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LINEAR GOURSAT PROBLEMS FOR ENTIRE FUNCTIONS WHEN THE COEFFICIENTS ARE VARIABLE

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1. Introduction.

The Cauchy-Kovalevsky theorem for analytic functions, [4], is a central theorem in the theory of partial differential equations. It has a local form. As a local theorem it has been extended to theorems for different function classes when data are given on intersecting hyperplanes. See Lednev [5], Hörmander [3], p. 116, Friedmann [1], Gårding [2], and Persson [6], [8], and [10].

One main feature is new techniques of majorization and approximation. In this respect all authors above have given their contribution. See also Rosenbloom [14]. Another feature is the substitution of analyticity by «continuity» in some time variables. See [5], [8], [11], and also Talenti [15]. In [1] and [8] analyticity in the space variables is substituted by Gevrey differentiability. Systems are treated in [5], [2], and [8] under the hypothesis introduced in [5].

The necessity of the hypothesis in the Cauchy-Kovalevsky theorem is the subject in [7]. See also the second part of theorem 1 in [9].

It may also be noted that the exponential majorization introduced in [8] has been used to give simple proofs and generalizations of classical theorems for the equation

$$D_1 D_2 u = f(x, u, D_1 u, D_2 u), \; u(0, x_2) = u(x_1, 0) = 0.$$

See [10], [12], and [13].

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The subject of the present paper is global Goursat problems. A global version of the Cauchy-Kovalevsky theorem for linear operators with constant coefficients in the class of entire functions is proved in Treves [16]. The majorization used in [16] is that of [14], and [3]. Various refinements are also given in [16]. Global Goursat problems is the subject in [9], and [10]. In [9] the functions are considered as formal power series in the time variables. The theorems in [9] are stated for Gevrey differentiability in all variables with the restriction that the functions are entire analytic functions in the time variables. Theorem 1 in [10] may be seen as a global version of theorem 3 in [8] with « continuity » in some time variable, analyticity in the others and Gevrey differentiability in the space variables. The common feature of the theorems in [9], and [10] mentioned above is that the problems are linear with constant coefficients in the operators.

The question of non-linearity and global Goursat problems is answered by the following simple example. Let $D = d/dx$. The solution u of

$$Du = e^u, \; u(0) = 0$$

is $u = -\log(1 - x)$. So there is no hope for a global existence theorem in the non linear case. The following example given below for linear operators with variable coefficients shows that even in this case some restrictions must be imposed. The analytic solution u of

$$D_1 u = e^{x_1 + x_2} D_2 u, \; u(0, x_2) = e^{x_2}$$

is $u = (1 - e^{x_1} + e^{-x_2})^{-1}$ in a neighbourhood of the origin. It is obvious that u is not an entire function, i. e. analytic in all C^2. If may also be pointed out that $u = e^{e^{x_1} - e^{-x_2}}$ solves

$$D_1 u = e^{x_1 + x_2} D_2 u.$$

Thus there is at least one entire non-zero solution of that equation.

The solution of

$$D_1 u = e^{x_1} u, \; u(0) = 1$$

is $u = e^{e^{x_1} - 1}$ so there is some hope if the variable coefficients do not appear in the principal part.

These facts together with the experience from [6], [8], [9], have been used in the framing of the theorem in section 4. This theorem is the subject of the present paper. Partly by necessity and partly from expediency it only treats entire functions without any conditions of growth. See the example above.

The proof of the theorem is based on a presumably new characterization of the entire functions in C^n. This and some notation are given in section 2. Section 3 contains lemma 1, essentially due to Friedmann [1], but used in the present form in [8], and (11]. In section 3 are also lemma 2 and lemma 3. Lemma 2 is the cornerstone of the proof of the theorem. It is applicable to the proof when a special kind of entire functions is treated Lemma 3 says that this special kind is all entire functions, so lemma 2 is universially applicable.

The proof of the theorem in section 4 is rather short with the use of the results in section 3. The estimates for the coefficients of the solution of the problems are obtained by induction. This is much easier than the method used in [9].

In [9], p. 47, there is an incorrect remark concerning variable coefficients.

It should be mentioned here that the theorem specialized to a non-characteristic linear Cauchy problem says that the analytic solution is entire if the coefficient in the principal part are constants and if other functions involved are entire.

