta_k),$$

defined for $\zeta \in U$ and $z \in \mathbb{C}^n$, is called the *Levi polynomial* of ϱ. The function ϱ is called *q-convex* on U $(1 \leq q \leq n)$ if the Levi matrix $(\partial^2\varrho(z)/\partial z_j \partial \bar z_k)_{j,k=1}^n$

has at least q positive eigenvalues for each $z \in U$ ("n-convex" means then "strictly plurisubharmonic"). The function ϱ will be called *normalized q-convex* if, for all $z \in U$, the matrix $(\partial^2\varrho(z)/\partial z_j\partial\bar{z}_k)_{j,k=1}^q$ is positive-definite (i.e., ϱ is strictly plurisubharmonic with respect to $z_1, \dots, z_q$) and, moreover, there are some constants $\beta > 0$ and $B < \infty$ such that, for all $z, \varsigma \in U$,

$$\text{Re } F_\varrho(z,\varsigma) \geq \varrho(\varsigma) - \varrho(z) + \beta \cdot \sum_{j=1}^{q} |\varsigma_j - z_j|^2 - B \cdot \sum_{j=q+1}^{n} |\varsigma_j - z_j|^2. \tag{1.1}$$

DEFINITION 1.1. (see [10], Definition 2.1) A quadruple $[U, \varrho, \varphi, D]$ is called *q-convex configuration* in $\mathbb{C}^n$ if the following conditions are fulfilled:

(i) $U \subseteq \mathbb{C}^n$ is a convex open set, and $\varphi : U \to \mathbb{R}$ is a convex C^2 function such that $\emptyset \neq \{\varphi < 0\} \subset\subset U$;

(ii) $\varrho : \tilde{U} \to \mathbb{R}$ is a normalized $(q+1)$-convex function in some neighbourhood $\tilde{U}$ of $\overline{U}$;

(iii) $d\varrho(z) \wedge d\varphi(z) \neq 0$ for all $z \in \{\varrho = 0\} \cap \{\varphi = 0\}$;

(iv) $D = \{z \in U : \varrho(z) < 0,\ \varphi(z) < 0\}$ and $\emptyset \neq D \neq \{\varphi < 0\}$.

In this case, D is called the domain of the configuration, and we say that $[U, \varrho, \varphi]$ *defines* the q-convex configuration $[U, \varrho, \varphi, D]$.

DEFINITION 1.2. $[U, \varrho, \varphi, H, D]$ will be called a *q-concave configuration* of class $C^{2+\alpha}$ in $\mathbb{C}^n$, $0 \leq \alpha \leq 1/2$, $1 \leq q \leq n-1$, if $[U, -\varrho, \varphi]$ defines a q-convex configuration in $\mathbb{C}^n$, where $\varrho \in C^{2+\alpha}$ and the following conditions (a)-(d) are fulfilled:

(a) $H = H(z)$, $z \in \mathbb{C}^n$, is a function of the form

$$H(z) = H'(z) + M \cdot \sum_{j=q+2}^{n} |z_j|^2, \tag{1.2}$$

where $H'(z)$ is a holomorphic polynomial in $z \in \mathbb{C}^n$ and M is a positive number;

(b) $\varphi(z) < 0$ for all $z \in U$ with $\text{Re}\, H(z) = \varrho(z) = 0$;

(c) $D = \{z \in U : \varrho(z) < 0,\ \varphi(z) < 0,\ \text{Re}\, H(z) < 0\} \neq \emptyset$;

(d) $d\,\text{Re}\, H(z) \neq 0$ for all $z \in U$ with $\text{Re}\, H(z) = 0$,
$d\,\text{Re}\, H(z) \wedge d\varphi(z) \neq 0$ for all $z \in U$ with $\text{Re}\, H(z) = \varphi(z) = 0$,
$d\,\text{Re}\, H(z) \wedge d\varrho(z) \neq 0$ for all $z \in U$ with $\text{Re}\, H(z) = \varrho(z) = 0$.

In this case, D is called the *domain* of the configuration and we say that $[U, \varrho, \varphi, H]$ *defines* the q-concave configuration $[U, \varrho, \varphi, H, D]$.

LEMMA 1.3. *Let $\varrho : X \to \mathbb{R}$ be a $(q+1)$-concave function (i.e., $-\varrho$ is a $(q+1)$-convex function) on an n-dimensional complex manifold $(1 \le q \le n-1)$, and let $y \in X$ be such that $\varrho(y) = 0$.*

Then there exists a holomorphic map h, from some neighbourhood V of y onto the unit ball, such that $h(y) = 0$ and the following statement holds true: We can find a function $\tilde{\varrho}$ on $h(V)$ and a function H of the form (1.2), a number r with $0 < r < 1$ and neighbourhoods $y \in V_1 \subset\subset V_2 \subset\subset V$ such that

$$[h(V),\ \tilde{\varrho},\ \varphi = |z|^2 - r^2, H]$$

defines a q-concave configuration in $\mathbb{C}^n$,

$$\tilde{\varrho} = \varrho \circ h^{-1} \quad \text{on} \quad h(V_1)$$

$$\tilde{\varrho} \ge \varrho \circ h^{-1}$$

$\tilde{\varrho}$ has not degenerate critical points in $h(V \setminus V_2)$.

