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# WEAK CONVERGENCE TO THE LAW OF THE BROWNIAN SHEET

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Abstract. We obtain the Brownian sheet as a weak limit of processes constructed from one-parameter Poisson processes. Previously we prove several criteria of tightness for some families of probability measures in $C([0,1]^2)$.

## 1. INTRODUCTION.

The aim of this paper is to give an approximation of the Brownian sheet based on those obtained by Stroock (see Stroock, 1982, Chap. II, Sec. 2). In this book Stroock approximates the one parameter Brownian motion by processes obtained from a standard Poisson process. A similar construction with the two parameter Poisson process does not seem adequate for the approximation of the Brownian sheet. We take two independent sequences of independent copies of the processes given by Stroock and prove in Proposition 1 the desired convergence for a sum of products of these processes. We need before some results on weak compactness of some classes of processes in $C([0,1]^m)$.

Here we apply general criteria of weak compactness of measures on $C([0,1]^m)$, space of all real valued continuous functions on $[0,1]^m$ with the topology of uniform convergence, in order to obtain a criterion of weak compactness for the family of processes $\{Z_{n,\lambda}\}_{n\in\mathbb{N},\lambda\in\Lambda}$ where

$$Z_{n,\lambda}(t_1,\ldots,t_m) = n^{-1/2}\sum_{i=1}^{n} X_\lambda^i(t_1,\ldots,t_m) \tag{1}$$

with $\Lambda$ a set of parameters, $\{X_\lambda^i\}_{i\in\mathbb{N}}$ independent copies of $X_\lambda$, and $\{X_\lambda\}_{\lambda\in\Lambda}$ a family of zero mean $C([0,1]^m)$-valued random variables in some probability space $(\Omega,\mathcal{F},P)$, this criterion is given in Theorem 3.

There is a similar idea in a work of M. Yor (see Yor, 1983) where he gives an alternative proof for a result of Nualart (see Nualart,1981) that we obtain also in Corollary 2.

The applications of Theorem 3 include sufficient conditions ensuring the validity of the central limit theorem for a continuous process (see Theorem 4) and an approximation for the two–parameter Wiener process (Proposition 1).

The structure of the paper is as follows. In section two we state the preliminary results and prove the criterion of tightness for families of processes defined in (1). In section three we give some applications of Theorem 3. Finally, in section four the approximation of the Brownian sheet is derived.

## 2. THE RESULTS.

We will use the following generalization of Theorem 12.3 of Billingsley (see Billingsley, 1968) for multiparameter continuous processes, with conditions on the moments. Since the notation for the general multiparameter case is complicated, we only state the two-parameter version. See Bickel and Wichura, 1971 and Centsov, 1969 for similar results.

THEOREM 1. *A family $(Q_\lambda)_\lambda$ of probability measures on $C([0,1]^2)$ is tight if there exists $p_i > 0, i = 1,2,3,4$, $\alpha,\beta,\gamma > 1$, $F, G$ continuous increasing functions and $\mu$ a finite nonnegative measure on $[0,1]^2$ with continuous marginals such that :*

a) $\sup\limits_\lambda E_{Q_\lambda}(|x(0,0)|^{p_1}) < \infty$

b) $\sup\limits_\lambda E_{Q_\lambda}(|x(0,t_2) - x(0,t_1)|^{p_2}) \leq (F(t_2) - F(t_1))^\alpha$ *for all* $t_1 \leq t_2$

c) $\sup\limits_\lambda E_{Q_\lambda}(|x(s_2,0) - x(s_1,0)|^{p_3}) \leq (G(s_2) - G(s_1))^\beta$ *for all* $s_1 \leq s_2$

d) $\sup\limits_\lambda E_{Q_\lambda}(|x([s_1,s_2] \times [t_1,t_2])|^{p_4}) \leq \mu([s_1,s_2] \times [t_1,t_2])^\gamma$ *for all* $(s_1,t_1) \leq (s_2,t_2)$,*where* $x([s_1,s_2] \times [t_1,t_2]) = x(s_1,t_1) + x(s_2,t_2) - x(s_1,t_2) - x(s_2,t_1)$.

