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The $\oplus_c$-topology on abelian p-groups

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The $\oplus_c$-Topology on Abelian p-Groups (*).

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Introduction.

In this paper we investigate the topology of an abelian p-group G which admits as a base of neighborhoods of 0 all the subgroups X of G such that G/X is a direct sum of cyclic groups. We call this topology the *$\oplus_c$-topology* of G. If G with the $\oplus_c$-topology is a complete Hausdorff topological group, then G is said to be *$\oplus_c$-complete.* The Hausdorff completion of G with respect to the $\oplus_c$-topology is called the *$\oplus_c$-completion* of G and is denoted by $\check{G}$.

In section 1 we prove that the $\oplus_c$-completion $\check{G}$ of a p-group G is a $\oplus_c$-complete group; moreover the completion topology of $\check{G}$ and its own $\oplus_c$-topology are the same. The group $\check{G}$ coincides with the completion of G with respect to the inductive topology if and only if G is thick.

In section 2 we study the class of $\oplus_c$-complete groups. This class of separable p-groups is very large, containing the groups which are direct sums of torsion-complete p-groups, as well as the groups which are the torsion part of direct products of direct sums of cyclic p-groups. But the most interesting result in this direction perhaps is that every separable p^σ-projective p-group is $\oplus_c$-complete. There are a lot of these groups: in fact Nunke proved in [12] that, for every ordinal σ, there exists a p^σ-projective p-group which fails to be p^τ-projective for every $\tau < \sigma$. Moreover the class of $\oplus_c$-complete groups has many closure properties typical of both the classes of p^ω-projective and p^ω-injective p-groups.

In section 3 we study the $\oplus_c$-completion with respect to basic subgroups and we prove the inadequacy of the socle in determining the $\oplus_c$-complete groups; finally we give some applications in connection with the class of thick groups.

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1. – The $\oplus_c$-completion.

All groups considered in the following are abelian groups. Notations and terminology are those of [4]. In particular p is a prime number and the symbol $\oplus_c$ denotes a direct sum of cyclic p-groups. If G is any group and G' is a pure subgroup of G, then we write $G' \leqslant_* G$. A p-group G may be equipped with various topologies. The p-adic topology has the subgroups $p^n G$ with $n \in \mathbb{N}$ as a base of neighborhoods of 0; the inductive topology has the family of large subgroups as a base of neighborhoods of 0. Throughout the paper, for every p-group G, the group $\bar{G}$ stands for the completion of G with respect to the inductive topology. If λ is a limit ordinal, then the generalization of the p-adic topology is the λ-adic topology. This topology, studied by Mines in [11], has the subgroups $p^\sigma G$ with $\sigma < \lambda$ as a base of neighborhoods of 0. In [13] Salce has studied the λ-inductive topology introduced by Charles in [3]; a base of neighborhoods of 0 for this topology consists of all subgroups $G(\boldsymbol{u})$ where $G(\boldsymbol{u}) = \{x \in G: h(p^n x) \geqslant \sigma_n,\ n \in \mathbb{N}\}$ and $\boldsymbol{u} = (\sigma_n)_{n\in\mathbb{N}}$ is an increasing sequence of ordinals $\sigma_n < \lambda$ for all $n \in \mathbb{N}$. In the following, unless otherwise indicated, every p-group G is endowed with the $\oplus_c$-topology. If we are dealing with some other topology, then the group G equipped with its $\oplus_c$-topology is denoted by $(G, \oplus_c)$.

Let G be a p-group and let L be a large subgroup of G. Since $G/L = \oplus_c$ ([4] Proposition 67.4), L is open with respect to the $\oplus_c$-topology of G and so the $\oplus_c$-topology is finer than the inductive topology. The next statement immediately follows from this result and the fact that a p-group G is thick if and only if $G/X = \oplus_c$ implies $L \leqslant X$ for some large subgroup L of G.

PROPOSITION 1.1. *Let G be a p-group. Then G is thick if and only if the $\oplus_c$-topology coincides with the inductive topology and a thick group G is $\oplus_c$-complete if and only if it is torsion-complete.*

Since quasi-complete groups are thick ([4] Theorem 74.1, Corollary 74.6; [1] Theorem 3.2), the quasi-complete and non torsion-complete group constructed by Hill and Megibben in ([7] Theorem 7) is an example of a group which is not $\oplus_c$-complete. Let us note the following facts.

1) A p-group G is discrete in the $\oplus_c$-topology if and only if $G = \oplus_c$ and G is Hausdorff if and only if $p^\omega G = 0$.

2) Every homomorphism $f: G \to H$ with G and H p-groups is continuous with respect to the $\oplus_c$-topologies. In fact if $H/X = \oplus_c$, the same holds for $G/f^{1-}(X)$.

3) For every p-group G the $\oplus_c$-topology of $G/p^\omega G$ coincides with the quotient topology of the $\oplus_c$-topology of G. By property 2, it is enough to observe that the natural homomorphism $G \to G/p^\omega G$ is open.

Therefore in the study of the $\oplus_c$-completion it is not restrictive to confine ourselves to separable non thick groups. In order to show that the $\oplus_c$-completion of a p-group is $\oplus_c$-complete, we need two lemmas.