PROOF. Analogously to Lemma 2.2 in [10], it can be shown that there exist neighbourhoods $y \in V_1 \subset\subset V_2 \subset\subset V$, holomorphic coordinates $h : V \to \mathbb{C}^n$, and a q-convex function $-\tilde{\varrho}$ on $h(V)$, such that $\tilde{\varrho} \le -\varrho \circ h^{-1}$ on $h(V_1)$, $\tilde{\varrho}$ has not degenerate critical points in $h(V \setminus V_2)$, $\tilde{\varrho} = \varrho \circ h^{-1}$ on $h(V_1)$, $h(V)$ is the unit ball, $h(y) = 0$ and, for some r with $0 < r < 1$, $[h(V), -\tilde{\varrho}, |z|^2 - r^2]$ defines a q-convex configuration. It remains to construct the function H. Since $-\tilde{\varrho}$ is normalized $(q+1)$-convex, there are constants $C < \infty$, $\beta > 0$ with

$$-\operatorname{Re} F_{\tilde{\varrho}}(z, 0) \ge \tilde{\varrho}(z) + \beta \cdot \sum_{j=1}^{q+1} |z_j|^2 - C \cdot \sum_{j=q+2}^{n} |z_j|^2$$

for all $z \in h(V)$. Setting

$$\tilde{H}(z) = -F_{\tilde{\varrho}}(z, 0) + (C + \beta) \sum_{j=q+2}^{n} |z_j|^2$$

we obtain a function of the form (1.2) such that $\operatorname{Re} \tilde{H}(z) \ge \beta |z|^2$ for all $z \in h(V)$ with $\tilde{\varrho}(z) \ge 0$.

Hence, $\left[h(V),\ \tilde{\varrho},\ |z|^2 - r^2,\ \tilde{H} - \frac{\beta}{2} r^2\right]$ fulfils all conditions in order to define a q-concave configuration, except for (possibly) condition (d). By a lemma of Morse (cf., for instance, [5], Lemma 0.3 in Appendix B), for almost all complex linear maps $L : \mathbb{C}^n \to \mathbb{C}$, the function

$$\operatorname{Re} \left(\tilde{H} - \frac{\beta}{2} r^2 + L(z) \right) \qquad z \in \mathbb{C}^n$$

has not degenerate critical points. The same is true for the restriction of this function to the surface $\{\varphi = 0\}$.

Furthermore, the hypersurface $\{\tilde{\varrho} = 0\}$ has in $h(V \setminus V_2)$ only non-degenerate and, hence, isolated singularities. The neighbourhood $V_2 \ni y$ can be chosen small enough, so that $\operatorname{Re} H(z) \neq 0$ in $\overline{V}_2$. Then, without loss of generality, we can assume that $L(z)$ has been chosen such that the restriction of $\operatorname{Re}\left(\tilde{H} - \frac{\beta}{2} r^2 + L(z)\right)$ on $\{\tilde{\varrho} = 0\} \cap h(V \setminus V_2)$ has not degenerate critical points either. This implies that, for almost all real numbers ε, the function $H(z) := \left(\tilde{H} - \frac{\beta}{2} r^2 + L(z)\right) + \varepsilon$, $z \in \mathbb{C}^n$, fulfils condition (d) in Definition (1.2). If, moreover, L and ε are sufficiently small, then H fulfils also the other conditions in this definition. ∎

The set $\operatorname{Div}(\varrho)$. Let $[U, -\varrho, \varphi, D]$ be a fixed q-convex configuration $(1 \leq q \leq n-1)$.

Choose C^1 functions $a_{jk} : U \to \mathbb{C}$, $j, k = 1, \ldots, n$, such that

$$\left| a_{jk}(z) - \frac{\partial^2 \varrho(z)}{\partial z_j \partial z_k} \right| < \frac{\beta}{2n^2} \tag{1.3}$$

for all $z \in U$. For $z, \zeta \in U$ we define

$$w_1^j(z, \zeta) := 2 \frac{\partial \varrho}{\partial z_j} + \sum_{k=1}^{n} a_{jk}(z)(\zeta_k - z_k) \tag{1.4}$$

if $1 \leq j \leq q+1$,

$$w_1^j(z, \zeta) := 2 \frac{\partial \varrho}{\partial z_j} + \sum_{k=1}^{n} a_{jk}(z)(\zeta_k - z_k) + (B + \beta)(\overline{\zeta}_j - \overline{z}_j)$$

if $q + 2 \leq j \leq n$, and we set $w_1 = (w_1^1, \ldots, w_1^n)$.

Then, it follows from (1.1) and (1.3) that

$$\operatorname{Re} \langle w_1(z, \zeta), \zeta - z \rangle \geq \varrho(\zeta)\varrho(z) + \frac{\beta}{2} |\zeta - z|^2 \tag{1.5}$$

for all $\zeta, z \in U$.

Further, we define

$$w_2^j = w_2^j(z, \zeta) = \frac{\partial \varphi(\zeta)}{\partial \zeta_j}$$

for $j = 1, \ldots, n$ and $z, \zeta \in U$, and we set $w_2 = (w_2^1, \ldots, w_2^n)$. Since φ is convex and, for fixed $\zeta \in \{\varphi = 0\}$, $\{z : \langle w_2, \zeta - z \rangle = 0\}$ is the complex tangent plane of $\{\varphi = 0\}$ at ζ, we have the relation

$$\langle w_2, \zeta - z \rangle \neq 0 \tag{1.6}$$

for all $\zeta \in \{\varphi = 0\}$ and $z \in \{\varphi < 0\}$.

The set Div(H). Let H be of the form (1.2), i.e.,

$$H(z) = H'(z) + M \cdot \sum_{j=q+2}^{n} |z_j|^2, \quad z \in \mathbb{C}^n,$$

where H' is a holomorphic polynomial and M a positive number.

DEFINITION. By Div(H) we denote the set of all n-tuples $v = (v^1, \ldots, v^n)$ of complex valued C^1 functions $v^j : \mathbb{C}^n \times \mathbb{C}^n \to \mathbb{C}$ which are obtained by the following

Construction: Take holomorphic polynomials $v'_j = v'_j(z, \varsigma)$, $j = 1, \ldots, n$, in $(z, \varsigma) \in \mathbb{C}^n \times \mathbb{C}^n$ such that

$$H'(\varsigma) - H'(z) = \sum_{j=1}^{n} v'_j(z, \varsigma)(\varsigma_j - z_j),$$

set $v^j = v'_j$ for $j = 1, \ldots, q+1$, and $v^j = v'_j + M(\overline{\varsigma}_j + \overline{z}_j)$ for $j = q+2, \ldots, n$.