The criterion of weak compactness that we obtain in Theorem 3 for the families of processes given in (1) is a consequence of the following inequality. This inequality was obtained by Kai Lai Chung as an immediate corollary of Theorem 13 of the classical paper " Sur les fonctions independantes " by Marcinkiewicz and Zygmund (see Marcinkiewicz and Zygmund, 1937 and Kai Lai Chung, 1951), and can be also obtained from Burkholder's inequality.

THEOREM 2. *Let $\xi_1, \xi_2, \ldots, \xi_n, \ldots$ be zero mean, independent, real valued random variables. Then, for all $r \geq 1$ there exists a constant $C$, which depends only on $r$ such that*

$$E(|\xi_1 + \cdots + \xi_n|^{2r}) \leq Cn^{r-1} \sum_{i=1}^{n} E(|\xi_i|^{2r})$$

THEOREM 3. *Let $(X_\lambda)_\lambda$ be a family of zero mean $C([0,1]^2)$-valued random variables. Let $(Q_{n,\lambda})$ be the laws of $(Z_{n,\lambda})$ defined in (1) and let $(Q_\lambda)$ be the laws of $(X_\lambda)$. Then, in order to ensure that $(Q_{n,\lambda})$ is a tight set of measures on $C([0,1]^2)$ it is sufficient that the family $(Q_\lambda)$ satisfies the conditions of Theorem 1, with the additional assumption that $p_i \geq 2, i = 1, \ldots, 4$.*

PROOF: We will show that $(Q_{n,\lambda})$ verifies conditions (a) to (d) with the same $p_i, i = 1, \ldots, 4$, $\alpha, \beta$ and $\gamma$ and modifying $F, G$ and $\mu$ by a constant.

a) $\sup_{n,\lambda} E(|Z_{n,\lambda}(0)|^{p_1}) = \sup_{n,\lambda} E(n^{-1/2}|\sum_{i=1}^{n} X_\lambda^i(0)|)^{p_1} \leq$

$\sup_{n,\lambda}\{n^{-p_1/2} C_{p_1} n^{p_1/2-1} \sum_{i=1}^{n} E(|X_\lambda^i(0)|^{p_1})\} = C_{p_1} \sup_{\lambda} E(|X_\lambda(0)|^{p_1})$

b) $\sup_{n,\lambda} E(|Z_{n,\lambda}(0,t_2) - Z_{n,\lambda}(0,t_1)|^{p_2}) =$

$\sup_{n,\lambda} E(n^{-1/2}|\sum_{i=1}^{n}[X_\lambda^i(0,t_2) - X_\lambda^i(0,t_1)]|)^{p_2} \leq$

$\sup_{n,\lambda}\{n^{-p_2/2} C_{p_2} n^{p_2/2-1} \sum_{i=1}^{n} E(|X_\lambda^i(0,t_2) - X_\lambda^i(0,t_1)|^{p_2})\} =$

$C_{p_2} \sup_{\lambda} E(|X_\lambda(0,t_2) - X_\lambda(0,t_1)|^{p_2}) \leq C_{p_2}(F(t_2) - F(t_1))^\alpha$

d) $\sup_{n,\lambda} E(|Z_{n,\lambda}([s_1,s_2] \times [t_1,t_2])|^{p_4}) =$

$\sup_{n,\lambda} E(n^{-1/2}|\sum_{i=1}^{n} X_\lambda^i([s_1,s_2] \times [t_1,t_2])|)^{p_4}$

and from this expression we follow as in condition (b). ∎

## 3. APPLICATIONS.

Observe that in the case in which the family $(X_\lambda)$ is reduced to a single $X$ these criteria provide sufficient conditions for $X$ to verify the central limit theorem in $C([0,1]^2)$ ( or in $C([0,1]^m)$ ) because, in particular, conditions (a) to (d) imply that $X(s,t)$ has second order moment in all $(s,t)$, and by the ordinary central limit theorem we have that the finite dimensional distributions of $Z_n$ converge to a Gaussian distribution when $n \to \infty$. So, we can state the following theorem.