LEMMA 1.2. *Let G be a p-group. Then the $\oplus_c$-completion $\check{G}$ of G is a p-group.*

PROOF. By definition $\check{G} = \varprojlim G/X$ where X ranges over the subgroups X of G such that $G/X = \oplus_c$. Let $\hat{G}$ denote the p-adic completion of G. Since $\hat{G} = \varprojlim G/p^n G$ where $n \in \mathbb{N}$, there is a canonical homomorphism $\varphi: \check{G} \to \hat{G}$ such that $\varphi((g_X + X)_X) = (g_{p^n} + p^n G)_n$ for all $(g_X + X)_X \in \check{G}$. Since the completion of G in the inductive topology is the group $\bar{G} = \varprojlim G/L$ with L running over the large subgroups of G, there exists a natural homomorphism $\psi: \check{G} \to \bar{G}$ that takes $(g_X + X)_X$ to $(g_L + L)_L$ for all $(g_X + X)_X \in \check{G}$. To show that $\check{G}$ is a p-group, it suffices to check that ψ is an embedding, and this clearly holds if φ is injective. We shall now prove that if $(g_X + X)_X \in \operatorname{Ker} \varphi$, then $g_X \in X$ for all X. To see this, fix X. Let $m \in \mathbb{N}$; if $Y = X \cap p^m G$, then $G/Y = \oplus_c$. By hypothesis $g_{p^m G} \in p^m G$ and, by the choice of Y, $g_Y + p^m G = g_{p^m G} + p^m G$; consequently $g_Y \in p^m G$. On the other hand $g_X + X = g_Y + X$ and so the height of $g_X + X$ in G/X is at least m. Since m is any natural number and $G/X = \oplus_c$, we conclude that $g_X \in X$, as claimed. This completes the proof that $\check{G}$ is a p-group. $\square$

From now on we shall identify $\check{G}$ with the subgroup $\psi(\check{G})$ of $\bar{G}$ and, if G is separable, then we shall view G as a subgroup of $\check{G}$.

LEMMA 1.3. *Direct summands of $\oplus_c$-complete groups are $\oplus_c$-complete.*

PROOF. Let G' be a direct summand of a $\oplus_c$-complete group G. Since the inclusion $G' \to G$ is continuous, every Cauchy net in G' is a Cauchy net in G. Therefore the hypothesis that G is $\oplus_c$-complete and the continuity of the projection of G onto G' assure that G' is $\oplus_c$-complete. $\square$

We are now ready to establish the main result of this section.

THEOREM 1.4. *Let G be a p-group. Then the $\oplus_c$-completion γ of G is $\oplus_c$-complete.*

PROOF. Without loss of generality we may assume that G is separable. For every ordinal λ we define a group G_λ as follows: if $\lambda = 0$, then $G_\lambda = G$; if $\lambda > 0$ and λ is not a limit ordinal, then G_λ is the $\oplus_c$-completion of $G_{\lambda-1}$; if λ is a limit ordinal, then $G_\lambda \bigcup\limits_{\sigma<\lambda} G_\sigma$. To prove the theorem, we shall use three facts:

(i) The $\oplus_c$-topology of $\check{G}$ is finer than the completion topology.

Let $\mathfrak{B}$ be the family of all subgroups X of G such that $G/X = \oplus_c$. Then $\check{G} = \varprojlim\limits_{X\in\mathfrak{B}} G/X$ and $\check{G}$ with the completion topology is a topological subgroup of the group $\prod\limits_{X\in\mathfrak{B}} G/X$ equipped with the product topology of the discrete topologies on every G/X. Thus a base of neighborhoods of 0 for the completion topology of $\check{G}$ consists of all subgroups $U_F = \check{G} \cap \prod\limits_{X\in\mathfrak{B}\setminus F} G/X$ where F is a finite subset of $\mathfrak{B}$. Since

$$\check{G}/U_F \cong \check{G} + \prod_{X\in\mathfrak{B}\setminus F} G/X \Big/ \prod_{X\in\mathfrak{B}\setminus F} G/X \leqslant \prod_{X\in\mathfrak{B}} G/X \Big/ \prod_{X\in\mathfrak{B}\setminus F} G/X = \oplus_c\,,$$

every U_F is a neighborhood of 0 for the $\oplus_c$-topology of $\check{G}$, and so (i) is proved.

(ii) G_λ is a subgroup of $\bar{G}$ for all λ.

We shall prove by transfinite induction that $G_\lambda \underset{*}{\leqslant} \bar{G}$ for all λ. If $\lambda = 0$ the assertion is obvious. Let $\lambda > 0$ and assume $G_\sigma \underset{*}{\leqslant} \bar{G}$ for every $\sigma < \lambda$. If λ is a limit ordinal, then evidently $G_\lambda \underset{*}{\leqslant} \bar{G}$. If λ is not a limit ordinal and $\lambda = \sigma + 1$, then the hypothesis that $G \leqslant G_\sigma \underset{*}{\leqslant} \bar{G}$ implies that $G_\sigma \leqslant G_\lambda \leqslant \bar{G}_\sigma = \bar{G}$. Since $G_\sigma \underset{*}{\leqslant} G_\lambda$, we get $\bar{G}_\sigma = \bar{G} \leqslant \bar{G}_\lambda$ and therefore $G_\lambda \underset{*}{\leqslant} \bar{G}$, as required.

(iii) G_1 is a direct summand of G_λ for all $\lambda \geqslant 1$.

Assume by transfinite induction that G_1 is a summand of G_σ for all $1 \leqslant \sigma < \lambda$. Write $G_\sigma = G_1 \oplus G'_\sigma$ for all $1 \leqslant \sigma < \lambda$. If λ is a limit ordinal, G_1 is a direct summand of G_λ, because $G_\lambda = \bigcup\limits_{\sigma<\lambda} G_\sigma = G_1 \oplus \Big(\bigcup\limits_{1\leqslant\sigma<\lambda} G'_\sigma\Big)$. If λ is not a limit ordinal and $\lambda = \sigma + 1$ then, by the induction hypothesis, $G_\sigma = G_1 \oplus G'_\sigma$. Let $\pi\colon (G_\sigma, \oplus_c) \to (G_1, \mathcal{T})$ be the canonical projection where $(G_1, \mathcal{T})$ is the $\oplus_c$-completion of G. To check that π is continuous, let U be an open subgroup of $(G_1, \mathcal{T})$. Then, by property (i), there is some $W \leqslant U$ such that $G_1/W = \oplus_c$. Since $G_\sigma/\pi^{-1}(W) = G_1 \oplus G'_\sigma/W \oplus G'_\sigma \cong G_1/W = \oplus_c$, we see that π is continuous. This result guarantees the existence of a homomorphism $\bar{\pi}$ making the following diagram commute

$$\begin{array}{ccc} (G_\sigma, \oplus_c) & \xrightarrow{\ \pi\ } & (G_1, \mathcal{T}) \\ \downarrow & & \downarrow \\ (G_\lambda, \mathcal{T}) & \xrightarrow{\ \bar{\pi}\ } & (G_1, \mathcal{T}) \end{array}$$

where the vertical maps are the natural ones and $(G_\lambda, \mathcal{T})$ is the $\oplus_c$-completion of $(G_\sigma, \oplus_c)$. Consequently $G_\lambda = G_1 \oplus \operatorname{Ker} \bar{\pi}$ and so G_1 is a direct summand of G_λ, as claimed.