REMARK. For any $v \in$ Div(H), we have the relation

$$\begin{aligned}\langle v(z, \varsigma), \varsigma - z \rangle = H'(\varsigma) - H'(z)& \\ + M \cdot \sum_{j=q+2}^{n} (|\varsigma_j|^2 - |z_j|^2 + \overline{z}_j \varsigma_j \overline{\varsigma}_j z_j)&\end{aligned}$$

and, hence,

$$\operatorname{Re} \langle v(z, \varsigma), \varsigma - z \rangle = \operatorname{Re} H(\varsigma) - \operatorname{Re} H(z). \tag{1.7}$$

CANONICAL LERAY DATA AND MAPS 1.4. (cf. [5], Section 13.4). Let $[U, \varrho, \varphi, H, D]$ be a q-concave configuration in $\mathbb{C}^n$, $1 \leq q \leq n-1$. Set $\psi_1 = \varrho$, $\psi_2 = \varphi$, $\psi_3 = \operatorname{Re} H$ and

$$Y_j = \{z \in U : \psi_j = 0\}, \ D_j = \{z \in U : \psi_j < 0\} \text{ for } j = 1, 2, 3.$$

Then $D = D_1 \cap D_2 \cap D_3$ is a domain with piecewise almost C^1 boundary, and (Y_1, Y_2, Y_3) is a frame for D (cf. [5], Sect. 3.1).

PROPOSITION. *Let* (w_1, w_2, w_3) *be such that*

$$\begin{aligned}&w_1 \in \operatorname{Div}(\varrho)\\ &w_2 = \nabla^{\mathbb{C}} \varphi, \ \textit{where } \nabla^{\mathbb{C}} = \left(\frac{\partial}{\partial \varsigma_1}, \ldots, \frac{\partial}{\partial \varsigma_n}\right)\\ &w_3 \in \operatorname{Div}(H).\end{aligned} \tag{1.8}$$

Then

$$\langle w_j(z,\varsigma),\, \varsigma - z\rangle \neq 0 \tag{1.9}$$

for all $\varsigma \in Y_j$, $z \in D_j$ *and* $j = 1,2,3$.

PROOF. This follows from (1.5), (1.6) and (1.7). ∎

DEFINITION. We say (w_1, w_2, w_3) is a *canonical Leray datum* for $[U, \varrho, \varphi, H, D]$ (or for D) if (1.9) holds true.

We set

$$S_j = Y_j \cap \partial D,\ \ S_{ij} = S_i \cap S_j,\ \ (i,j = 1,2,3).$$

Then, by (1.9), the following *Leray maps* are correctly defined

$$\eta_j = \eta_j(z,\varsigma) = \frac{w_j(z,\varsigma)}{\langle w_j(z,\varsigma), \varsigma - z\rangle}$$

for all $z \in D$ and $\varsigma \in S_j$ ($j = 1,2,3$). Further, let

$$\Delta = \{\lambda = (\lambda_0, \lambda_1, \lambda_2, \lambda_3) \in \mathbb{R}^4 : \lambda_0 + \lambda_1 + \lambda_2 + \lambda_3 = 1,\ \ \lambda_j \geq 0,\ \ j = 0,1,2,3\}$$

be the 3-dimensional standard simplex, $\Delta_{ij} = \{\lambda \in \Delta : \lambda_i + \lambda_j = 1\}$, and $\Delta_{0ij} = \{\lambda \in \Delta : \lambda_0 + \lambda_i + \lambda_j = 1\}$. We set

$$\eta_{ij} = \eta_{ij}(z,\varsigma,\lambda) = \lambda_i \eta_i(z,\varsigma) + \lambda_j \eta_j(z,\varsigma)$$

for $z \in D$, $\varsigma \in S_{ij}$, $\lambda \in \Delta_{ij}$;

$$\eta_{0j} = \eta_{0j}(z,\varsigma,\lambda) = \lambda_0 \frac{\bar{\varsigma} - \bar{z}}{|\varsigma - z|^2} + \lambda_j \eta_j(z,\varsigma)$$

for $z \in D$, $\varsigma \in S_j$, $\lambda \in \Delta_{0j}$, $j = 1,2,3$; and

$$\eta_{0ij} = \eta_{0ij}(z,\varsigma,\lambda) = \lambda_0 \frac{\bar{\varsigma} - \bar{z}}{|\varsigma - z|^2} + \lambda_i \eta_i(z,\varsigma) + \lambda_j \eta_j(z,\varsigma)$$

for $z \in D$, $\varsigma \in S_{ij}$, $\lambda \in \Delta_{0ij}$.

Further, we shall use the following notations: if $A = (a_{ij})_{i,j=1}^n$ is a matrix of differential forms, then

$$\det{}' A := \frac{1}{(2\pi i)^n} \sum_{\sigma} \operatorname{sgn}(\sigma)\ a_{\sigma(1),1} \wedge \ldots \wedge a_{\sigma(n),n},$$

where the summation is over all permutations σ of $\{1,\ldots,n\}$. If $a_1,\ldots,a_m$ are vectors of length n of differential forms and $s_1,\ldots,s_n \geq 0$ are integers with $s_1 + \ldots + s_m = n$, then

$$\det{}'_{s_1,\ldots,s_m}(a_1,\ldots,a_m) := \det{}'\ (\underbrace{a_1,\ldots,a_1}_{s_1 \text{ times}},\ldots,\underbrace{a_m,\ldots,a_m}_{s_m \text{ times}}).$$

We set $\omega = \omega(\varsigma) = \mathrm{d}\varsigma_1 \wedge \ldots \wedge \mathrm{d}\varsigma_n$.