THEOREM 4. *Let $X$ be a zero mean $C([0,1]^2)$-valued random variable, suppose there are $p_i \geq 2, i = 1,...,4$, $\alpha, \beta, \gamma > 1$, $F$ and $G$ increasing continuous functions, and $\mu$ finite nonnegative measure on $[0,1]^2$ with continuous marginals, such that :*

a) $E(|X(0,0)|^{p_1}) < \infty$

b) $E(|X(0,t_2) - X(0,t_1)|^{p_2}) \leq (F(t_2) - F(t_1))^\alpha, \quad t_1 \leq t_2$

c) $E(|X(s_2,0) - X(s_1,0)|^{p_3}) \leq (G(s_2) - G(s_1))^\beta, \quad s_1 \leq s_2$

d) $E(|X([s_1,s_2]\times[t_1,t_2])|^{p_4}) \leq (\mu([s_1,s_2]\times[t_1,t_2]))^\gamma, (s_1,t_1) \leq (s_2,t_2).$

*Then $X$ satisfies the central limit theorem, i.e. if $X_1, \ldots, X_n \ldots$ are independent copies of $X$, the probability measures induced on $C([0,1]^2)$ by $n^{-1/2}(X_1 + \cdots + X_n)$ converge weakly to the centered Gaussian measure on $C([0,1]^2)$ whose covariance is that of $X$.*

The $m$-parameter version of this result includes another one of Araujo ( see Corallary of Section 2 in Araujo, 1978). In that result the hypotheses are : $EX(t) = 0$, $EX^2(0) < \infty$, and $E|X(s) - X(t)|^r \leq K|s-t|^{m+\alpha}$ for all $s$ and $t$ in $[0,1]^m$, some $\alpha > 0$, $K > 0$ and $r > 2$. Is not difficult to show that these conditions imply those of Theorem 4.

On the other hand Jain and Marcus gave, in a well-known result (see Theorem 1 of Jain and Marcus, 1975), sufficient conditions for $X$ to verify the central limit theorem in $C(S)$, where $(S,d)$ is a compact metric space. These conditions are : there exist a nonnegative random variable $M$ , $E(M^2) = 1$, and a metric $\rho$ on $S$, which is continuous with respect to $d$ such that given $s,t \in S$, $\omega \in \Omega$

$$|X(s,\omega) - X(t,\omega)| \leq M(\omega)\rho(s,t) \tag{2}$$

and $\rho$ also satisfies a condition relative to its entropy. When $S = [0,1]^m$, this criterion does not include Theorem 4, since if $X$ is a continuous process satisfying the hypotheses of Theorem 4 is not always possible to find a metric $\rho$ and a square integrable random variable $M$ satisfying inequality (2).

We can also state the following result of approximation for the Brownian sheet.

THEOREM 5. *Let $(X_k)_{k\in\mathbb{N}}$ be a sequence of zero mean $C([0,1]^2)$-valued random variables that verifies the conditions of Theorem 3 and such that its finite dimensional distributions converge weakly to those of a zero mean process $Y$ with covariance function $E[Y(s,t)Y(s',t')] = (s\wedge s')(t\wedge t')$. Then, the laws of the processes*

$$Z_{n,k}(s,t) = n^{-1/2}\sum_{i=1}^{n} X_k^i(s,t),$$

*where $(X_k^i)_{i\in\mathbb{N}}$ are independent copies of the process $X_k$, converge weakly, when $n$ and $k$ tend to $\infty$, to the Wiener measure in $C([0,1]^2)$.*

PROOF: By Theorem 3 the laws of processes $Z_{n,k}$ form a family of tight measures in $C([0,1]^2)$. In order to see that they converge weakly to the Wiener measure we will prove that any convergent subsequence converges to this measure. We will show that the finite dimensional distributions of this subsequence converge weakly to those of the Brownian sheet.

Let $\{(n',k')\}$ be the sequence of indices such that the probability measures associated with $Z_{n',k'}$ converge weakly.