2. Preliminaries.

Let $(x_1, \dots, x_n) \in C^n$. By $\alpha = (\alpha_1, \dots, \alpha_n)$ we denote a multi-index with non-negative integers as components. If $D = (\partial/\partial x_1, \dots, \partial/\partial x_n)$, then we write $D^\alpha = (\partial/\partial x_1)^{\alpha_1} \dots (\partial/\partial x_n)^{\alpha_n}$. We also write $|\alpha| = \alpha_1 + \dots + \alpha_n$, $\alpha! = \alpha_1! \dots \alpha_n!$, $\binom{\alpha}{\beta} = \alpha!\,((\alpha - \beta)!\,\beta!)^{-1}$, and

$$\alpha \leq \beta \Longleftrightarrow \alpha_j \leq \beta_j\,, \ 1 \leq j \leq n\,.$$

If $b = (b_1, \dots, b_n) \in R^n$, then we define $b\alpha = b_1 \alpha_1 + \dots b_n \alpha_n$.

We also define $|\xi|^{|\xi|-1} = 1$ and $\xi! = 1$ for $\xi = 0$.

If $u(x)$ is a function analytic in all C^n then it is well known that to every $t > 0$ there exists a constant $C > 0$ such that

$$|D^\xi u(0)| \leq C t^{|\xi|} |\xi|^{|\xi|-1}, \text{ all } \xi. \tag{2.1}$$

Conversely, an analytic function defined in some neighbourhood of the origin, which satisfies the condition above can be extended to a function analytic in all C^n. Such functions will be called entire functions in the following.

The characterization of entire functions given above has been used in a somewhat generalized sense in [5], p. 146, [9], and [11]. It is, however, not suited for our present needs. An equivalent condition is the following. The real-valued function $p(t) > 0$, is defined for $t \geq 0$, and $p(t) \to +\infty$, when $t \longrightarrow +\infty$. The condition given before is then equivalent to the existence of a function p as above, and a constant $C > 0$ such that

$$(2.2) \qquad |D^{\xi} u(0)| \leq C [p(|\xi|)]^{-|\xi|} |\xi|^{|\xi|-1}, \text{ all } \xi.$$

We shall use the condition expressed in (2.2) later on.

Let u be an entire function and let β be a multi-index. We define

$$u = O(x^{\beta}) \Longleftrightarrow D_j^k u(x) = 0, \ x_j = 0, \ 0 \leq k < \beta_j, \ 1 \leq j \leq n.$$

3. Three lemmas.

LEMMA 1. *There exists a constant $c > 0$, independent of ξ such that*

$$(3.1) \qquad \sum_{0 \leq \nu \leq \xi} \binom{\xi}{\nu} |\xi|^{-|\xi|+1} |\xi - \nu|^{|\xi-\nu|-1} |\nu|^{|\nu|-1} \leq c, \text{ all } \xi.$$

For a proof of lemma 1 see the proof of lemma 1 in [8].

LEMMA 2. *The non-negative function $p(t)$ is defined and differentiable for $t > 0$. It satisfies the following four conditions,*

(3.2) $\quad 0 < p(t) \leq \log(t+4)$,

(3.3) $\quad 0 < p'(t) \leq (t+4)^{-1}$,

(3.4) $\quad p(t)$ *tends monotonically to* $+\infty$, *when* $t \longrightarrow +\infty$,

(3.5) $\quad p'(t)$ *tendes monotocally to zero, when* $t \longrightarrow +\infty$.

Let $C \geq \exp(2 + (p(p(0))^{-1})$. *The function* $g_n(t)$ *is defined by*

$$(3.6) \qquad g_n(t) = [p(p(n))]^n [p(p(n-t-1))]^{-(n-t-1)} [Cp(t)]^{-t} (n-t)^{-1}.$$

Then it follows that

$$(3.7) \qquad \lim_{n \to +\infty} \sup_{0 \leq t \leq n-1} g_n(t) = 0.$$

It also follows from (3.6), *and* (3.7) *that for a fixed* $\varepsilon > 0$ *there exists a* $C > 0$ *such that*

$$(3.8) \qquad C^{-1} g_n(t) < \varepsilon, \ 0 \leq t \leq n-1, \ n = 1, 2, 3, \ldots.$$