Now, for each continuous differential form f on $\overline{D}$, we define the following integral operators (integration over S_1 means integration over the regular part of S_1, which is well defined by Lemma 1.3 in [10]):

$$L_j f = \int_{S_j} f(\varsigma) \wedge \det{}'_{1,n-1}\left(\eta_j(\cdot,\varsigma),\ \overline{\partial}_{z,\varsigma}\eta_j(\cdot,\varsigma)\right) \wedge \omega(\varsigma),\ \ j = 1,2,3,$$

$$L_{ij} f = \int_{S_{ij}\times\Delta_{ij}} f(\varsigma) \wedge \det{}'_{1,n-1}\left(\eta_{ij}(\cdot,\varsigma,\lambda),\ (\overline{\partial}_{z,\varsigma} + \mathrm{d}_\lambda)\eta_{ij}(\cdot,\varsigma,\lambda)\right) \wedge \omega(\varsigma),$$

$$R_j f = \int_{S_j\times\Delta_{0j}} f(\varsigma) \wedge \det{}'_{1,n-1}\left(\eta_{0j}(\cdot,\varsigma,\lambda),\ (\overline{\partial}_{z,\varsigma} + \mathrm{d}_\lambda)\eta_{0j}(\cdot,\varsigma,\lambda)\right) \wedge \omega(\varsigma),$$

$$R_{ij} f = \int_{S_{ij}\times\Delta_{0ij}} f(\varsigma) \wedge \det{}'_{1,n-1}\left(\eta_{0ij}(\cdot,\varsigma,\lambda),\ (\overline{\partial}_{z,\varsigma} + \mathrm{d}_\lambda)\eta_{0ij}(\cdot,\varsigma,\lambda)\right) \wedge \omega(\varsigma),$$

$$Bf = \int_D f(\varsigma) \wedge \frac{\det{}'_{1,n-1}(\overline{\varsigma}\overline{z}, \mathrm{d}\overline{\varsigma} - \mathrm{d}\overline{z})}{|\varsigma - z|^{2n}} \wedge \omega(\varsigma).$$

LEMMA 1.5. (cf. Lemma 13.7 in [5]). *Let $[U,\varrho,\varphi,H,D]$ be a q-concave configuration in $\mathbb{C}^n$, $1 \le q \le n-1$, let D_2, D_3 be as in Section 1.4, and let η be a canonical Leray map for $[U,\varrho,\varphi,H,D]$. Then, for any integer r with $0 \le r \le n-2$, there exists a linear operator*

$$M_r : Z^0_{0,r}(\overline{D}) \to Z^0_{0,r}(D_2 \cap D_3)$$

which is continuous with respect to the Banach space topology of $Z^0_{0,r}(\overline{D})$ and the Fréchet space topology of $Z^0_{0,r}(D_2 \cap D_3)$, such that

$$M_r f|_D = L_{23} f$$

for all $f \in Z^0_{0,r}(D)$.

PROOF. The proof is analogous to the proof of Lemma 13.7 in [5]. ∎

THEOREM 1.6. (cf. Theorem 13.10 in [5]). *Let $[U,\varrho,\varphi,H,D]$ be a q-concave configuration in $\mathbb{C}^n$, $1 \le q \le n-1$. Then, for each $1 \le r \le q-1$, the following assertions hold true:*

(i) *For any $\overline{\partial}$-closed continuous $(0,r)$-form f in $\overline{D}$, there exists a continuous $(0,r-1)$-form u in D with $\overline{\partial} u = f$.*

(ii) *Set*

$$\tilde{D} := \{z \in U : \varphi(z) < 0,\ \operatorname{Re} H(z) < 0\}.$$

Let $T := B + R_1 + R_2 + R_3 + R_{13} + R_{23}$ be the Cauchy-Fantappiè operator for the Leray map η on $D\,(R_{12} = 0$ since $S_{12} = \emptyset)$ and

$$M_r : Z^0_{0,r}(\overline{D}) \to Z^0_{0,r}(\tilde{D})$$

the continuous operator from Lemma 1.5. Then for any $f \in Z^0_{0,r}(\overline{D})$, we have the representation

$$f = \overline{\partial} T f + M_r f \quad \text{in } D. \tag{1.10}$$

Moreover, there exists a continuous $(0, r-1)$-form g on $\tilde{D}$ with $M_r f = \overline{\partial} g$ on $\tilde{D}$. Hence, $u := Tf + g$ solves the equation $\overline{\partial} u = f$ in D.

PROOF. Part (i) is included in part (ii). For the proof of part (ii), let $f \in Z^0_{0,r}(\overline{D})$. Then it can be proved analogously to [5], Lemma 13.6, that $L_1 f = L_2 f = L_3 f = L_{13} f = 0$, and hence the piecewise Cauchy-Fantappiè formula (cf. [5], Theorem 3.12) takes the form

$$f = \overline{\partial} T f + L_{23} f \quad \text{in } D.$$

Since $L_{23} f = M_r f$ on D (cf. Lemma 1.5), this implies (1.10). Since, $\tilde{D}$ is completely pseudoconvex, which can be proved analogously to Lemma 13.5(i) in [5], and by Theorem 5.3 in [5], it follows, for instance from Theorem 12.16 in [5], that $M_r f = \overline{\partial} g$ for some continuous $(0, r-1)$-form g on $\tilde{D}$. ∎

2. - Uniform estimates for the local solutions of the $\overline{\partial}$-equation and finiteness of the Dolbeault cohomology of order r with uniform estimates on strictly q-concave domains with $1 \le r \le q-1$

EXAMPLE 2.1. The following example shows that the Leray maps used in the present paper admit only to find solutions of the $\overline{\partial}$-equation which is Hölder continuous with exponent α.