We can now show that $\check{G} = G_1$ is $\oplus_c$-complete. Suppose this were not true. Then, from Lemma 1.3 and property (iii), we deduce that G_λ is not $\oplus_c$-complete for any λ, and therefore the groups G_λ are all distinct. But this is clearly impossible, because, by property (ii), they are all subgroups of $\bar{G}$. This contradiction establishes that $\check{G}$ is $\oplus_c$-complete and the theorem is proved. □

The next proposition describes the topological structure of the $\oplus_c$-completions.

PROPOSITION 1.5. *For every p-group G the $\oplus_c$-topology of $\check{G}$ coincides with the completion topology.*

PROOF. It is not restrictive to assume $p^\omega G = 0$. As before $\mathcal{T}$ denotes the completion topology of $\check{G}$. By property (i) of Theorem 1.4 we know that the $\oplus_c$-topology of $\check{G}$ is finer than $\mathcal{T}$. On the other hand, by a well known result of general topology ([2] Chapter III § 3, No. 4 Proposition 7), a base of neighborhoods of 0 for the completion topology $\mathcal{T}$ is formed by the closures in $\check{G}$ with respect to $\mathcal{T}$ of the neighborhoods of 0 for the $\oplus_c$-topology of G. Therefore, to end the proof, it is enough to show that if U is an open subgroup of $(\check{G}, \oplus_c)$ and $U' = U \cap G$, then the closure V of U' in $(\check{G}, \mathcal{T})$ is a subgroup of U. To prove this, let $\{g_i\}$ be a Cauchy net in $(G, \oplus_c)$ with $g_i \in U'$ for all i. Since the natural embedding $G \to \check{G}$ is continuous with respect to the $\oplus_c$-topologies, $\{g_i\}$ is a Cauchy net in $(\check{G}, \oplus_c)$. Thus, by Theorem 1.4, it converges to some x in $(\check{G}, \oplus_c)$ and clearly $x \in U$, because U is closed in $(\check{G}, \oplus_c)$ and $g_i \in U$ for all i. Since $\mathcal{T}$ is smaller than the $\oplus_c$-topology of $\check{G}$, the given net converges to x in $(\check{G}, \mathcal{T})$; so $x \in V$, by the definition of V. This means that $V \leqslant U$ and therefore the $\oplus_c$-topology of $\check{G}$ coincides with the completion topology, as claimed. □

COROLLARY 1.6. *Let G be a separable p-group. Then G is a pure topological subgroup with divisible cokernel of a $\oplus_c$-complete group.*

PROOF. By Theorem 1.4 and Proposition 1.5, G is a pure dense topological subgroup of the $\oplus_c$-complete group $\check{G}$. Consequently G is a dense subgroup of $\check{G}$ equipped with the p-adic topology. Hence $\check{G}/G$ is divisible and the proof is complete. □

Before comparing the $\oplus_c$-completion and the completion with respect to the inductive topology, we prove the following lemma.

LEMMA 1.7. *Let G be a separable p-group and let $G \leqslant X \leqslant \bar{G}$. Then $\check{G} \leqslant \check{X}$ and $\check{X} \leqslant \bar{G}$.*

PROOF. Since $G \underset{*}{\leqslant} X$, we may assume $\bar{G} \leqslant \bar{X}$. To show that $\check{G} \leqslant \check{X}$, select $\bar{g} \in \check{G}$. Then, by Proposition 1.5, there exists a net $\{g_i\}$ with $g_i \in G$ for all i which converges to $\bar{g}$ in $(\check{G}, \oplus_c)$. Since $\{g_i\}$ is also a Cauchy net in $(X, \oplus_c)$ and all the canonical maps $\check{G} \to \bar{G}$, $\bar{G} \to \bar{X}$, $\check{X} \to \bar{X}$ are continuous with respect to the $\oplus_c$-topologies, $\bar{g}$ is the limit of $\{g_i\}$ in $(\check{X}, \oplus_c)$ and so $\bar{g} \in \check{X}$. This proves the inclusion $\check{G} \leqslant \check{X}$. To see that $\check{X} \leqslant \bar{G}$, take $\bar{x} \in \check{X}$. As before, there is a net $\{x_i\}$ with $x_i \in X$ for all i which converges to $\bar{x}$ in $(\check{X}, \oplus_c)$. Since $\{x_i\}$ is a Cauchy net in $(\bar{G}, \oplus_c)$ and all the natural embeddings $\check{X} \to \bar{X}$, $\bar{G} \to \bar{X}$ are continuous with respect to the $\oplus_c$-topologies, $\bar{x}$ is the limit of $\{x_i\}$ in $(\bar{G}, \oplus_c)$ and so $\bar{x} \in \bar{G}$. Consequently $\check{X} \leqslant \bar{G}$ and the lemma is proved. □

PROPOSITION 1.8. *Let G be a separable p-group. The following facts hold:*

(*i*) *If G is not thick, then the group $\bar{G}/\check{G}$ has uncountable rank.*

(ii) *If G is not $\oplus_c$-complete, then the group $\check{G}/G$ may have finite rank.*

PROOF (i). We first show that $\check{G} \neq \bar{G}$. Since $\bar{G}$ is thick, it has the same inductive and $\oplus_c$-topologies. Moreover, by ([13] Theorem 2.3), the inductive topology of $\bar{G}$ induces on G its own inductive topology. On the other hand, by Proposition 1.5, the $\oplus_c$-topology of $\check{G}$ induces on G its own $\oplus_c$-topology. Therefore, if G is not thick, then $\check{G}$ must be a proper subgroup of $\bar{G}$. We now prove that $\bar{G}/\check{G}$ is uncountable. Suppose this were not true. Since $\check{G}$ is a pure subgroup of $\bar{G}$ with countable divisible cokernel, we deduce from ([10] Theorem 3.5) that $\check{G}$ is thick, and this is impossible. In fact $\check{G}$ is $\oplus_c$-complete, but it is not torsion-complete. This contradiction shows that $\bar{G}/\check{G}$ is uncountable.