Proof of lemma 2. The function $f_n(t)$ is defined by

$$f_n(t) = \log g_n(t) = n \log p(p(n)) - (n-t-1) \log p(p(n-t-1)) -$$

$$- t \log C - t \log p(t) - \log(n-t).$$

The goal is to determine the maximum point of f_n, $0 \leq t \leq n-1$. We differentiate f_n and get

$$(3.9) \quad f_n'(t) = \log p(p(n-t-1)) +$$

$$+ (n-t-1) \cdot [p(p(n-t-1)]^{-1} p'(p(n-t-1)) \cdot p'(n-t-1) -$$

$$- \log C - \log p(t) - t \cdot (p(t))^{-1} p'(t) + (n-t)^{-1}.$$

All reasoning in the following is based on the assumption that n is sufficiently great. We note that for $t \geq 2^{-1} n$, using (3.3), (3.4) and (3.5),

$$f_n'(t) \leq \log p(p(n)) + [p(p(0))]^{-1} - \log C - \log p(2^{-1} n) + 1.$$

We choose C such that $\log C > 1 + [p(p(0))]^{-1}$ then it follows from (3.2) that

$$f_n'(t) < 0, \quad t \geq 2^{-1} n.$$

We also note that

$$f_n'(0) > \log p(p(n-1)) - \log C - (p(0))^{-1} - \log p(0).$$

It is obvious that, for a fixed C, $f_n'(0) > 0$ for n great. So the maximum point of f_n will be found in

$$(3.10) \qquad 0 < t < n/2.$$

From now on we denote the maximum point by t. We define

$$a_1 = (n-t-1)[p(p(n-t-1)]^{-1} \cdot p'(p(n-t-1)) \cdot p'(n-t-1),$$

$$a_2 = t(p(t))^{-1} p'(t),$$

$$a_3 = (n-t)^{-1}.$$

It follows from $f_n'(t) = 0$ and (3.9) that

$$(3.11) \qquad \log p(p(n-t-1)) = \log p(t) + \log C + a_2 - a_1 - a_3.$$

The mean value theorem says that for some $\theta, 0 < \theta < 1$,

$$\begin{aligned}\log p\,(p\,(n)) &= \log p\,(p\,(n-t-1)) + \\ &+ (t+1)\,[p\,(p\,(n-\theta\,(t+1)))]^{-1}\cdot p'\,(p\,(n-\theta\,(t+1)))\cdot \\ &\quad p'\,(n-\theta\,(t+1))) = \log p\,(p\,(n-t-1) + b.\end{aligned} \tag{3.12}$$

Here b is defined as

$$b = (t+1)\,[p\,(p\,(n-\theta\,(t+1)))]^{-1}\,p'\,(p\,(n-\theta\,(t+1))\,p'\,(n-\theta\,(t+1)).$$

A look at (3.3), (3.4), and (3.5) shows that

$$b \leq (t+1)\,[p\,(p\,(n-t-1))]^{-1}\,p'\,(p\,(n-t-1))\cdot p'\,(n-t-1). \tag{3.13}$$

From (3.9), (3.11) and (3.12) it follows that

$$\log\, p\,(p\,(n)) = \log p\,(t) + \log\, C + a_2 - a_3 - a_1 + b.$$

It follows from the properties of p and from $t < 2^{-1}\,n$ that

$$0 < a_1 \leq 1,\;\; 0 < a_3 \leq 1,\;\; a_2 > 0.$$

So we now have

$$\log\, p\,(t) \leq \log p\,(p\,(n)) + 1 + 1 - \log\, C \leq \log p\,(p\,(n)).$$