Let $\varphi_{(Z_{n',k'}(s_1,t_1),\dots,Z_{n',k'}(s_d,t_d))}(u_1,\dots,u_d)$ be the characteristic function of the random vector $\overrightarrow{Z} = (Z_{n',k'}(s_1,t_1),\dots,Z_{n',k'}(s_d,t_d))$, $(u_1,\dots,u_d) \in \mathbb{R}$. Put $z_i = (s_i,t_i)$, $i = 1,\dots,d$. Since

$$\lim_{n',k' \to \infty} \varphi_{(Z_{n'k'}(z_1),\dots,Z_{n'k'}(z_d))}(u_1,\dots,u_d)$$

exists we can compute this limit as $\lim_{n'}[\lim_{k'} \varphi_{\overrightarrow{Z}}(\overrightarrow{u})]$. But

$$\lim_{k'} \varphi_{(Z_{n'k'}(z_1),\dots,Z_{n'k'}(z_d))}(u_1,\dots,u_d) =$$
$$\lim_{k'}[\varphi_{(X_{k'}(z_1),\dots,X_{k'}(z_d))}(\frac{u_1}{\sqrt{n'}},\dots,\frac{u_d}{\sqrt{n'}})]^{n'} =$$
$$[\varphi_{(Y(z_1),\dots,Y(z_d))}(\frac{u_1}{\sqrt{n'}},\dots,\frac{u_d}{\sqrt{n'}})]^{n'},$$

where $Y$ is a zero mean process with the same covariance function as the Wiener process. And from this expression the Theorem follows easily. ∎

COROLLARY 1. *Let $(X_k)_{k\in\mathbb{N}}$, $(Y_k)_{k\in\mathbb{N}}$ be independent sequences of zero mean $C([0,1])$-valued random variables. Suppose that there exist $p \geq 2$ and $\delta > 1$ and $F_1,F_2$ increasing functions such that*

(i) $\sup_k E[|X_k(0)|^p] + \sup_k E[|Y_k(0)|^p] < \infty$

(ii) $\sup_k E[|X_k(s_2) - X_k(s_1)|^p] \leq (F_1(s_2) - F_1(s_1))^\delta$ *for all* $s_1 \leq s_2$

(iii) $\sup_k E[|Y_k(t_2) - Y_k(t_1)|^p] \leq (F_2(t_2) - F_2(t_1))^\delta$ *for all* $t_1 \leq t_2$.

*Suppose also that the finite dimensional distributions of $(X_k)$, $(Y_k)$ converge weakly to those of $X, Y$, processes with the same covariance function as the Wiener process in $[0,1]$. Then the laws of*

$$Z_{n,k}(s,t) = n^{-1/2} \sum_{i=1}^{n} X_k^i(s) Y_k^i(t)$$

*converge weakly to the Wiener measure in $C([0,1]^2)$, as $n \to \infty$ and $k \to \infty$.*

PROOF: (i), (ii), and (iii) imply (a), (b), (c) and (d) of Theorem 3 for $(X_k(s)Y_k(t))$, then the laws of $Z_{n,k}$ form a tight set of measures. On the other hand we have that the finite dimensional distributions of $X_k(s)Y_k(t)$ converge weakly to those of $X'(s)Y'(t)$, where $X'$ and $Y'$ are independent with the same law as $X$ and $Y$. Since $X'(s)Y'(t)$ has the same covariance function as a two-parameter Wiener process, applying Theorem 5 the laws of $Z_{n,k}$ converge weakly, when $k \to \infty$, $n \to \infty$, to the Wiener measure in $C([0,1]^2)$. ∎

COROLLARY 2. (Nualart) *Let $\{X^i(t); t \in [0,1], i \in \mathbb{N}\}$ and $\{Y^i(t); t \in [0,1], i \in \mathbb{N}\}$ be independent sequences of independent copies of two* *independent Brownian motions $X$ and $Y$. Then, the sequence of two-parameter continuous processes*

$$Z_n(s,t) = n^{-1/2} \sum_{i=1}^{n} X^i(s) Y^i(t)$$

*converges weakly to a two-parameter Wiener proces.*

PROOF: It is an immediate consequence of Corollary 1. Since $X$, $Y$ vanish on the axes it is enough to see (ii) and (iii), and these are a consequence of the fact that for $X$ Brownian motion $E(X(t_2)-X(t_1))^4 = 3(t_2 - t_1)^2$. This proves tightness. ∎