(ii) Assume the rank of $\check{G}/G$ is not finite. Choose a pure subgroup H of $\check{G}$ such that $G \leqslant H$ and $\check{G}/H \cong \mathbb{Z}(p^\infty)$. Then Lemma 1.7 tells us that $\check{H} = \check{G}$. Since the rank of $\check{H}/H$ is 1, the proof is complete. □

2. – $\oplus_c$-complete groups.

In this paragraph we study the $\oplus_c$-complete groups. As the results of section 1 suggest, the class of $\oplus_c$-complete groups is very large.

First we prove a statement that we shall often use.

PROPOSITION 2.1. *Direct sums of $\oplus_c$-complete groups are $\oplus_c$-complete.*

PROOF. Let $G = \oplus G_i$ where G_i is $\oplus_c$-complete for all i. To show that G is $\oplus_c$-complete, we notice the following properties:

(i) The groups $X = \bigoplus_{i \in I} X_i$ where $X_i \leqslant G_i$ and $G_i/X_i = \oplus_c$ for every i are a base of neighborhoods of 0 for the $\oplus_c$-topology of G.

This assertion is obvious.

(ii) G is a closed topological subgroup of the group $\prod_{i \in I} G_i$ equipped with the box topology of the $\oplus_c$-topology on each component.

We recall that the box topology considered on $\prod_{i \in I} G_i$ admits the subgroups of the form $\prod_{i \in I} X_i$ with $X_i \leqslant G_i$ and $G_i/X_i = \oplus_c$ for all i as a base of neighborhoods of 0. Thus the conclusion that G is a topological subgroup of $\prod_{i \in I} G_i$ follows from (i). To complete the proof, let $\bar{g} = (g_i)_{i \in I}$ with $g_i \in G_i$ for every i be an element of the closure of G in $\prod_{i \in I} G_i$. Let S be the support of $\bar{g}$, that is let $S = \{i \in I : g_i \neq 0\}$. Then for each $i \in S$ we can choose a subgroup X_i of G_i such that $g_i \notin X_i$ and $G_i/X_i = \oplus_c$. Our assumption on $\bar{g}$ assures that $\bar{g} \in G + \left(\prod_{i \in S} X_i + \prod_{i \in I \setminus S} G_i\right)$; consequently S is finite and so $\bar{g} \in G$. This proves that G is a closed subgroup of $\prod_{i \in I} G_i$, as required.

The hypothesis that every G_i is $\oplus_c$-complete implies that $\prod_{i \in I} G_i$ with the box topology is complete ([4] Proposition 13.3). Hence, by property (ii), G is $\oplus_c$-complete. □

COROLLARY 2.2. *Direct sums of torsion-complete p-groups are $\oplus_c$-complete.*

PROOF. Since torsion-complete p-groups are $\oplus_c$-complete, the corollary follows from Proposition 2.1. □

We shall obtain another large class of $\oplus_c$-complete groups by means of the next lemmas.

LEMMA 2.3. *Let G be a separable p-group and let G' be a subgroup of G with bounded cokernel. Then G is $\oplus_c$-complete if and only if G' is $\oplus_c$-complete.*

PROOF. We first show that G' is a topological subgroup of G. Let X be a subgroup of G' such that $G'/X = \oplus_c$. Since $(G/X)/(G'/X) \cong G/G'$ is bounded and $G'/X = \oplus_c$, we have $G/X = \oplus_c$. This proves that the restriction to G' of the $\oplus_c$-topology of G is finer than the $\oplus_c$-topology of G'. Therefore the two topologies coincide, because the natural injection $G' \to G$

is continuous. Assume now that G is $\oplus_c$-complete. Since G' is a closed topological subgroup of G, we conclude that G' is $\oplus_c$-complete. Conversely, suppose G' is $\oplus_c$-complete. Since G' is an open complete topological subgroup of G, evidently G is $\oplus_c$-complete and the proof is finished. □

LEMMA 2.4. *Let G be a separable p-group and let P be a bounded subgroup of G with separable cokernel. Then G is $\oplus_c$-complete if and only if G/P is $\oplus_c$-complete.*

PROOF. Assume first G/P is $\oplus_c$-complete and choose $n \in \mathbb{N}$ such that $p^n P = 0$. Let us verify that $\check{G} \leqslant G + \bar{G}[p^n]$. Take $\bar{g} \in \check{G}$; then there is a net $\{g_i\}$ in G which converges to $\bar{g}$ in $\check{G}$. The hypothesis that G/P is $\oplus_c$-complete guarantees that $\{g_i + P\}$ has a limit $g + P \in G/P$. Since the canonical homomorphisms $G/P \to p^n G$ and $p^n G \to G$ are continuous, $\{p^n g_i\}$ converges to $p^n g$ in G and obviously $p^n \bar{g} = p^n g$. Thus $\bar{g} \in G + \bar{G}\ [p^n]$ and therefore $\check{G} \leqslant G + \bar{G}[p^n]$. By Theorem 1.4 and Lemma 2.3, this implies that G is $\oplus_c$-complete. Conversely, suppose G is $\oplus_c$-complete; then Lemma 2.3 says that $p^n G$ is $\oplus_c$-complete. Since $(G/P)/(G[p^n]/P) \simeq p^n G$, the first part of the proof assures that G/P is $\oplus$-complete and the lemma follows. □

Observe that the class of $\oplus_c$-complete groups is a full p^ω-class in the sense of [6]. Indeed, by Proposition 2.1 and Lemma 2.3, the class of $\oplus_c$-complete groups is a p^ω-class. Moreover, if G is separable and G/P is $\oplus_c$-complete for some $P \leqslant G[p]$, then, by Lemma 2.4, G is $\oplus_c$-complete.
We can now prove the following