Let D be the domain of a 2-concave configuration in $\mathbb{C}^3$ $[U, \varrho, \varphi, H, D]$ such that

(i) For any $z \in U$ and $\alpha > 0$, $\dfrac{\partial^2 \varrho}{\partial z_1 \partial \overline{z}_1}$ is not Hölder continuous with exponent α,

(ii) $\dfrac{\partial^2 \varrho}{\partial z_i \partial \overline{z}_j} \in C^{1/2}$ for $(i, j) \neq (1, 1)$.

Furthermore, let f be a continuous $(0, 1)$-form such that

$$\operatorname{supp} f \subseteq G_\varepsilon = \{\varphi < -\varepsilon\} \cap \overline{D}.$$

Then

$$R_1 f = \frac{\partial^2 \varrho}{\partial z_1 \partial \overline{z}_1} K_1(f,z) + K_2(f,z),$$

where $K_i(f,z) \in C^{1/2}$. Hence, we can choose a form f such that $K_1(f,z) \not\equiv 0$, and therefore $R_1 f \notin C^\alpha$ for any $\alpha > 0$. Since the other operators from the integral representation admit an 1/2-Hölder estimate, it follows that the solution defined in Theorem 1.6 is in general not Hölder continuous.

LEMMA 2.2. (cf. Lemma 5.2 in [10]). *If* $w(z,\varsigma)$ *is of the form* (1.4) *and* $t(z,\varsigma) = \operatorname{Im}\langle w(z,\varsigma), \varsigma - z\rangle$, *then the following assertions hold true*:

(i) $\|\mathrm{d}_\varsigma t(z,\varsigma)|_{\varsigma=z}\| = \|\mathrm{d}\varrho(z)\|$ *for all* $z \in U$;

(ii) $\|\mathrm{d}\varrho(z)\| \leq n^{1/2}\ \|\mathrm{d}\varrho(z) \wedge \mathrm{d}_\varsigma t(z,\varsigma)|_{\varsigma=z}\|$ *for all* $z \in U$;

(iii) *If* $x_j = x_j(\varsigma)$ *are the real coordinates of* $\varsigma \in \mathbb{C}^n$, *with*

$$\varsigma_j = x_j(\varsigma) + \sqrt{-1}\, x_{j+n}(\varsigma),$$

then there is a constant $K < \infty$ *such that*

$$\left| \frac{\partial t(z,\cdot)}{\partial x_j}(\varsigma) - \frac{\partial t(z,\cdot)}{\partial x_j}(z) \right| \leq K\ |\varsigma - z| \quad \textit{for all } \varsigma, z \in D_2,\ j = 1, \ldots, 2n;$$

(iv) *If* $\varsigma \in \partial D$ *with* $\varrho(\varsigma) = \mathrm{d}\varrho(\varsigma) = 0$, *then there exists a constant* $K < \infty$ *such that*

$$|t(z,\varsigma)| \leq K\ (\|\mathrm{d}\varrho(\varsigma)\|\ |\varsigma - z| + |\varsigma - z|^2) \quad \textit{for all } z, \varsigma \in D_2.$$

PROOF. The proof is analogous to the proof of Lemma 5.2 in [10]. ∎

THEOREM 2.3. (cf. Theorem 14.1 in [5]). *Let* $[U, \varrho, \varphi, H, D]$ *be a q-concave configuration of class* $C^{2+\alpha}$ *in* $\mathbb{C}^n$, $1 \leq q \leq n-1$, $0 \leq \alpha \leq 1/2$, η *a canonical Leray map for* D, $\varepsilon > 0$, *and*

$$D_\varepsilon := \{z \in U : \varphi(z) < -\varepsilon,\ \operatorname{Re} H(z) < -\varepsilon\}.$$

Then there exist constants $C_\alpha < \infty$ *such that: for all continuous differential forms* f *on* $\overline{D}$

$$\|T^v f\|_{\alpha, D_\varepsilon} \leq C \|f\|_{0,D}. \tag{2.1}$$

Moreover, the operator T *is compact as operator from* $C^0_r(\overline{D})$ *into* $C^0_{r-1}(D_\varepsilon)$ (*for* $\alpha > 0$, *this follows by Ascoli's theorem from* (2.1)).

PROOF. The proof of (2.1) is a repetition of the proof of Theorem 5.1 in [10] with the following exceptions:

(i) The operator T is now of the form

$$T = B + R_1 + R_2 + R_3 + R_{13} + R_{23}$$

and we have to add the remark that, since $S_3 \cap \overline{D}_\varepsilon = S_{13} \cap \overline{D}_\varepsilon = \emptyset$, estimates (2.1) holds true also with R_3, R_{13}, R_{23} instead of T.

(ii) Instead of Lemma 5.2 in [10], we have to use Lemma 2.2 from this paper.

(iii) Furthermore, w_1 is only $C^{1+\alpha}$ with respect to z and therefore Lemma 4.1 in [10] gives the result (2.1) with the exponent α.