Since p is increasing and $\log\, x$ is increasing we see that

$$t \leq p\,(n).$$

It follows from (3.2) that

$$t \leq \log\,(n+4). \tag{3.14}$$

The maximum point is inserted in g_n. Since

$$p\,(p\,(n-t-1)) = Cp\,(t)\exp\,(a_2 - a_1 - a_3),$$

and since

$$p\,(p\,(n)) = Cp\,(t)\exp\,(a_2 - a_1 - a_3 + b),$$

we obtain from (3.10) that

$$\begin{aligned}g_n\,(t) &\leq 2n^{-1}\,(Cp\,(t)\exp\,(a_2 + b - a_1 - a_3))^{n-1}\,p\,(p\,(n))\cdot \\ &\cdot (Cp\,(t)\exp\,(a_2 - a_1 - a_3))^{-(n-t-1)}\,(Cp\,(t))^{-t} = \\ &= 2n^{-1}\exp\,(b\,(n-1) + t\,(a_2 - a_1 - a_3))\,p\,(p\,(n)).\end{aligned}$$

But from the definitions and from (3.2)-(3.4) and (3.13) it follows that

$$b(n-1) + t(a_2 - a_1 - a_3) \leq$$

$$\leq [(n-1)(t+1) - t(n-t-1)]\,[p(p(n-t-1))]^{-1} \cdot p'(p(n-t-1)) \cdot$$

$$\cdot\, p'(n-t-1) + t^2 \cdot (p(t))^{-1}\, p'(t) - t(n-t)^{-1} \leq$$

$$\leq (n + t^2 - 1)\,[p(p(n-t-1)]^{-1} \cdot p'(p(n-t-1)) \cdot p'(n-t-1) +$$

$$+ t \cdot [p(t)]^{-1}.$$

Since $t < \log(n+4)$, (3.11), it is now obvious that, for sufficiently great n, $g_n(t)$ can be estimated by

$$g_n(t) \leq 2n^{-1}\, p(p(n)) \exp(1 + [p(t)]^{-1} \log(n+4)) =$$

$$= 2en^{-1}\, p(p(n))\,(n+4)^{[p(t)]^{-1}}.$$

If we can prove that $p(t) \to +\infty$ when $n \to +\infty$, then the lemma is proved. We know from (3.11) and (3.12) that

$$\log p(t) = \log p(p(n)) - b - a_2 - \log C + a_1 + a_3 \geq$$

$$\geq \log p(p(n)) - 1 - (p(0))^{-1} - \log C.$$

So for a fixed C we have proved that $p(t) \to +\infty$, when $n \to +\infty$. By that we have proved that (3.7) is true. Then (3.8) is obvious. The proof of the lemma is finished.

LEMMA 3. *Let $(a_j)_{j=0}^{\infty}$ be a sequence of numbers with $a_j > 0$, $j = 0, 1, 2, \ldots$ such that*

$$a_j \to +\infty, \text{ when } j \to +\infty.$$

Then it follows that there exists a function p satisfying the hypothesis of lemma 2 such that

$$p(j) \leq a_j,\ j = 0, 1, 2, \ldots.$$

PROOF OF LEMMA 3. It is geometrically evident that we can construct a piece-wise linear strictly increasing continuous function $q(t)$ such that

$$0 < q(j) \leq a_j,\ j = 0, 1, 2, \ldots,\ q(t) \to +\infty,\ t \to +\infty,$$

with corners only in integer points. It is also obvious that we may choose q such that

$$q'(t_1) \geq q'(t_2), \; t_1 < t_2,$$

if both $q'(t_1)$ and $q'(t_2)$ are defined.

We choose

$$q(0) < \log 4.$$

Let

$$t_1 = \inf\{t \mid q'(t) \geq (t+4)^{-1}, \text{ all } t \text{ where } q(t) \text{ is defined}\}.$$

Perhaps the set is empty and t_1 does not exist. Then we choose $q_1 = q$, see below. If t_1 exists then we define

$$q_1(t) = q(t), \; t \leq t_1,$$

$$q_1(t) = q(t_1) + \log(t+4) - \log(t_1+4), \; t_1 \leq t \leq t_1'.$$

Here

$$t_1' = \sup\{s \mid q(t_1) + \log(t+4) - \log(t_1+4) \leq q(t), \; t_1 \leq t \leq s\}.$$