THEOREM 2.5. *Let σ be any ordinal. If G is a p^σ-projective separable p-group, then G is $\oplus_c$-complete.*

PROOF. The proof is by induction on σ. If $\sigma \leqslant \omega$ the assertion is obvious, because $G = \oplus_c$. Let $\sigma > \omega$ and assume the assertion is true for all $\lambda < \sigma$. By ([4] § 82 Ex. 13), G is a summand of the group $\operatorname{Tor}(H_\sigma, G)$, where H_σ is the generalized Prüfer group of length σ. To see that G is $\oplus_c$-complete, we first suppose σ is a limit ordinal. Then, by ([4] § 82 Ex. 2 and 8; Lemma 64.1) and by the induction hypothesis, G is a summand of a direct sum of $\oplus_c$-complete groups. Hence the conclusion that G is $\oplus_c$-complete follows from Lemma 1.3 and Proposition 2.1. Assume now σ is not a limit ordinal. From the exact sequence

$$0 \to p^{\sigma-1} H_\sigma \simeq \mathbb{Z}(p) \to H_\sigma \to H_\sigma/p^{\sigma-1} H \simeq H_{\sigma-1} \to 0\ ,$$

one obtains the long exact sequence

$$0 \to \operatorname{Tor}\big(\mathbb{Z}(p), G\big) \cong G[p] \to \operatorname{Tor}(H_\sigma, G) \xrightarrow{\varphi} \operatorname{Tor}(H_{\sigma-1}, G) \xrightarrow{\psi}$$
$$\to \mathbb{Z}(p) \otimes G \cong G/pG \to H_\sigma \otimes G \to H_{\sigma-1} \otimes G \to 0 .$$

Thus the following sequences are exact:

$$(1) \qquad 0 \to G[p] \to \operatorname{Tor}(H_\sigma, G) \to \operatorname{Im}\varphi \to 0 ,$$

$$(2) \qquad 0 \to \operatorname{Im}\varphi \to \operatorname{Tor}(H_{\sigma-1}, G) \to \operatorname{Im}\psi \to 0 .$$

Evidently in (2) the group $\operatorname{Tor}(H_{\sigma-1}, G)$ is $\oplus_c$-complete, by the induction hypothesis, and $\operatorname{Im}\psi$ is bounded; therefore, by Lemma 2.3, $\operatorname{Im}\varphi$ is $\oplus_c$-complete. From Lemma 2.4 and the exactness of (1), we deduce that $\operatorname{Tor}(H_\sigma, G)$ is $\oplus_c$-complete and, by Lemma 1.3, the same applies to its summand G. □

Proposition 2.1 indicates that the class of $\oplus_c$-complete groups has a closure property analogous to a closure property of the class of direct sums of cyclis groups. This projective property can be regarded as dual of the following injective property, which is similar to a closure property of the class of torsion-complete groups ([4] Corollary 68.6).

PROPOSITION 2.6. *The torsion part of a direct product of $\oplus_c$-complete groups is $\oplus_c$-complete.*

PROOF. Let $G = t\Big(\prod_{i\in I} G_i\Big)$ where G_i is $\oplus_c$-complete for all i. Since $G_i \leqslant_* \bar{G}_i$ for every i, it is easy to check that G is a pure subgroup of the torsion-complete group $T = t\Big(\prod_{i\in I} \bar{G}_i\Big)$. Therefore, by the first part of Lemma 1.7, we may assume $\check{G} \leqslant T$. Let now $t = (t_i)_{i\in I} \in \check{G}$ with $t_i \in \bar{G}_i$ for all i. Then t is the limit of a net $\{g_j\}_{j\in J}$ where $g_j = (g_{ij})_{i\in I} \in G$ and $g_{ij} \in G_i$ for all $i \in I$, $j \in J$. Fix $i \in I$; to end the proof, it is enough to show that $t_i \in G_i$. Since $\check{G} \leqslant T$ and the canonical projection $T \to \bar{G}_i$ is continuous, $\{g_{ij}\}_{j\in J}$ converges to t_i in $\bar{G}_i$. From the hypothesis that G_i is $\oplus_c$-complete and $g_{ij} \in G_i$ for all j, we get $t_i \in G_i$. This completes the proof. □

As the next corollary shows, Proposition 2.6 gives some information about $\oplus_c$-complete groups which is not contained in Corollary 2.2 and Theorem 2.5.

COROLLARY 2.7. *There is a $\oplus_c$-complete group which is not a direct sum of torsion-complete p-groups and p^σ-projective separable p-groups.*

PROOF. Let $G = t\left(\prod_{n\in\mathbb{N}} G_n\right)$ where $G_n = \bigoplus_{k>n} \mathbb{Z}(p^k)$ for all n. By Proposition 2.6, G is $\oplus_c$-complete. We observe now the following facts:

(i) G is not a direct sum of torsion-complete p-groups.

This can be easily proved.

(ii) A proper p^σ-projective separable p-group G' with $\sigma > \omega$ cannot be a direct summand of G.

Assume the contrary. Then $G' = t\left(\prod_{n\in\mathbb{N}} C_n\right)$ where $C_n = \oplus_c$ for all n ([9] Theorem 3). Since $\sigma > \omega$, there is no $k \in \mathbb{N}$ such that $p^k C_n = 0$ for almost all n. Hence, by ([8] Proposition 1.6), G' has an unbounded torsion-complete group T as a summand, but this is impossible. Indeed, by ([12] Proposition 6.7), a p^σ-projective p-group cannot contain an unbounded torsion-complete group. This contradiction shows that (ii) holds.

The corollary is now obvious. □

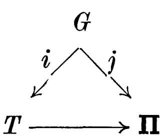
Let us note that the group G defined in the proof of Corollary 2.7 is pure-complete ([9] Theorem 2). Another application of Proposition 2.6 enables us to characterize all the $\oplus_c$-complete groups.