We prove that T is compact for $\alpha = 0$. Let $\chi(z,\varsigma)$ be a C^∞-function such that $\chi = 0$ if $|z-\varsigma| > 2\varepsilon$, and $\chi = 1$ if $|z-\varsigma| < \varepsilon$. Set

$$R_1^\varepsilon f = \int \chi\, f(\varsigma)\, \det{}'_{1,n-1}\left(v_{01}(\cdot,\varsigma,\lambda), (d_{z,\varsigma} + d_\lambda)v_{01}(\cdot,\varsigma,\lambda)\right) \wedge \omega(\varsigma).$$

Then, by standard arguments, the operator $R_1 - R_1^\varepsilon$ is compact. Therefore it is sufficient to show that $\|R_1^\varepsilon f\| \to 0$ for $\varepsilon \to 0$. Analogously to the proof of Theorem 5.1 in [10]

$$|R_1^\varepsilon f(z)| \leq C_3 \|f\|_{0,D} \int\limits_{\substack{\varsigma\in S_1\\ |z-\varsigma|<2\varepsilon}} \frac{\|d\varrho\|\ dS_1}{(|\operatorname{Im}\Phi(z,\varsigma)| + |\varsigma - z|^2)\ |\varsigma - z|^{2n-3}}$$

$$+ C_3\|f\|_{0,D} \int\limits_{\substack{\varsigma\in S_1\\ |z-\varsigma|<2\varepsilon}} \frac{dS_1}{|\varsigma - z|^{2n-2}}.$$

The second integral is estimated in [10], (3.3), by $C\varepsilon$ (set there $y = z$). The first integral can be estimated analogously to (3.17) in [10] with one exception: since $d\varrho(y)$ does not necessarily vanish, we get only the following estimate for $|t(y,x)|$

$$|t(y,x)| \leq C\,\varepsilon \qquad \text{(instead of } C\,\varepsilon^2\text{)}.$$

This implies the estimate $\|R_1^\varepsilon\| \leq C\,\varepsilon^{1/2}$ which is sufficient to prove the compactness of R_1. ■

3. - Invariance of the Dolbeault cohomology of order $0 \leq r \leq q-1$ with respect to the boundary

DEFINITION 3.1. (cf. [5], Def. 14.3). Let $D \subset\subset X$ be some domain in an n-dimensional complex manifold X and q an integer with $1 \leq q \leq n-1$. The boundary of D will be called *strictly q-concave with respect to X* if the intersection of D with any connected component of X is non-empty, and there exists a strictly $(q+1)$-concave function $\varrho : U \to \mathbb{R}$ in some neighbourhood of ∂D such that

$$D \cap U = \{z \in U : \varrho(z) < 0\}.$$

THEOREM 3.2. *Let X be an n-dimensional complex manifold and $D \subset\subset X$ a domain with strictly q-concave boundary with respect to X such that the defining function $\varrho \in C^{2+\alpha}$ $(0 \leq \alpha \leq 1/2)$. Furthermore let E be a holomorphic vector bundle over X. Then*

$$E_{0,r}^{\alpha\to 0}(\overline{D}, E) = E_{0,r}^{0}(D, E) \cap Z_{0,r}^{0}(\overline{D}, E)$$

for all $1 \leq r \leq q-1$.

(Here $E_{0,r}^{\alpha\to 0}(\overline{D}, E)$ is the space of continuous E-valued $(0,r)$-forms f on $\overline{D}$ such that there exists on $\overline{D}$ an α-Hölder-continuous E-valued $(0, r-1)$-form u on $\overline{D}$ with $\bar\partial u = f$. $E_{0,r}^{0}(D, E)$ is the same with D instead of $\overline{D}$ and $\alpha = 0$. By $Z_{0,r}^{0}(\overline{D}, E)$, we denote the space of $\bar\partial$-closed continuous E-valued $(0,r)$-forms on $\overline{D}$).

PROOF. This follows by well-known arguments (see, e.g., the proof of Theorem 2.3.5 in [4]) from Theorems 1.6 and 2.1 as well as Theorem 15.11 in [5]. ∎

4. - The Andreotti-Vesentini separation theorem

CONSTRUCTION 4.1. Let U be a ball in $\mathbb{C}^n$ and ϱ a normalized $(q+1)$-convex function defined in some neighbourhood of $\overline{U}$.

We set

$$(4.1)\qquad \begin{aligned} w^j &= -2\frac{\partial \varrho}{\partial z_j} - \sum_{k=1}^{n} a_{jk}(z)(\varsigma_k - z_k), \quad 1 \leq j \leq q+1, \\ w^j &= -2\frac{\partial \varrho}{\partial z_j} - \sum_{k=1}^{n} a_{jk}(z)(\varsigma_k - z_k) + (B+\beta)(\overline{\varsigma}_j - \overline{z}_j), \quad q+2 \leq j \leq n, \end{aligned}$$

where a_{jk} are some C^1 functions on U such that

$$\left| a_{jk}(z) - \frac{\partial^2 \varrho(z)}{\partial z_j \partial z_k} \right| < \frac{\beta}{2n^2}.$$

Analogously to (1.5), it follows that

$$\operatorname{Re}\langle w(z,\varsigma), \varsigma - z\rangle \geq \varrho(z) - \varrho(\varsigma) + \frac{\beta}{2}\,|\varsigma - z|^2 \quad \text{for all } z, \varsigma \in U.$$

This implies that $\langle w(z,\varsigma), \varsigma - z\rangle \neq 0$ for $\varrho(\varsigma) = 0$, $\varrho(z) > 0$. Hence w is a Leray datum for $D = U \cap \{\varrho > 0\}$. The corresponding Leray map for D is defined by

$$(4.2)\qquad \eta = \frac{w(z,\varsigma)}{\langle w(z,\varsigma), \varsigma - z\rangle}, \qquad \tilde\eta = \lambda\,\frac{\overline{\varsigma} - \overline{z}}{|\varsigma - z|^2} + (1-\lambda)\eta.$$

As in Section 1.4, we define the operators L, R and B with respect to the Leray map (4.2) for differential forms which are continuous on $\overline{D}$. Since we consider special differential forms (cf. Lemma 4.2), we need not define other Leray maps and L and R operators. Set $T = B+R$. Then we have the following