## 4. APPROXIMATION OF THE BROWNIAN SHEET FROM ONE-PARAMETER POISSON PROCESSES.

Let $\{N(t), t \geq 0\}$ be a standard Poisson prosess. Define $\theta(t) = (-1)^{N(t)}$ and $x(t) = \int_0^t \theta(u)\,du$. For every $\varepsilon > 0$ set $x_\varepsilon(t) = \varepsilon x(t/\varepsilon^2)$, $t \in [0,1]$. Stroock proved (see Stroock, 1982, Chap. II, Sec. 2) that the laws of the processes $x_\varepsilon$ converge weakly to the Wienner measure in $C([0,1])$ when $\varepsilon \to 0$. On the other hand we have the result of Nualart given in Corollary 2.

By using Corollary 1 we can prove the following proposition.

PROPOSITION 1. *Let $(x_\varepsilon^i(t))_{i\in\mathbb{N}}$, $(y_\varepsilon^i(t))_{i\in\mathbb{N}}$ be independent sequences of independent copies of the process $x_\varepsilon(t)$. For $(s,t) \in [0,1]^2$ define*

$$Z_{n,\varepsilon}(s,t) = n^{-1/2} \sum_{i=1}^{n} (x_\varepsilon^i(s) - Ex_\varepsilon^i(s))(y_\varepsilon^i(t) - Ey_\varepsilon^i(t))$$

*and let $P_{n,\varepsilon}$ be the law of $Z_{n,\varepsilon}(s,t)$ in $C([0,1]^2)$.*

*Then $P_{n,\varepsilon} \xrightarrow{w} W$ when $n \to \infty$, $\varepsilon \to 0$, where $W$ is the Wiener measure in $C([0,1]^2)$.*

LEMMA 1. *For all $0 \leq t \leq t' \leq 1$ we have that:*

(a) $Ex_\varepsilon(t) = \varepsilon \dfrac{1 - exp(-2t/\varepsilon^2)}{2}$
(b) *There exists $C > 0$ such that $E(X_\varepsilon(t') - X_\varepsilon(t))^4 \leq C(t'-t)^2$, where $X_\varepsilon(t) = x_\varepsilon(t) - Ex_\varepsilon(t)$.*

PROOF OF LEMMA: (a) $Ex(t) = E(\int_0^t \theta(u)\,du) = \int_0^t E(\theta(u))\,du = \int_0^t [P\{N(u) = \dot{2}\} - P\{N(u) = \dot{2}+1\}]\,du = \int_0^t e^{-2u}\,du = (1-e^{-2t})/2$, so $Ex_\varepsilon(t) = \varepsilon(1 - e^{-2t/\varepsilon^2})/2$.

(b)$E[X_\epsilon(t') - X_\epsilon(t)]^4 = \varepsilon^4 E\{x(t'/\varepsilon^2) - x(t/\varepsilon^2) - E[x(t'/\varepsilon^2) - x(t/\varepsilon^2)]\}^4 \leq M\varepsilon^4 E[x(t'/\varepsilon^2) - x(t/\varepsilon^2)]^4$.

We find a bound for $E[x(t') - x(t)]^4$ for all $0 \leq t \leq t' < \infty$.
$E[x(t') - x(t)]^4 = E[\int_t^{t'} \theta(u)\,du]^4 = E[\prod_{i=1}^4 \int_t^{t'} \theta(t_i)\,dt_i] =$
$E\int_t^{t'}\int_t^{t'}\int_t^{t'}\int_t^{t'} \theta(t_1)\theta(t_2)\theta(t_3)\theta(t_4)\,dt_1\,dt_2\,dt_3\,dt_4$ and by applying Fubini's Theorem we can introduce the mathematical expectation into the integral sign.