THEOREM 2.8. *Let G be a p-group. The following statements are equivalent:*

(i) *G is $\oplus_c$-complete.*

(ii) *G is a closed topological subgroup of the torsion part of a direct product of a direct sums of cyclic p-groups.*

PROOF (i) ⇒ (ii). By hypothesis $G = \varprojlim_{X\in\mathcal{B}} G/X$ where $\mathcal{B}$ is a base of neighborhoods of 0 for G and $G/X = \oplus_c$ for all $X \in \mathcal{B}$. Let $\mathbf{\Pi} = \prod_{X\in\mathcal{B}} G/X$ and let $T = t(\mathbf{\Pi})$. If G and T are equipped with the $\oplus_c$-topology and $\mathbf{\Pi}$ is regarded as the topological product of the discrete groups G/X, then all the natural inclusions in the commutative diagram



are continuous. Evidently the groups of the form $j^{-1}(U)$ where U ranges over the open subgroups of $\mathbf{\Pi}$ are a base of neighborhoods of 0 for G. Thus the same holds for the groups $i^{-1}(V)$ with V running over the open subgroups of T. Hence G is a topological subgroup of T. Since G is $\oplus_c$-complete, G must be closed in T and (ii) is proved.

(ii) $\Rightarrow$ (i). This immediately follows from Proposition 2.6. $\square$

It is now clear that the class of $\oplus_c$-complete groups is the smallest class of separable p-groups $\mathcal{C}$ with the following properties:

(1) $0 \in \mathcal{C}$ and a group isomorphic to a member of $\mathcal{C}$ belongs to $\mathcal{C}$.

(2) If $S \leqslant G[p]$ and $G/S \in \mathcal{C}$, then $G \in \mathcal{C}$.

(3) $\mathcal{C}$ is closed under direct sums and the torsion part of a direct product of groups of $\mathcal{C}$ belongs to $\mathcal{C}$.

(4) $\mathcal{C}$ contains every group that, endowed with its $\oplus_c$-topology, is a closed topological subgroup of a group determined by the above conditions.

3. – Some applications.

In this last section we discuss some consequences of the preceding results.

The next proposition investigates the connection between $\oplus_c$-complete groups and basic subgroups.

PROPOSITION 3.1. *The following facts hold:*

(i) *If two separable p-groups have isomorphic $\oplus_c$-completions, then they have isomorphic basic subgroups.*

(ii) *There exist $2^{\aleph_0}$ pairwise nonisomorphic $\oplus_c$-complete groups with isomorphic basic subgroups.*

PROOF (i). Let G and H be separable p-groups such that $\check{G} \simeq \check{H}$. Since $\bar{G}$ is isomorphic to $\bar{H}$, we conclude that G and H have isomorphic basic subgroups.

(ii) Let $B = \bigoplus_{n \geqslant 1} \mathbb{Z}(p^n)$. We want to prove that there exist $2^{\aleph_0}$ pairwise nonisomorphic $\oplus_c$-complete subgroups of $\bar{B}$ whose basic subgroup is B. To see this, let I be a set of cardinality $2^{\aleph_0}$ and let $\{X_i\}_{i \in I}$ be a family of subsets of positive integers such that if $i \neq j$ then $(X_i \setminus X_j) \cup (X_j \setminus X_i)$ is not finite. Let $G_i = t\Big(\prod_{n \in X_i} \mathbb{Z}(p^n)\Big) \oplus \Big(\bigoplus_{\substack{n \geqslant 1 \\ n \notin X_i}} \mathbb{Z}(p^n)\Big)$ for all i; then every G_i is a $\oplus_c$-complete group admitting B as a basic subgroup. To complete the proof, it is enough to show that if $|X_i \setminus X_j| = \aleph_0$, then G_i is not isomorphic to G_j. Suppose this were not true. Then, by ([4] Theorem 73.6; Lemma 71.1), there exist an isometry $\varphi\colon G_i[p] \to G_j[p]$, a finite subset $F \subseteq \mathbb{N}\setminus X_j$ and some $k \in \mathbb{N}$ such that $\varphi\Big(p^k\Big(\prod_{n\in X_i}\mathbb{Z}(p^n)\Big)[p]\Big) \leqslant t\Big(\prod_{n\in X_j}\mathbb{Z}(p^n)\Big) \oplus \Big(\bigoplus_{n\in F}\mathbb{Z}(p^n)\Big)$. Consequently there is a finite subset $F' \subseteq X_i$ such that $X_i\setminus F' \subseteq F \cup X_j$, while, by hypothesis, $X_i \setminus X_j$ is not finite. This contradiction proves that G_i is not isomorphic to G_j, as claimed. □

The following statement shows that socles, viewed as valued vector spaces, do not give much information in the study of $\oplus_c$-complete groups.

PROPOSITION 3.2. *The following facts are true*:

(i) *There exists a $\oplus_c$-complete group whose socle is isometric to the socle of a non $\oplus_c$-complete group.*

(ii) *There exist nonisomorphic $\oplus_c$-complete groups with isometric socles.*

PROOF (i). Let G be a separable p-group which is neither $\oplus_c$-complete nor thick (for instance, let G be an infinite direct sum of quasi-complete non torsion-complete p-groups) and let $S = \breve{G}[p]$. Singe $\breve{G} \underset{*}{\leqslant} \bar{G}$, we can choose $x \in \bar{G}[p]\setminus \breve{G}$. Let y be an element of order p^2 of $\breve{G}$ and let $z = x + y$. Take a subgroup A of $\bar{G}$ such that $\langle G, z\rangle \leqslant A$ and $A/G \simeq \mathbb{Z}(p^\infty)$. Since $A \underset{*}{\leqslant} \bar{G}$ and $A[p] \leqslant S$, there exists a pure subgroup H of $\bar{G}$ such that $A \leqslant H$ and $H[p] = S$. We want to prove that H is not $\oplus_c$-complete. Assume the contrary. Since $G \leqslant H \leqslant \bar{G}$ and, by hypothesis, H is $\oplus_c$-complete, Lemma 1.7 implies that $\breve{G}$ is a pure subgroup of H. Using this fact and the equality $H[p] = S = \breve{G}[p]$, one obtains $\breve{G} = H$. This is a contradiction, because $x \notin \breve{G}$, $y \in H$ and $z = x + y \in H$. Hence H is not $\oplus_c$-complete and (i) is proved.