LEMMA 4.2. *Let U, ϱ, D be as above and f a continuous $\overline{\partial}$-closed $(0,q)$-form on $\overline{D}$ such that* $\operatorname{supp} f \subseteq U'$ *for some $U' \subset\subset U$. Then $Lf = 0$ and, hence, the Cauchy-Fantappiè-formula takes the form*

$$(-1)^q f = \overline{\partial} T f.$$

Moreover, if $\varrho \in C^{2+\alpha}$, with $0 \le \alpha \le 1/2$, then there exists a constant C which depends only on U' and D such that for all f

$$\|Tf\|_{\alpha,D} \le C\ \|f\|_{0,D}.$$

PROOF. We can find a C^2 function $\tilde{\varrho}$ such that $\tilde{\varrho} \le \varrho$ on U, $\varrho = \tilde{\varrho}$ on U' and $[U, \tilde{\varrho}, \varphi]$ (where $\{\varphi < 0\}$ is some ball U'' with $U' \subset\subset U'' \subset\subset U$) defines some q-convex configuration (cf. Definition 1.1). Therefore we have the following Cauchy-Fantappiè formula (cf. [10])

$$(-1)^q f = \tilde{L} f + \overline{\partial}\tilde{T} f + \tilde{T}\overline{\partial} f$$

for all f such that f and $\overline{\partial} f$ are continuous on the closure of $\tilde{D} = U'' \cap \{\tilde{\varrho} > 0\}$, where $\tilde{L}$ and $\tilde{T}$ are some integral operators with the following properties

$$\tilde{L} f = L f, \text{ and}$$
$$\tilde{T} f = T f \text{ for all } f \text{ with } \operatorname{supp} f \subseteq U'.$$

From Theorem 2.3 we get an estimate

$$\|Tf\|_{\alpha,D} \le C\,\|f\|_{0,D}.$$

It remains to prove that $Lf = 0$. Since f is of bidegree $(0,q)$, the kernel of the integral defining Lf takes the form

$$\Lambda_q(z,\zeta) = (-1)^q \binom{n-1}{q} \det{}'_{1,q,n-q-1}\ (\eta, \overline{\partial}_z \eta, \overline{\partial}_\zeta \eta) \wedge \omega(\zeta)$$

(here $\omega(\zeta) = d\zeta_1 \wedge \ldots \wedge d\zeta_n$). The Leray map η depends holomorphically on $\zeta_1, \ldots, \zeta_{q+1}$ and therefore $\overline{\partial}_\zeta \Lambda_q = 0$ for $(U \backslash D) \times D$. Moreover, for fixed $z \in D$, Λ_q is $\overline{\partial}$-closed in some neighbourhood of $(U \backslash D)$ and, by Theorem 8.1 in [5], it can be uniformly approximated on $S = \partial\tilde{D} \cap \{\varrho = 0\}$ by a sequence $(g_p)_{p \in \mathbb{N}}$ with $g_p \in Z^0_{n,n-q-1}(\mathbb{C}^n)$. Then

$$Lf(z) = \int_S f(\zeta) \wedge \Lambda_q(z,\zeta) = \lim_{p\to\infty} \int_S f(\zeta) \wedge g_p(\zeta).$$

Since f and g_p are $\overline{\partial}$-closed in D, it follows that $f \wedge g_p$ is also $\overline{\partial}$-closed in D. Therefore, by Stokes' Theorem,

$$0 = \int_{\partial \tilde{D}} f \wedge g_p = \int_{S} f \wedge g_p + \int_{\partial \tilde{D} \cap \partial U''} f \wedge g_p = \int_{S} f \wedge g_p$$

(since $\partial U'' \cap \operatorname{supp} f = \emptyset$). Hence $Lf(z) = 0$. ∎

DEFINITION 4.3. Let K be a closed subset of the n-dimensional complex manifold X. Then we say that X is a *q-convex extension of class $C^{2+\alpha}$* of K, $0 \leq \alpha \leq 1/2$, $1 \leq q \leq n-1$, if there exist constants c, C,

$$-\infty < c < C \leq +\infty,$$

and a $(q+1)$-convex function of class $C^{2+\alpha}$, $\varrho : U \to (-\infty, C]$, in a neighbourhood of $\overline{X \backslash K}$, such that $K \cap U = \{\varrho \leq c\}$ and $\{c \leq \varrho \leq t\}$ is compact for all $t < C$.

Now we go to prove the following

THEOREM 4.4. (cf. Theorem 2.1 in [7]). *Let E be a holomorphic vector bundle over the n-dimensional complex manifold X and let $\Omega \subseteq X$ be an open (not necessarily relatively compact in X) such that its boundary in X is compact, and X is a q-convex extension of class $C^{2+\alpha}$ of $\overline{\Omega}$, $1 \leq q \leq n-1$. Then for each $f \in Z^0_{0,q}(X \backslash \Omega, E)$ with compact support, there exists $u \in C^{\alpha}_{0,q-1}(X \backslash \Omega, E)$ with compact support such that $\overline{\partial} u = f$ on $X \backslash \overline{\Omega}$.*

PROOF. The proof is analogous to the proof of Theorem 2.1 in [7]. Instead of Lemma 2.2 of [7] we have to use the following

LEMMA 4.5. *Let E, X, Ω, q, α be as in Theorem 4.4 and ϱ some function which realizes the q-convex extension. Then, for each point $\xi \in \partial \Omega$, there exists a neighbourhood Θ of ξ such that the following holds: For each open set $\Omega_0 \subseteq X$ with $\Omega \subseteq \Omega_0$ such that the defining function ϱ_0 of Ω_0 is of class $C^{2+\alpha}$ and sufficiently close to the function ϱ with respect to the C^2-topology and, for each $f \in Z^0_{0,q}(X \backslash \Omega, E)$ with compact support, there exists a form $u \in C^{\alpha}_{0,q-1}(\Theta \cap (X \backslash \Omega_0), E)$ such that $\overline{\partial} u = f$ on $\Theta \cap (X \backslash \overline{\Omega}_0)$.*

PROOF. By [5] Lemma 7.3, there exist holomorphic coordinates $h : W \to \mathbb{C}^n$ in a neighbourhood W of ξ such that $\varrho \circ h^{-1}$ is normalized $(q+1)$-convex on $h(W)$. Without loss of generality, we can assume that $h(W) = U$ is a ball. We fix some neighbourhoods $V_1 \subset\subset V_2 \subset\subset V_3 \subset\subset V_4 \subset\subset U$. Let $\Omega_0 \subseteq X$ be a domain defined by a function ϱ_0 sufficiently close to ϱ in the C^2-topology. Then X is a q-convex extension of $\overline{\Omega}_0$ too, and $\varrho_0 \circ h^{-1}$ is normalized $(q+1)$-convex on U.