By this construction we surpass at least one corner to the right of t_1. We see that for $0 \leq t \leq t_1'$, q_1 is strictly increassing, differentiable except in a finite number of points

$$0 < q_1'(t) \leq (t+4)^{-1}, \; q_1'(t') \geq q_1'(t''), \; t' \leq t'',$$

where q' exists. The process is continued. Let

$$t_2 = \inf\{t \mid q'(t) \geq (t+4)^{-1}, \; t \geq t_1'\},$$

$$q_1(t) = q(t), \; t_1' \leq t \leq t_2,$$

$$q_1(t) = q(t_2) + \log(t+4) - \log(t_2+4), \; t_2 \leq t \leq t_2'.$$

Here

$$t_2' = \sup\{s \mid q(t_2) + \log(t+4) - \log(t_2+4) \leq q(t), \; t_2 \leq t \leq s\}.$$

It follows from above that q_1 can be defined by induction for all t so that

$$q_1(t) \leq q(t), \; t \geq 0,$$

and so that (3.2)-(3.5) are true. The only exception is in those points where q_1' is not defined. Except the integer points there can be at most two exceptional points between two integers, since $\log(t+4)$ is a concave function.

Since q_1 is concave a suitable regularization p_1 of q_1 exists, such that $p = 2^{-1} p_1$, and

$$p(t) \leq q_1(t), \quad t \geq 0.$$

The function p satisfies all conditions in lemma 1. The definition $p = 2^{-1} p_1$, is made so that p does not violate (3.3) since $(t + 4)^{-1}$ is a convex function. A regularization might make $p_1' > (t + 4)^{-1}$ in certain points. The proof of lemma 3 is finished.

4. A Goursat problem for entire functions.

THEOREM. *Let $\beta, \alpha^k, 1 \leq k \leq N$, be multi-indices. They are restricted by*

$$|\alpha^k| \leq |\beta|, 1 \leq k \leq N. \tag{4.1}$$

There exists a vector $b = (b_1, \dots, b_n)$,

$$b_j > 0, \quad 1 \leq j \leq n,$$

such that

$$b\beta > b\alpha^k, \quad 1 \leq k \leq N. \tag{4.2}$$

The functions f and a_k, $1 \leq k \leq N$, are entire functions. The functions a_k, $1 \leq k \leq N$, are restricted by

$$|\alpha^k| = |\beta| \Longrightarrow a_k \text{ is a constant.} \tag{4.3}$$

It follows that there exists a unique entire function u such that

$$D^\beta u = \sum_{1 \leq k \leq N} a_k D^{\alpha^k} u + f, \quad u = O(x^\beta). \tag{4.4}$$

PROOF OF THE THEOREM. It follows from (2.2) and lemma 3 that there exists a function p that satisfies the hypothesis of lemma 2 with the additional property that for some $C' > 0$,

$$|D^\xi f(0)| \leq C' (p(|\xi|))^{-|\xi|} |\xi|^{|\xi|-1}, \tag{4.5}$$

and

$$|D^\xi a_k(0)| \leq C' (p(|\xi|))^{-|\xi|} |\xi|^{|\xi|-1}, \quad 1 \leq k \leq N, \text{ all } \xi. \tag{4.6}$$

It is known from [6] or theorem 1 in [8], that the analytic solution u and (4.4) exist and is unique in some neighbourhood of the origin. This also follws from the proof below.

It is easily seen that in the new coordinate system

$$x'_j = e^{\tau b j} x_j\,,\ 1 \leq j \leq n,$$

the new coefficients are of the form

$$a'_k(x') = e^{\tau b(\alpha^k - \beta)}\, a_k(x).$$

It follows from (4.2) that we may choose τ so great that

(4.7) $$\sum_{|\alpha^k| = |\beta|} |a'_k| \leq 2^{-1}$$

We may also choose τ so great that $C' = 1$ can be used in (4.5) and (4.6), since

$$f'(x') = e^{-\tau b \beta} f(x),$$

See also [6]. With $C' = 1$ and deleting the primes from the variables we now assert that

(4.8) $$|D^{\xi+\beta} u(0)| \leq [C/p(p(|\xi|))]^{|\xi|}\, |\xi|^{|\xi|-1}$$

is true for all ξ. The constant C will be defined later. Of course $|0|^{-1} = 1$ here.