Now we compute $E(\theta_{t_1}\theta_{t_2}\theta_{t_3}\theta_{t_4})$. Let $t'_1, t'_2, t'_3$ and $t'_4$ be such that $t'_1 \leq t'_2 \leq t'_3 \leq t'_4$ and $\{t'_1, t'_2, t'_3, t'_4\} = \{t_1, t_2, t_3, t_4\}$.Then
$E(\theta_{t_1}\theta_{t_2}\theta_{t_3}\theta_{t_4}) = E[(-1)^{\sum_{i=1}^4 N(t_i)}] = E[(-1)^{\sum_{i=1}^4 N(t'_i)}] =$
$E[(-1)^{N(t'_2)-N(t'_1)+N(t'_4)-N(t'_3)}]$. Since $N(t'_i) - N(t'_{i-1})$ are independent random variables with Poisson distribution, $N(t'_2) - N(t'_1) + N(t'_4) - N(t'_3)$ will be a random variable with Poisson distribution with parameter $t'_2 - t'_1 + t'_4 - t'_3$ and therefore $E(\theta_{t_1}\theta_{t_2}\theta_{t_3}\theta_{t_4}) = e^{-2(t'_2 - t'_1 + t'_4 - t'_3)}$.

We can obtain now $\int_t^{t'}\int_t^{t'}\int_t^{t'}\int_t^{t'} E(\prod_{i=1}^4 \theta_{t_i})\,dt_1\,dt_2\,dt_3\,dt_4$. The 4-dimensional cube $[t,t']^4$ can be subdivided in 24 regions of the form $A_\sigma = \{(t_1,t_2,t_3,t_4) : t_{\sigma(1)} \leq t_{\sigma(2)} \leq t_{\sigma(3)} \leq t_{\sigma(4)}\}$ with $\sigma$ permutation of $\{1,2,3,4\}$. These regions verify that their intersections have Lebesgue measure zero in $\mathbb{R}^4$, and by the above arguments we can write $E(\prod_{i=1}^4 \theta_{t_i}) = \sum_\sigma 1_{A_\sigma} e^{-2(t_{\sigma(2)} - t_{\sigma(1)} + t_{\sigma(4)} - t_{\sigma(3)})}$ except for $(t_1,t_2,t_3,t_4)$ belonging to a set of null Lebesgue measure. Therefore
$\iiiint_{[t,t']^4} E(\prod_{i=1}^4 \theta_{t_i})\,dt_1\,dt_2\,dt_3\,dt_4 =$
$\sum_\sigma \iiiint_{A_\sigma} e^{-2(t_{\sigma(2)} - t_{\sigma(1)} + t_{\sigma(4)} - t_{\sigma(3)})} dt_1\,dt_2\,dt_3\,dt_4$. By the change of variables theorem it is immediate to show that the 24 integrals have the same value, and then we have to compute

$$24\int_t^{t'}\int_t^{u}\int_t^{z}\int_t^{y} e^{-2(y-x+u-z)}\,dx\,dy\,dz\,du =$$
$$3(t'-t)^2 - 6(t'-t) - 3(t'-t)e^{-2(t'-t)} + \frac{9}{2} - \frac{9}{2}e^{-2(t'-t)}.$$

In order to see that this later expression is bounded by $K(t'-t)^2$, consider the function $f(x) = 3x^2 - 6x - 3xe^{-2x} + \frac{9}{2} - \frac{9}{2}e^{-2x}$. This function has second derivative bounded in $\mathbb{R}_+$ and verifies that $f(0) = f'(0) = 0$, so for all $x \in \mathbb{R}_+$, $f(x) \leq Kx^2$. This gives us the desired bound. ∎

PROOF OF PROPOSITION 1: We will prove that the hypotheses of Corollary 1 are verified. Since $x_\epsilon(t) - Ex_\epsilon(t)$, $y_\epsilon(t) - Ey_\epsilon(t)$ vanish at the origin it is enough to verify (ii) and (iii). These conditions are satisfied by Lemma 1, with $p = 4$, $\delta = 2 > 1$. This gives tightness.

Finally, we will show that the finite dimensional distributions of $X_\epsilon$ and $Y_\epsilon$ converge weakly to those of the Wiener process in $[0,1]$. This is an immediate consequence of the result of Strook and Theorem 4.1 of Billingsley's book (see Billingsley, 1968) by using part (a) of Lemma 1. ∎

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