Now we can find a function ϱ_0' which fulfils the following conditions:

i) ϱ_0' is sufficiently close to ϱ_0 such that $\varrho_0' \circ h^{-1}$ is normalized $(q+1)$-convex;

ii) $\varrho_0 = \varrho_0'$ in some neighbourhood of $U \backslash (V_4 \backslash V_1)$;

iii) $\varrho_0' < \varrho_0$ on $\overline{V}_3 \backslash V_2$.

Then there exists a neighbourhood Ω_0' of $\overline{\Omega}_0$ so small that $\varrho_0' < 0$ on $h(W \cap \overline{\Omega}_0') \cap (\overline{V}_3 \backslash V_2)$ and therefore

$$h(W \cap \overline{\Omega}_0') \cap (\overline{V}_3 \backslash V_2) \cap \{\varrho_0' > 0\} = \emptyset. \tag{4.3}$$

Let $f \in Z^0_{0,q}(X \backslash \Omega_0, E)$, with compact support, be given. Since the manifold X is a q-convex extension of Ω_0, by Theorem 16.1 in [5] and the regularity of $\overline{\partial}$, we can find a form

$$v \in \bigcap_{0<\alpha<1} C^\alpha(X \backslash \Omega_0, E)$$

with compact support such that $f = \overline{\partial} v$ on $X \backslash \overline{\Omega}_0$.

We choose a C^∞ function χ on X, with $\chi = 0$ in a neighbourhood of $\overline{\Omega}_0$ and $\chi = 1$ in a neighbourhood of $X \backslash \overline{\Omega}_0'$, and set $f' = f - \overline{\partial}(\chi v)$ on $X \backslash \Omega_0$. Then $\operatorname{supp} f' \subset\subset \Omega_0' \backslash \Omega$ and, in view of (4.3), by

$$\varphi := \begin{cases} (h^{-1})^* f & \text{on } D_0' \cap V_3 \\ 0 & \text{on } D_0' \backslash V_2, \end{cases}$$

a form $\varphi \in Z^0_{0,q}(\overline{D}_0', (h^{-1})^* E)$ is correctly defined. Since $(h^{-1})^* E$ is trivial over U (U is a ball), φ can be viewed as a vector of forms from $Z^0_{0,q}(\overline{D}_0')$ which fullfils the condition of Lemma 4.2. Therefore there exists $w \in C^\alpha_{0,q-1}(\overline{D}_0')$ such that $\varphi = \overline{\partial} w$. Let us set $\Theta = h^{-1}(V_1)$ and $u = \chi v + h^* w$ on $\Theta \cap (X \backslash \Omega_0)$. Then $u - w \in C^\alpha_{0,q-1}(\Theta \cap (X \backslash \Omega_0), E)$ and $\overline{\partial} u = f$ on $\Theta \cap (X \backslash \Omega_0)$. ■

Theorem 4.4 implies the following two versions of the Andreotti-Vesentini separation theorem.

THEOREM 4.6. *Let E be a holomorphic vector bundle over the n-dimensional complex manifold X and let $\Omega \subset\subset X$ be a relatively compact open set. Suppose, for some q, with $1 \leq q \leq n-1$, the following two conditions are fulfilled*:

i) *There exist a neighbourhood of $\partial\Omega$ and a strictly $(q+1)$-convex function ϱ of class $C^{2+\alpha}$ such that $\Omega \cap U = \{\varrho < 0\}$,*

ii) *X is $(n-q)$-convex.*

Then the space $Z^0_{0,q}(X \backslash \Omega, E) \cap \overline{\partial} C^\alpha_{0,q+1}(X \backslash \Omega, E)$ is closed in $C^0_{0,q}(X \backslash \Omega, E)$ with respect to the uniform convergence on compact subsets of $X \backslash \Omega$.

PROOF. The proof is the same as the proof of Theorem 2.4 in [7], with one exception. Instead of Theorem 2.1 of [7], we have to use Theorem 4.3 of the present paper. ■

THEOREM 4.7. *Let n, q be integers, with $1 \leq q \leq n-1$, let E be a holomorphic vector bundle over the n-dimensional $(n-q)$-convex complex manifold X and let $\Omega \subset\subset \Omega' \subset\subset X$ be two open sets, where Ω is defined* *by some $(q+1)$-convex function $\varrho : U \to \mathbb{R}$ of class $C^{2+\alpha}$, (here U is some neighbourhood of $\partial\Omega$ and $0 \leq \alpha \leq 1/2$), Ω is strictly q-convex and Ω' is strictly $(n-q)$-convex. Set $D = \Omega' \setminus \overline{\Omega}$.*

Then the space

$$Z^0_{0,q}(\overline{D}, E) \cap \overline{\partial} C^\alpha_{0,q-1}(\overline{D}, E)$$

is closed in $C^0_{0,q}(\overline{D}, E)$ with respect to the uniform convergence.

PROOF. The proof is the same as the proof of Corollary 2.5 of [7], with the following exceptions: instead of Theorem 2.4 of [7], we have to use Theorem 4.5 of the present paper, and instead of the local solutions with C^k estimates from Lieb and Range, we have to use the local solutions with Hölder estimates given in [10]. ∎

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