(ii) A result of ([5] Corollary 1) guarantees the existence of two nonisomorphic groups G_1 and G_2 with isometric socles such that G_1 is a direct sum of torsion-complete p-groups and G_2 is a $p^{\omega+1}$-projective p-group. On the other hand, by Corollary 2.2 and Theorem 2.5, G_1 and G_2 are $\oplus_c$-complete. Therefore (ii) holds and the proof is finished. □

Now we point out some relations between $\oplus_c$-complete groups and thick groups. As Proposition 1.8 suggests and as we shall see in the following, these two classes have completely different properties.

PROPOSITION 3.3. *Let G be a separable p-group which is neither $\oplus_c$-complete nor thick. The following facts hold*:

(i) *There exists a thick group K such that $K + \breve{G} = \bar{G}$; $K \cap \breve{G} = G$ and $K \underset{*}{\leqslant} \bar{G}$.*

(ii) *Condition* (i) *does not necessarily determine the group K up to isomorphisms.*

PROOF (i). By Corollary 1.6 we can choose a subgroup K of $\bar{G}$ such that $\breve{G}/G \oplus K/G = \bar{G}/G$. Therefore $K + \breve{G} = \bar{G}$; $K \cap \breve{G} = G$ and $K \underset{*}{\leqslant} \bar{G}$. It remains to check that K is thick. Since $G \leqslant K \leqslant \bar{G}$, Lemma 1.7 implies that $\breve{G} \leqslant \breve{K}$ and clearly $\bar{G} = \bar{K}$. Consequently $\breve{K} = \bar{K}$ and so, by Proposition 1.8, K is thick, as required.

(ii) Let $G = G_1 \oplus G_2$ where $G_1 = \bigoplus_{n \geqslant 1} \mathbb{Z}(p^n)$ and G_2 is a quasi-complete non torsion-complete group whose basic subgroup is G_1 ([4] § 74 Example). Then G is neither $\oplus_c$-complete nor thick, $\breve{G} = G_1 \oplus \bar{G}_2$ and a suitable choice for K is that of $K = \bar{G}_1 \oplus G_2$. We shall prove that there exist $2^{\aleph_0}$ pairwise nonisomorphic subgroups of $\bar{G}$ satisfying condition (i). To see this, let I be a set of cardinality $2^{\aleph_0}$. Take a subgroup H of $\bar{G}$ with $G \leqslant H$ and elements x_n, $y_{in} \in \bar{G}/G$, where $n \in \mathbb{N}$, with the following properties:

(1) $\breve{G}/G = \langle x_n : n \in \mathbb{N} \rangle \oplus H/G$ where $\langle x_n : n \in \mathbb{N} \rangle \cong \mathbb{Z}(p^\infty)$, $px_0 = 0$ and $px_{n+1} = x_n$ for all $n \in \mathbb{N}$.

(2) $K/G = \bigoplus_{i \in I} \langle y_{in} : n \in \mathbb{N} \rangle$ where $\langle y_{in} : n \in \mathbb{N} \rangle \cong \mathbb{Z}(p^\infty)$, $py_{i0} = 0$ and $py_{in+1} = y_{in}$ for all $n \in \mathbb{N}$; $i \in I$.

For every $J \subseteq I$, let K_J be the subgroup of $\bar{G}$ such that $G \leqslant K_J$ and

$$K_J/G = \bigoplus_{i \in J} \langle y_{in} + x_n : n \in \mathbb{N} \rangle \oplus \bigoplus_{i \in I \setminus J} \langle y_{in} : n \in \mathbb{N} \rangle .$$

We claim that every K_J satisfies condition (i). In fact let J be any subset of I. Since $K_J/G + \breve{G}/G = \langle x_n, y_{in} : n \in \mathbb{N}, i \in I \rangle + H/G = K/G \oplus \breve{G}/G = \bar{G}/G$, evidently $K_J + \breve{G} = \bar{G}$. The definition of K_J guarantees that $K_J \underset{*}{\leqslant} \bar{G}$, because $G \underset{*}{\leqslant} K_J$ and K_J/G is a divisible subgroup of $\bar{G}/G$. To verify that $K_J \cap \breve{G} = G$, select $z \in K_J/G \cap \breve{G}/G$. Then there exist $v \in H/G$; n, $n_i \in \mathbb{N}$ and a, $a_i \in \mathbb{Z}$ where $0 \leqslant a$, $a_i < p$ and $a_i = 0$ for almost all i such that

$$z = \sum_{i \in J} a_i(y_{in_i} + x_{n_i}) + \sum_{i \in I \setminus J} a_i y_{n_i} = ax_n + v .$$

Since $K/G \cap \breve{G}/G = 0$, we get $a_i = 0$ for all i; hence $z = 0$. Therefore $K_J/G \cap \breve{G}/G = 0$ and so $K_J \cap \breve{G} = G$. Consequently every K_J satisfies condition (i) and it is easy to show that the groups K_J are all distinct. To end the proof, we apply an argument similar to that used in the last part of ([4] Theorem 66.4). Let B be a basic subgroup of G; then B is countable.

Since $|\mathrm{Hom}\,(K, \bar{G})| \leqslant |\mathrm{Hom}\,(B, \bar{G})| = 2^{\aleph_0}$, the subgroups of $\bar{G}$ isomorphic to K are at most $2^{\aleph_0}$. This means that $2^{2^{\aleph_0}}$ groups of the form K_J are pairwise nonisomorphic and so (ii) holds. □

From Propositions 1.8 and 3.3 we deduce that, if G is neither $\oplus_c$-complete nor thick, then there exist a lot of thick groups between G and $\bar{G}$. The following result indicates that, under the same hypotheses on G, there exist also a lot of $\oplus_c$-complete groups between G and $\bar{G}$.