The number $D^{\eta} u(0)$ is zero if some $\eta_j < \beta_j$, since $u = O(x^{\beta})$. If $\eta \geq \beta$, then $\eta = \xi + \beta$, where ξ is a multi-index with non-negativ components. The vector b has positive components. For a given n there is only a finite number of multi-indices ξ such that $b\,\xi \leq n$. The possible values of $b\,\xi$ can be arranged in a strictly increasing sequence that tends to infinity. We form the set

(4.9) $$\{C_k\,;\ k = 0, 1, 2, \ldots, C_k < C_{k+1}\} = \{b\xi;\ \xi \text{ multi-index}\}.$$

It follows from (4.2) that for an arbitrary $k, \alpha^k_j < \beta_j$ for some j. Thus

$$D^{\alpha^k} u(0), = 0,\ 1 \leq k \leq N,$$

since $u = O(x^{\beta})$.

Therefore (4.4) and (4.5) give

$$|D^{\beta} u(0)| \leq 1.$$

So (4.8) is true for $|\xi| = 0 = C_0$. Assume that it is true for all ξ, $b\xi < C_k$. We shall prove that it is true for ξ, $b\xi = C_k$, too. Take such a ξ.

$$D^{\xi+\beta} u = \Sigma D^{\xi} (a_k D^{\alpha^k} u) + D^{\xi} f = \tag{4.10}$$

$$= \sum_{|\alpha^k| = |\beta|} a_k D^{\xi+\alpha^k} u + \sum_{|\alpha^k| < |\beta|} \sum_{0 \leq \nu \leq \xi} \binom{\xi}{\nu} (D^{\nu} a_k) D^{\xi-\nu+\alpha^k} u + D^{\xi} f.$$

If $(\xi + \alpha^k - \nu)_j < \beta_j$ for some j then

$$D^{\xi+\alpha^k-\nu} u(0) = 0.$$

If $\xi + \alpha^k - \nu \geq \beta$, then, because of (4.2), we obtain

$$\xi + \alpha^k - \nu = \xi + \alpha^k - \nu - \beta + \beta, \text{ and } b(\xi + \alpha^k - \nu - \beta) \leq$$
$$\leq b\xi + b(\alpha^k - \beta) < b\xi = C_k.$$

So all derivatives in the right member of (4.10) satisfies (4.8) in the origin, or is zero there. Therefore we can estimate $D^{\xi-\nu+\alpha^k} u(0)$, $\nu \neq \xi$ by

$$|D^{\xi-\nu+\alpha^k} u(0)| \leq [C/p(p(t))]^t t^{t-1},$$

where $t = |\xi|$, when $\nu = 0$ and $|\alpha^k| = |\beta|$. When $|\alpha^k| < |\beta|$, then $t = |\xi - \nu| - 1$. This we can do since we have now chosen C so great that $[C/p(p(t)]^t t^{t-1}$ is increasing when t increases.

It follows from (4.10) and (4.7) letting $|\xi| = j$, and $|\nu| = k$, that

$$|D^{\xi+\beta} u(0)| \leq 2^{-1} [C/p(p(j))]^j j^{j-1} +$$
$$+ N \sum_{\substack{0 \leq \nu \leq \xi \\ \nu \neq \xi}} \binom{\xi}{\nu} [C/p(p(j-k-1))]^{j-k-1} (j-k-1)^{j-k-2} \cdot k^{k-1} (p(k))^{-k} +$$
$$+ (p(j))^{-j} j^{j-1}.$$

Rewriting this inequality we get

$$|D^{\xi+\beta} u(0)| \leq [C/p(p(j))]^j j^{j-1} \cdot$$
$$\cdot (2^{-1} + N \sum_{\substack{0 \leq \nu \leq \xi \\ \nu \neq \xi}} \binom{\xi}{\nu} [C/p(p(j))]^{-j} [C/p(p(j-k-1))]^{j-k-1} p(k)^{-k} \cdot$$
$$\cdot j^{-j+1} k^{k-1} (j-k)^{j-k-2} + C^{-j} [p(p(j))/p(j)]^j).$$

It now follows from lemma 2 and lemma 1 that for a sufficiently great C, independent of ξ, the number inside the parantheses is less than 1. Thus (4.8) is true for every ξ. Since

$$p(p(t))/C \to +\infty, \text{ when } t \to +\infty,$$

we have now proved that $D^{\beta} u$ is an entire function. Since $u = O(x^{\beta})$, u itself is also an entire function. The proof is finished.

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