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Erratum Invariant measures for Markov processes arising from iterated function systems with place-dependent probabilities

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Invariant measures for Markov processes arising from iterated function systems with place-dependent probabilities

by

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The proof given for Lemma 2.5, pg. 374, while correct for certain spaces, *e.g.* $\mathscr{R}^1$, is incorrect in general, as it assumes special properties of the modulus of continuity. A correct proof is obtained by replacing from the beginning of the proof through lines 13, page 375, by the following:

Proof. – Note that each φ_i is non-decreasing, and $\varphi_i(t) \leqq 1$ for all t, since $|p_i(x) - p_i(y)| \leqq 1$ for all x, y. Let

$$\varphi_0(t) = \begin{cases} t, & 0 \leqq t \leqq 1 \\ 1, & t > 1. \end{cases}$$

Let $\varphi = \varphi_0 \vee \varphi_1 \vee \ldots \vee \varphi_N$, where $t \vee u$ denotes $\max\{t, u\}$. It is clear that φ also satisfies Dini's condition.

Sublemma. – Let $\varphi : [0, 1] \to [0, \infty)$ be non-decreasing, with

$$\int_0^1 \frac{\varphi(t)}{t} dt < \infty.$$

Then there exists $\psi : [0, 1] \to [0, \infty)$ such that $\psi(t) \geqq \varphi(t)$ for all t, $\frac{\psi(t)}{t}$ is non-increasing, and

$$\int_0^1 \frac{\psi\ t)}{t} dt < \infty.$$

Proof. – W.L.O.G. assume $\varphi(^+) = \varphi(t)$, $\forall t$. Let $f(t) = \varphi(t)/t$. We shall use the "rising sun" lemma of F. Riesz (Boas, page 134): Let E be the "shadow region" for the sun rising in the direction of the positive x-axis; that is, $E = \{t \in (0, 1) : \exists x > t \text{ with } f(x) > f(t)\}$. Then E is an open set and if (a, b) is any one of the open intervals comprising E, $f(x) \leq f(b)$ for $x \in (a, b)$, and $f(a) = f(b)$ since f is right-continuous and has only upward jumps.

Let C be countable collection of non-overlapping open intervals such that $E=\bigcup C$. Define

$$g(x)=\begin{cases} f(x), & x\notin E \\ f(b), & x\in(a, b)\in C. \end{cases}$$

Thus $g(x)$ is non-increasing and $g\geqq f$. Now if $(a, b)\in C$,

$$\int_a^b [g(x)-f(x)]\,dx=\int_1^b [f(b)-f(x)]\,dx$$
$$=\int_a^b \left[\frac{\varphi(b)}{b}-\frac{\varphi(x)}{x}\right]dx\leqq\int_a^b \left[\frac{\varphi(b)}{b}-\frac{\varphi(a)}{b}\right]dx$$

since φ is non-decreasing. Thus

$$\int_a^b [g(x)-f(x)]\,dx\leqq[\varphi(b)-\varphi(a)]\frac{b-a}{b}\leqq\varphi(b)-\varphi(a),$$

so

$$\int_0^1 [g(x)-f(x)]\,dx=\int_E [g(x)-f(x)]\,dx$$
$$=\sum_{(a,\, b)\in C}\int_a^b [g(x)-f(x)]\,dx\leqq\sum_{(a,\, b)\in C}\varphi(b)-\varphi(a)\leqq\varphi\, 1)$$

since φ is non-decreasing, so $\int_0^1 g(t)\,dt<\infty$. Finally, let

$$\psi(t)=tg(t)\geqq tf(t)=\varphi(t), \qquad \text{and} \qquad \frac{\psi(t)}{t}=g(t)$$

is non-increasing. □

So by the sublemma, increasing φ if necessary, we may assume in what follows that $\varphi(t)/t$ is non-increasing and φ is Dini.

Let $f\in C_c(X)$, $\|f\|\leqq 1$, and also assume $f\in \mathrm{Lip}_1$, so

$$|f(x)-f(y)|\leqq C\,d(x, y), \qquad \forall x, y\in X.$$

We may take $C\geqq 2$.

Without loss of generality, we may assume $q\leqq 1$ in the hypothesis of the lemma.

Define

$$\beta^*(t)=\frac{N\vee C}{1-r^q}\int_0^{tr^{-q}}\frac{\varphi(u)}{u}\,du.$$

This is finite since φ is Dini. Then $\beta^*(0)=0$, and β^* is continuous and strictly increasing. Also, β^* is a *concave* function, since $\varphi(t)/t$ is non-increasing.

We thank Roger Nussbaum for pointing out that our earlier proof was incorrect.

REFERENCES

R. P. Boas, *A Primer of Real Functions*, Wiley, 1970.