PROPOSITION 3.4. *Let G be a separable group which is neither $\oplus_c$-complete nor thick and let K be as in Proposition 3.3. The following are true:*

(i) *There exists an increasing sequence of $\oplus_c$-complete non thick groups $\{X_n\}$ with $\check{G} \leqslant X_n$ for all n such that $\bar{G} = \bigcup_{n\in\mathbb{N}} X_n$.*

(ii) *There exists an increasing sequence of non $\oplus_c$-complete non thick groups $\{Y_n\}$ with $G \leqslant Y_n$ for all n such that $K = \bigcup_{n\in\mathbb{N}} Y_n$.*

PROOF (i). Let $X_0 = \check{G}$ and let $X_n = \check{G} + \bar{G}[p^n]$ for all $n \geqslant 1$. Then, by Lemma 2.3, every X_n is $\oplus$-complete and the other properties clearly hold.

(ii) Under the hypotheses of (i), let $Y_n = K \cap X_n$ for all n. Then $\{Y_n\}$ is an increasing sequence, $G \leqslant Y_n$ for every n and $K = \bigcup_{n\in\mathbb{N}} Y_n$. To prove (ii), fix $n \in \mathbb{N}$. Since $G \underset{*}{\leqslant} Y_n$ and $\check{G} \nleqslant Y_n$, Lemma 1.7 assures that Y_n is not $\oplus_c$-complete. Moreover, since Y_n/G is bounded and, by hypothesis, G is not thick, it is easily seen that Y_n is not thick. This completes the proof. □

The next statement gives some properties of $\oplus_c$-complete groups and thick groups with respect to intersection and group union.

PROPOSITION 3.5. *Let G be a separable p-group. The following facts hold:*

(i) *Let X be a pure subgroup of G and let $X = \bigcap_i X_i$ where $X_i \leqslant G$ for all i. If every X_i is $\oplus_c$-complete, then X is $\oplus_c$-complete; if every X_i is thick, then X is not necessarily thick.*

(ii) *Group unions of $\oplus_c$-complete or thick subgroups of G are not necessarily $\oplus_c$-complete or thick.*

PROOF (i). Suppose X_i is $\oplus_c$-complete for every i. Then, by the first part of Lemma 1.7, $\check{X} \leqslant X_i$ for all i; hence X is $\oplus_c$-complete, as claimed. Let now X be a $\oplus_c$-complete group which is not thick and let $G = \bar{X}$. Evidently X is the intersection of all $X_i \underset{*}{\leqslant} G$ such that $X \leqslant X_i$ and $G/X_i \cong \mathbb{Z}(p^\infty)$. Since all these groups are thick ([10] Theorem 3.5), (i) is proved.

(ii) First assume G is not $\oplus_c$-complete. Then $G = \bigcup_{n\geq 1} G[p^n]$ and every $G[p^n]$ is $\oplus_c$-complete. Finally let $G = \bigoplus_{n\geq 1} \mathbb{Z}(p^n)$. Since $G[p^n]$ is thick for all $n \geq 1$ and G is not thick, (ii) follows. □

This last proposition shows that in a $\oplus_c$-complete group the cardinality of nondiscrete $\oplus_c$-complete subgroups and that of non $\oplus_c$-complete thick subgroups may be as large as possible.

PROPOSITION 3.6. *There exists a $\oplus_c$-complete group G with the following properties:*

(i) *G has $2^{|G|}$ $\oplus_c$-complete nondiscrete non thick subgroups.*

(ii) *G has $2^{|G|}$ non $\oplus_c$-complete thick subgroups.*

PROOF. We claim that the $\oplus_c$-complete group $G = t\left(\prod_{n\in\mathbb{N}} G_n\right)$ where $G_n = \bigoplus_{k\geq 1} \mathbb{Z}(p^k)$ for all n satisfies the above conditions.

(i) View G as the group of all bounded maps $f\colon \mathbb{N} \to \bigcup_{n\in\mathbb{N}} G_n$ such that $f(n) \in G_n$ for every $n \in \mathbb{N}$. If $f \in G$, let $Z(f) = \{n \in \mathbb{N}: f(n) = 0\}$. For every free ultrafilter φ on $\mathbb{N}$, let G_φ be the $\oplus_c$-completion of the group $\boldsymbol{\Sigma}_\varphi = \{f \in G: Z(f) \in \varphi\}$. Fix φ; since $\boldsymbol{\Sigma}_\varphi \underset{*}{\leqslant} G$, the first part of the proof of Lemma 1.7 guarantees that $G_\varphi \leqslant G$ and clearly $G_\varphi \neq \oplus_c$, because $\boldsymbol{\Sigma}_\varphi \neq \oplus_c$. Also note that every G_n is a summand of G_φ; consequently G_φ is not thick. The next step is to show that if $\varphi \neq \psi$, then $G_\varphi \neq G_\psi$. To this end, pick $F \in \varphi \setminus \psi$. Let g be an element of $G[p]$ with the property that $Z(g) = F$ and $g(n) \in G_n \setminus pG_n$ for every $n \in \mathbb{N} \setminus F$. Obviously $g \in G_\varphi$, but we claim that $g \notin G_\psi$. Suppose this were not true. Then, from the hypothesis that $g \in G_\psi$, we deduce that g belongs to the closure of $\boldsymbol{\Sigma}_\psi$ with respect to the $\oplus_c$-topology of G. Hence $g = g_1 + pg_2$ for some $g_1 \in \boldsymbol{\Sigma}_\psi$ and $g_2 \in G$. Since $Z(g) \notin \psi$ and $Z(g_1) \in \psi$, there exists $n \in Z(g_1) \setminus Z(g)$ and so $0 \neq g(n) = pg_2(n)$, contrary to the choice of g. This contradiction proves that $g \notin G_\psi$; thus the groups G_φ are all distinct. Since $\mathbb{N}$ has $2^{2^{\aleph_0}}$ free ultrafilters and $|G| = 2^{\aleph_0}$, (i) holds.

(ii) This property immediately follows from the proof of (ii) in Proposition 3.3.

Indeed all the groups K_J used in that proof can be embedded in the group $T = t\left(\prod_{n\geq 1} (\mathbb{Z}(p^n) \oplus \mathbb{Z}(p^n))\right)$ and it is easy to see that G has a direct summand isomorphic to T. □

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